UWB Signal Bandwidth Expansion and Synthesis Using Prolate and Wavelet Functions

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Abstract

This paper discusses how to generate UWB signals and extend their bandwidths using Prolate Spheroidal and wavelet functions. It also compares the achievable bandwidths. Bandwidth synthesis and expansion in the frequency domain is proposed based on convolution, superposition, modulation and multiplication. It is shown that convolution and addition of signal spectra can be used to expand UWB bandwidth with some functions by orders of more than 4.5 times. Daubechies wavelets are used in this paper.

Introduction

The push towards wireless broadband networks is very strong and irresistible. UWB provides a means for achieving that. UWB communications is legislated in the US for imaging systems for ground penetrating and vehicular radar, wall imaging, through-wall imaging, medical and surveillance systems. UWB techniques are well known in radar technology [1]. The distinctive advantages are high resolution and higher range measurements. The range resolution Δr is inversely proportional to the bandwidth of the short pulses used and directly proportional to the speed of light in the medium. A signal at 4 GHz can resolve a 7.5 cm range. Other advantages of WWB are given in [1], [4] and [5]. UWB communication manifests immunity to multipath cancellation because they are time-orthogonal to each other and hence low interference between existing telecommunication systems. When UWB sensors are used for high density deployment, the low UWB signal level is a disadvantage because the signals from other sensors mask the low level signal. This may be mitigated using orthogonal spreading codes. UWB communications is used for RF identification tags, precision time measurements, ground probing radar, direct broadband line of sight communication in difficult terrain, short range radar sensors and multi-hop, vehicle to roadside communications, wideband personal area networks, intrusion detection and ad hoc wireless networks.

1 Definitions

An ultra wideband (UWB) system is any communication system that fits within the following defining bandwidth considerations. Given two frequencies f_h and f_l defining the upper and lower 10 dB cutoff frequencies respectively in the power spectrum of any signal, the UWB centre frequency f_c is:

$$f_c = 0.5\left(f_h - f_l\right) \qquad (1)$$

The fractional bandwidth f_p is:

$$f_p = 2\left(f_h - f\right) \times \left(f_h + f_l\right)^{-1} \qquad (2)$$

A signal is UWB *if* $f_p > 0.2$ and $f_h + f_l > 500\,MHz$. The band is typically in the range 3.1 GHz to 10.6 GHz. Several communication systems share this band including radar and satellite systems in the band 3.1 GHz to 4.7 GHz. Therefore UWB systems interfere with existing communication systems. However, UWB signal powers are very low and relatively imperceptible by devices separated from them by 10 metres.

2 UWB Signals

Historically, UWB signals are generated using the Max-generator, shock excitation of wideband antenna, time-gated oscillators, conventional heterodyning and gated power-amplifier, pulse modulation and Fourier techniques [2-4]. Gaussian, Hermite, prolate spheroidal and Wavelets functions are popular. The *Gaussian monocycle (single cycle) pulse* $p(t)$ of amplitude A is

$$p(t) = 2A\sqrt{\pi e}\,\frac{t}{\tau_p}\,e^{-2\pi\left(t/\tau_p\right)^2} \qquad (3)$$

τ_p is the width of the pulse. The received pulse modified by the antenna's differentiating properties is:

$$p_r(t) = A'\left(1 - \frac{4\pi t^2}{\tau_p^2}\right)e^{-2\pi\left(t/\tau_p\right)^2} \qquad (4)$$

UWB transmitter sends the pulse series:

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$$s(t) = \sum_{k=-\infty}^{\infty} p(t - kT_f) \qquad (5)$$

2.1 Prolate Functions

Prolate spheroidal wave functions (PSWFs) have had applications in telecommunications for encryption [5] and biomedical signal processing [7]. Recently Walter and Shen [8] proposed wavelets based on PSWFs, using PSWFs as basis functions instead of sinc functions with better time localization. The new wavelets inherited properties of the prolate functions by preserving high energy concentration in both the time and frequency domains. The PSWFs may be obtained using

$$\int_{-T/2}^{T/2} \psi(x) \frac{\sin \beta(t-x)}{\pi(t-x)} dx = \lambda \psi(t) \qquad (6)$$

PSWFs are doubly orthogonal, are orthogonal basis of $L^2(-T/2, T/2)$ and are orthonormal basis of a sub space β of $L^2(-\infty, \infty)$ and satisfy the equations:

$$\int_{-T/2}^{T/2} \psi_m(t) \psi_n(t) dt = \lambda_m \delta_{mn}; \int_{-\infty}^{\infty} \psi_m(t) \psi_n(t) dt = \delta_{mn} \qquad (7)$$

λ_m is the amount of energy of $\psi(t)$ in the interval $[-T/2, T/2]$. Therefore unique demodulation at the receiver [8] is guaranteed. From (7) the energy λ_n in $[-T/2, T/2]$ is estimated for m=n as:

$$\lambda_n = \int_{-T/2}^{T/2} |\psi_n(t)|^2 dt \Big/ \int_{-\infty}^{\infty} |\psi_n(t)|^2 dt \qquad (8)$$

PSWF also satisfy the differential equation

$$\frac{d}{dt}\left(1 - t^2\right)\frac{d\psi_n(t)}{dt} + \left(\chi_n - ct^2\right)\psi_n(t) = 0 \qquad (9)$$

$\psi_n(t)$ are prolate wave functions of order n, $\chi_n(t)$ is its eigenvector. The constant $c = \beta T / 2$ (β is bandwidth and T =time duration). By changing the values of χ_n different orders of the prolate pulses can be obtained. This is the basic equation for generating multiple pulses. The prolate angular functions $\psi_n(t)$ are required to compute the angular functions of the first kind [8]:

$$\psi_n(t) = \psi_n(\beta, T, t) = \frac{\sqrt{2\lambda_n(c)/T}}{\sqrt{\int_{-1}^{1} \left[S_{0n}^1(c,x)\right]^2 dx}} S_{0n}^1(c, 2t/T) \qquad (10)$$

$$\text{where} \quad \sqrt{\int_{-1}^{1} \left[S_{0n}^1(c,x)\right]^2 dx} = \frac{2}{2n+1} \qquad (11)$$

S_{0n}^1 is the prolate angular function of the first kind. Although no exact solution is specified, a method for calculating the PSWF is by calculating the coefficients and then the basic functions. The first kind of the prolate angular function is [5]:

$$S_{0n}^1(c,t) = \begin{cases} \sum_{k=even}^{\infty} d_k(c) P_k(c,t); & n \text{ even} \\ \sum_{k=odd}^{\infty} d_k(c) P_k(c,t); & n \text{ odd} \end{cases} \qquad (12)$$

$P_k(c,t)$ is a Legendre polynomial and $d_k(c)$ satisfy the recurrence relation:

$$a_k d_{k+2}^2(c) + (\beta_k - \chi_n(c)) d_n^k(c) + \gamma_k d_{k-2}^n(c) = 0 \qquad (13)$$

$$a_k = \frac{(k+1)(k+2)c^2}{(2k+3)(2k+5)}; \quad \gamma_k = \frac{k(k-1)c^2}{(2k-1)(2k-3)}$$

$$\beta_k = \frac{(2k^2 + 2k - 1)c^2}{(2k-1)(2k+3)} + k(k+1); \qquad (14)$$

We set $d_k(c) = 0$ for all $k > 2N+1$. To compute $d_k(c)$ solve the equation: $(\Theta - \chi_n I) d^n = 0$; where d^n are the

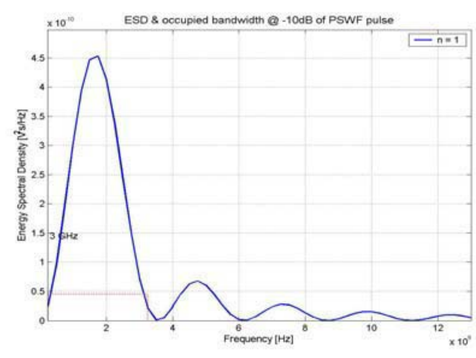

Figure 1: 10dB bandwidth-PSWF (Order 1)

eigenvectors and χ_n are the eigenvalues of Θ. Figure 1 is the spectrums of PSWF of order 1. The zeroth order function has low pass characteristics and the rest are band pass in nature. The pulse widths used are 0.4 ns at 50 GHz sampling rate. The approximation method for PSWF leads to low spectral ripples that are of no major concerns. The first order function is plotted in Figure 2. It shows that 3 GHz bandwidth can be achieved at the -10 dB point.

2.2 Wavelets as UWB Signals

Prolate functions share similar properties with wavelets including orthogornaility and finite support. Wavelets ("small waves") are localised in both the time and frequency domains. Wavelets oscillate a desirable property of UWB signals. They occur in very finite durations to result to ultra wideband. They are also localised in the frequency domain, a property that is particularly useful for UWB to ensure spectral leakage into the spectral regions of existing communication systems is limited. In fact the integral of a wavelet must be zero within its support and it has a finite energy. Furthermore, the construction of wavelets requires that the mother wavelet generates the basis functions by 'dilation' and 'translation'. Translation in time results to a proportional phase shift in the frequency domain. Similarly, it can be shown using Fourier series techniques that a frequency shift is the result of modulating a time series by a complex signal in the time domain. These properties facilitate bandwidth expansion.

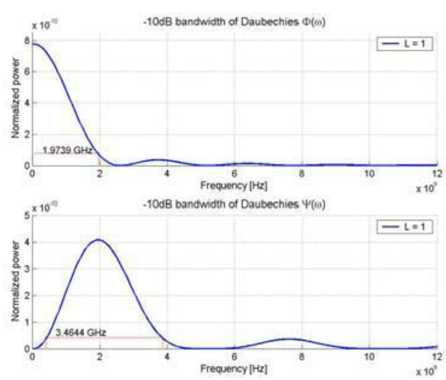

Figure 2: -10dB bandwidth Wavelets

Orthogonality of wavelets is used in wavelet analysis to facilitate fast computation, in UWB it is for separating multipath signals from two or more sources and to combat interference. In Figure 2 the achievable spectral bandwidth at the -10bB points for Daubechies wavelets of order1 are shown. Both the scaling and wavelet functions are compared. The scaling function has low pass characteristics and the wavelet function is band pass in nature. Their -10 dB bands are 1.9739 GHz and 3.4644 GHz respectively. The -10 dB bandwidth of the Daubechies wavelet of order 1 is at least 0.4644 GHz higher than with PSWF.

3 UWB Bandwidth Expansion

Occasionally the pulse width required for a particular UWB is too small to be generated with discrete components. However, by combining several smaller bandwidth UWB pulses, signal conditioning and processing for wider bandwidth can be done in the frequency domain. We have investigated four bandwidth expansion techniques including subband addition, convolution, product and modulation. There are several desirable properties that functions should have to facilitate UWB bandwidth expansion. The signals must have overlapping spectra, be combinations of LPF/HPF, LPF/BPF and BPF/HP. Therefore, the following combinations of orthogonal signals are recommended for bandwidth expansion. The combination of signals to create wider bandwidth signal must leave their orthogonality properties in tact. Therefore the wider bandwidth signal created should be orthogonal.

3.1 UWB Bandwidth Expansion

This section proposes four methods that can be used to expand the bandwidth from ultra wide band functions. The first method uses a sum of signal spectrums. We compose a UWB signal by adding two orthogonal signals either in time or frequency domains to generate a UWB signal. Assume that one of the signals is low pass, the second high pass, the two functions are mutually orthogonal and belong to an orthonormal set. The resulting sum function is also orthogonal to the basis. Such signals can be obtained using a function that is orthogonal to it's translates. A function is orthogonal to its integer translates if

$$\sum_{k=-\infty}^{\infty} \left|\Phi(w-2\pi k)\right|^2 = 1 \qquad (15)$$

Let the function $\phi(t)$ be orthogonal in general to it's integer translates $\phi(t-k)$, then impose the restriction that the set $\{\phi(t-k)\}$ are orthonormal if there is no overlap between $\phi(t-k)$ and $\phi(t-k')$ where $k \neq k'$ and the orthogonal signal is

$$\phi(t) = a_1\phi_1(t) + a_2\phi_2(t); \ \Phi(w) = a_1\Phi_1(w) + a_2\Phi_2(w) \qquad (16)$$

This method is extended to include the superposition of two or more orthogonal functions with spectrums:

$$\Phi(w) = \sum_k a_k \Phi_k(w) \qquad (17)$$

The result bandwidth is from sub bands.
The second method is the convolution of two orthogonal signals. By multiplying the two

signals in the time domain the result signal spectrum is the convolution of the spectra:

$$a_1\phi_1(t)a_2\phi_2(t) \Leftrightarrow a\Phi_1(w)*\Phi_2(w);\ a=\frac{a_1a_2}{2\pi} \quad (18)$$

$\phi_3(t)$ is orthogonal to the set since:

$$\langle\phi_3(t),\phi(t)\rangle = \int_{-\infty}^{\infty}\phi_3(t)\phi(t)dt = \left(\int_{-\infty}^{\infty}\phi_3(t)\phi_2(t)\right)dt.\phi_1(t)=0. \quad (19)$$

The third method is the product of the bandwidths of several orthogonal functions. By convolving orthogonal signals, the result is a product of spectra:

$$\phi_1(t)*\phi_2(t)\Leftrightarrow\Phi_1(w)\bullet\Phi_2(w); \quad (20)$$

Lastly, in many cases, the pulse families have DC components an undesirable condition leading to power wastage. We use modulation to shift the spectra of two orthogonal signals to regions that are of interest for a UWB application. The phase cancellation approach adopted is:

$$e^{\pm jw_0t}\phi_1(t)\Leftrightarrow\Phi_1(w\pm w_0) \quad (21)$$

The signals are then added and also multiplied to generate UWB signals.

4 Simulations

In each simulation we combined functions of orders zero and 1. Figure 4 is obtained using the PSWF. The -10 db bandwidth for convolution of the signals is about 14GHz. Bandwidth expansion is at least 4.67 times the bandwidth of the bandpass function and about 7 times the bandwidth of the lowpass signal we started with. The addition of spectrum results also to increase in the composite bandwidth, from 2 Ghz to more than 3 GHz and marginally for the high pass signal. Lastly we evaluated bandwidth expansion based on wavelet functions.

Functions	Compared with	Prolate(% increase)	Wavelets (% increase)
Spectral Convolution	LLP signal band	467	650
	HP signal band	700	430
Spectral Addition	LP signal band	33.3%	200
	HP signal band	marginal	33.3

Table 1: Summary of Bandwidth Increases

Daubechies wavelet and scaling functions of order L = 1 was evaluated. The -10 db bandwidth for the combined signals is about 13GHz. Expanded bandwidth is about 6.5 times the bandwidth of the low pass function and 4.3 times the bandwidth of the high pass signal. The resulting signal shows a reduction in ripples. The addition of spectrums also results to a slight increase in the bandwidth of about 4GHz. Table 1 is the results using the 2 UWB functions.

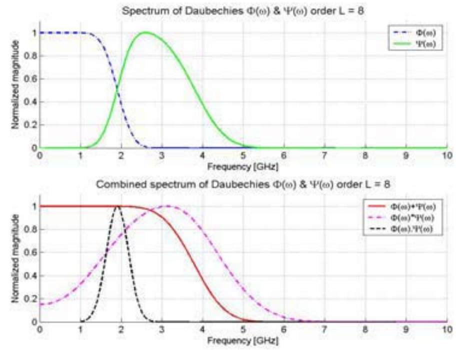

Figure 3: Bandwidth Expansion (Wavelets)

5 Conclusions

We have shown that convolution of spectrums achieves excellent bandwidth expansion. Bandwidth expansion is also achieved by superposition of spectrums with lesser efficiency than the convolution.

6 References

[1] T. W. Barrett, *"History of Ultra wideband (UWB) radar & Communications: Pioneers and Innovators,"* Progress in Electromagnetic Symposium 2000 (PIERS2000), Cambridge, MA, July 2000.

[2] G. S. Gill, "Ultra-Wideband Radar Using Fourier Synthesized Waveforms," IEEE Trans. On Electromagnetic Compatibility, Vol. 39, No. 2, May 1997, pp. 124 – 131.

[3] R. J. Fontana, *"Recent System Applications of Short-Pulse Ultra Wideband (UWB) Technology,"* IEEE Transactions on Microware Theory and Techniques, Vol. 52, No. 9, September 2004, pp. 2087 – 2104.

[4] M. Ghavami, L.B. Michael and R. Kohno, Ultra wideband signals and systems in communication engineering, John Wiley & Sons Ltd, 2004

[5] H. D. Truong, Evaluation of Ultra Wideband Signals and Modulation Techniques, MEng Graduate Project, University of Technology, Sydney, 2004.

[6] A.D. Wyner, "*Analog scrambling scheme which does not expand bandwidth P a r t - 1 : Discrete t i m e ,"* IEEE Trans. Inform. Theory, Vol IT-25, May 1979, pp. 415-425

[7] B. Feng, A. Sitek, G. T. Gullberg, " The Prolate Spheroidal Transform for gated SPECT," IEEE Trans. on Nuclear Science, 2001, Vol. 48 , Issue: 3 , June 2001, pp. 872 – 875.

[8] R. Dilmaghani, M. Ghavami, B. Allen and A.H. Aghvami, "Novel UWB Pulse Shaping Using Prolate Spheroidal Wave Functions", *14th IEEE Proc. on Personal Indoor Mobile Communications, Sept. 2003,* Vol.1, pp. 602 – 606.

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