Optimal Screening by Risk-Averse Principals

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Abstract

This paper studies the effects of principal’s risk aversion on principal-agent relationship under hidden information. It finds that the agent’s equilibrium effort increases and approaches the efficient level as the principal’s risk aversion increases and tends to infinity. Allowing for random participation by the agent, his effort can be efficient even when the principal’s risk aversion is finite. For the case of common agency with random participation, it is optimal for the principals to make the agent the residual claimant on profits and the principals’ net profits monotonically decrease to zero when their risk aversion tends to infinity.

KEYWORDS: principal-agent model, risk aversion, random participation, common agency

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1 Introduction

In contract designing, it is fundamentally important for principals to elicit the private information their agents possess. If there is no moral hazard in the principal-agent relationship, the taxation principle (Rochet, 1985) often allows to cast the problem of information revelation as equivalent to a supplier using price schedules to discover hidden demand characteristics. Existing literature in screening by nonlinear pricing, originating in papers of Rothschild and Stiglitz (1976), Mussa and Rosen (1978), and Maskin and Riley (1984), typically considers a certainty environment or assumes that principals are risk neutral and maximize expected payoffs. Though several papers and textbook treatments use a formulation of the screening problem that allows for risk aversion on the principals’ side, most of them do not study systematically the effects of principals’ risk aversion on equilibrium and efficiency. The reason for this omission in the theory development may be due to the recognition that the population of agents/consumers is sufficiently large, although the risk aversion of principals is well accepted as a more plausible and more realistic assumption than risk neutrality. With a large population (such as a continuum of consumers), the effect of uncertainty at the population level disappears or is negligible in spite of significant uncertainty at the individual level. Uncertainty and the risk attitude of a principal, however, become crucial when the principal faces a small and finite number of agents with hidden information. For example, at the time when a firm hires a CEO the firm may not know exactly or be not certain about the CEO’s ability of profit-making and work ethics. Consequently, the firm’s profits under the CEO’s management are uncertain. For another example, consider a small municipality signing a regulatory contract with a private "public services" operator, such as a large multinational firm (Gence-Creux (2000), Laffont and Rochet (1998)). In this case the principal, municipality, faces uncertainty because the monetary transfer from the principal to the operator depends on the unknown characteristics of the agent such as operating costs. In a financial market, when a liquidity provider (principal) posts a limit order on an electronic trading system, she does not know the size of next market order (agent) she will meet, although she can impose a limit on the maximum amount of buying or selling (Biais, Martinmort and Rochet (2000)). Under these circumstances, the payoff to a principal from the use of a particular contracting mechanism is uncertain and therefore the optimal mechanism will be affected by the degree of risk aversion of the principal. This paper focuses on the role played by the risk aversion of

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1The relevant literature is reviewed briefly at the end of this section.
principal(s) and its effects on the optimal screening and the equilibrium effort exerted by the agent.

The hidden information under the consideration, which is summarized by the type of an agent, include "vertical private information" such as an agent’s talent or capability as well as "horizontal private information" such as his valuation of outside opportunities. For concreteness, we investigate the case where a principal hires the agent to perform a task for her. To focus on screening, we abstract from the moral hazard issue by assuming that work effort is contractible by observing the output of the agent through a deterministic production function of effort, but the cost of effort is private information to the agent. In Section 2, we focus on vertical private information by considering a scenario where the set of possible types (i.e., marginal costs of effort) forms an interval on the Euclidean line but the value of the outside option is constant and known to the principal. Thus, in the spirit of Mussa and Rosen (1978), we develop a model of bilateral contracting, which can accommodate the risk aversion on the principal’s side. It is found that risk aversion makes pooling at equilibrium less likely. More importantly, the equilibrium effort provision increases in the degree of the principal’s risk aversion under certain relatively non-restrictive conditions. When the principal becomes infinitely risk averse, the effort reaches the efficient level. Total payment and marginal payment to the agent increase as well when the principal becomes more risk averse.

What are the driving forces behind these results? If the principal is risk neutral, her goal is to maximize the expected profits, net of compensation to the agent. However, if she is risk averse, the volatility of net profits, including the risk of breaking down the principal-agent relationship, matters too. Thus, she is willing to pay the agent more in exchange for lower volatility. Since the effort provision by the agent with the lowest marginal cost of effort is efficient and the equilibrium effort level declines in the marginal cost under risk neutrality, a risk-averse principal does not want to change the effort level provided by the most efficient agent but intends to make a less efficient agent work increasingly harder to reduce the volatility of net profits. For this purpose, the risk-averse principal either attracts more agents (those with higher marginal costs of effort) to participate or pays active agents higher with a flatter pay schedule. In the extreme, when the principal’s degree of risk aversion is infinitely high she aims for a non-random net profit. Then, she can optimally design a contract to make the agent residual claimant. In turn, the agent will exert an efficient effort. This section ends with a closed-form solution for the optimal wage designed by a risk-averse principal in a special case of the model.

Section 3 extends the model in Section 2 by including horizontal private information; i.e., allowing the agent’s private value of the outside option to
be random but independent of his marginal cost of effort. This extension parallels the Rochet and Stole’s (2002) extension of the Mussa-Rosen model (1978). Since the value of the outside option does not affect the marginal cost of effort, screening is possible only on the vertical dimension. If there is only one principal, we find once again that the equilibrium effort exerted by the agent approaches the efficient level when the principal is sufficiently risk averse. Moreover, there is a distribution of participation costs (or benefits) for which making the agent residual claimant on the profits is the optimal solution to the principal’s problem. With this distribution, the principal’s profits decline in her risk aversion. In the case of common agency, making the agent residual claimant on the profits is optimal in a full-participation symmetric equilibrium, for any distribution of participation costs. Similar to the case of single principal, the net profits of the principals monotonically decrease and converge to zero as their risk aversion increase and approaches infinity. However, there is an important difference between the single principal and common agency cases. In the common agency case, the profits left to the principals affect only the distribution of the surplus because the agent definitely works for a principal in equilibrium. But the agent may fail to be hired in the single principal equilibrium so that a decrease in the principal’s net profits can increase the probability of the agent to be hired and, in turn, the efficiency of the economy.

In sum, our analysis shows that the risk aversion on the principal(s)’ side generally plays a positive role in terms of improving efficiency and agent’s welfare. This finding is similar to Rochet and Stole’s (2002) results on the positive effects of random participation on nonlinear pricing, although the economic mechanisms behind them are different. Even though uncertainty and risk aversion are efficiency enhancing in our model, it is not necessary always to be the case. For instance, Salanie (1990, 1997) develops a model similar to ours but the contract is signed before the agent learns his type. By assuming risk aversion for the agent, but risk neutrality for the principal, it is found that incentives and outputs decrease with the agent’s risk aversion. Laffont and Rochet (1998) similarly show that risk aversion on the agent’s side leads to welfare reduction absent in the case of risk neutrality.

Risk aversion on the principal’s side has been considered in several studies, but they address issues different from this paper. In an early paper, Spier (1992) explores reasons for a risk-averse principal to offer strategically incomplete contracts in the presence of asymmetric information about her type and production output. In the standard principal-agent relationship, Lewis and Sappington (1995) introduce a financier who supplies the capital required by the agent when he produces for the principal. The paper examines the optimal
design of the agent’s capital structure and concludes that the principal’s risk aversion makes her strictly better off if she can dictate the capital structure. Page (1997) assumes contracting between a risk-averse principal and a risk-averse agent using a random production technology. Instead of investigating the effect of the risk attitude of the principal and/or the agent, the paper’s interest is in the existence of optimal contract. Celik (2003) also considers a risk-averse principal, who hires an agent to produce for her, but the utility function of the principal in Celik’s model differs from ours. It is the sum of the utility of consuming the agent’s production and the disutility of payment to the agent; i.e., the principal is risk averse with respect to the payments made to the agent rather than the risk aversion of net profits like in our model. More importantly, the agent has a supervisor who intermediates between him and the principal, and Celik’s focus is on the design of a contract that prevents collusion between the supervisor and the agent.

The models in the literature closest to ours are Gence-Creux (2000), and Laffont and Martimort (2002, Chapter 2). Different from our modeling, Laffont and Martimort assume that the contract is signed before the agent learns his type. Thus, the agent does not have information advantage over the principal at the time when the contract is signed and the contract needs only to satisfy ex ante participation constraint rather than interim participation constraints. It is found that the optimal contract implements the efficient outcome, independent of the principal’s risk attitude. Gence-Creux (2000) studies a joint problem of adverse selection and moral hazard when a regulator hires a private "public services" operator. Similar to Laffont and Martimort (2002, Chapter 2), the analysis is confined to the case where there are only two different types of agents and the agent’s uncertain production cost is additively separable from his effort variable. Moreover, because the principal in Gence-Creux’s model is a social planner, her objective is to maximize social welfare, which includes both consumer surplus and the operator’s profits. Gence-Creux shows that the equilibrium effort supplied by the high-type agent is efficient and the distortion in the low type’s effort is decreasing in the degree of the principal’s risk aversion. It is unclear, however, whether these results are due to the particular assumption of additive separability of uncertain cost from effort and/or due to the inclusion of the agent’s welfare in the principal’s objective. Almost all studies involving risk-averse principals consider only one-dimensional hid-

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2 For an infinitely risk-averse agent in the absence of hidden action a model with an interim participation constraint is equivalent to a model with ex-ante participation constraint.

3 Our model, similar to most models in the literature, assumes that the uncertain variable (marginal cost of effort) is multiplicatively separable from the effort variable.

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den information and no paper addresses both vertical and horizontal private information simultaneously. None of these studies addresses common agency either.

2 Agent with a known outside option

Consider a principal-agent relationship, where a risk-averse principal hires an agent to perform a task for her. The task involves exerting effort, $e$, by the agent, which produces profits $\pi(e)$ for the principal. The profit function $\pi(\cdot)$ is strictly increasing, strictly concave and deterministic. The assumption of non-random profits enables us to abstract from moral hazard since the principal can infer the effort exerted by the agent from the profit realization although she may not be able to directly observe the agent’s effort. Because the agent knows his type before contracting, he does not face any uncertainty in this model. Therefore, his risk attitude is irrelevant and his preferences are summarized by

$$u(w, e) = w - te - kx,$$

where $w$ is wage, $(t, x) \in (t_1, t_2) \times (x_1, x_2) \subset R^2$ indicates the type of the agent, and $k$ is a commonly known positive parameter. The value of $t$ can be interpreted as marginal cost of effort while $x$ reflects the agent’s preferences for non-monetary job characteristics (e.g. distance from home, flexibility, etc.) over an alternative job he can obtain on the market, which requires zero effort and pays zero wage. The type of the agent is private information, but the principal knows that $t$ and $x$ are independently distributed. Moreover, $t$ is strictly positive and its probability density function, $f(t)$, is strictly positive on $\Omega \equiv (t_1, t_2)$ and $x$ has a cumulative distribution function $N(\cdot)$. In this section we assume that

$$N(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}.$$  

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4The agent’s risk attitude can be made relevant in two ways. First, if effort is not contractible (for instance, profits are determined by not only the agent’s effort but also some external shocks), then the agent’s risk attitude does matter. The analysis in this case, however, does not have an unambiguous picture because the effects of adverse selection cannot be separated from the effects of moral hazard. Second, one can assume, following Salanie (1990), that the agent learns her type after signing the contract but before exerting effort. In that case risk aversion of the agent affects the participation decision but not the choice of the effort conditional on participation.

5Although the term $kx$ can be easily understood as disutility when $x$ is positive, it can also be interpreted as certain private benefits from a particular job when $x$ is negative.
i.e., the outside option is the same across agents and is commonly known. We will relax this assumption in the next section.

By the Taxation Principle (Rochet (1985)) one can without loss of generality restrict mechanisms used by the principal to be nonlinear wage schedule, $w(e)$. For a given wage schedule $w(·) : R \to R$, define the agent’s surplus by

$$s(t) \equiv \max_e (w(e) - te).$$  

Denote $e(t)$ the utility-maximizing effort of a type-$t$ agent. By the envelope theorem, we have

$$s'(t) = -e(t).$$

Since the agent’s outside choice is common knowledge in this section, the only source of uncertainty is the agent’s marginal cost of effort. The principal’s expected utility is given by

$$\int_{t_1}^{t_2} V(\pi(e(t)) - w(e(t)))f(t)dt,$$

where utility function $V(·)$ is strictly increasing, concave, and twice differentiable with normalization $V(0) = 0$.

Thus, the principal’s problem is to choose a wage schedule, $w(·)$, to maximize her expected utility, subject to (4) and the implementability constraint, which states that $e(·)$ is non-increasing.

Let us first concentrate on the relaxed problem; i.e., drop the implementability constraint. Then the principal solves

$$\max_{t_1}^{t_2} \int V(\pi(e(t)) - w(e(t)))f(t)dt$$

s.t. $s'(t) = -e, \quad s(t_2) = 0.$

Since the agent is uniquely characterized by his marginal cost of effort, the principal can always properly select a wage schedule to make the surplus of the most inefficient agent, agent $t_2$, equal to the value his outside option, which is

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6 We make a slightly stronger assumption that $V'(·) > 0$, so the Arrow-Prates coefficient of absolute risk aversion is always well defined.
normalized to be zero in this section. The Hamiltonian for this problem is
\[ H \equiv V(\pi(e) - s(t) - te(t))f(t) - \lambda(t)e(t). \] (7)
The Pontryagin maximum principle states that if \(\{e(\cdot), s(\cdot)\}\) is the solution to the principal’s problem then there exists \(\lambda(t) \in C^1(\Omega)\) such that
\[
\begin{align*}
s'(t) &= -e(t) \\
\lambda'(t) &= V'(\pi(e) - s(t) - te(t))f(t) \\
\lambda(t) &= V'(\pi(e) - s(t) - te(t))f(t)(\pi'(e) - t) \\
s(t_2) &= 0, \quad \lambda(t_1) = 0
\end{align*}
\] (8)
In particular, the above system implies that
\[ \pi'(e(t_1)) = t_1, \] (9)
i.e., the marginal profit of the principal is equal to the marginal effort cost of agent \(t_1\). In other words, we obtain the well-known result of no efficiency distortion at the top.\(^7\) On the other hand, the second equation of system (8) implies that \(\lambda'(t) > 0\) for all \(t \in (t_1, t_2)\). Since \(\lambda(t_1) = 0\), it is obvious that \(\lambda(t) > 0\) for all \(t \in (t_1, t_2)\). Recalling the third equation of system (8), the strict positivity of \(\lambda(t)\) means
\[ \pi'(e(t)) > t \] (10)
for all \(t \in (t_1, t_2]\). Thus, all agents, except for agent \(t_1\), undersupply effort.

This result still holds in the presence of pooling at equilibrium. Indeed, suppose at equilibrium all agents on segment \([t_a, t_b]\) exert the same effort \(e^*\) and assume that for some \(t \in [t_a, t_b]\) inequality (10) is violated. Then it will also be violated at point \(t_b\) belonging to the separating region, which is impossible.

Substituting the third equation of (8) into the second and after some rearrangement one obtains
\[
\frac{d}{dt} ((\pi' - t)f - F) = R(\pi' - t)^2 f e',
\] (11)
where \(R(\cdot) \equiv -V''(\cdot)/V'(\cdot)\) is the Arrow-Pratt’s coefficient of absolute risk aversion and \(F(\cdot)\) is the cumulative distribution function corresponding to \(f\). Note, we have dropped the arguments in (11) to simplify notations and

\(^7\)Note, \(t_1\) type (i.e. the type for whom the effort is least costly) generates the highest total surplus. Therefore, we call it the top type.
\( e' \equiv de(t)/dt \). The initial condition for (11) is (9). Two results are immediate from (11) and (9). First, if the principal is risk neutral, then \( R(\cdot) = 0 \), and (11) and (9) implies

\[
\pi'(e(t)) = v(t) \equiv t + \frac{F(t)}{f(t)}. \tag{12}
\]

Expression (12) is well known. A similar result for a risk-neutral principal can be found, for example, in Varian (1989). If virtual type, \( v(\cdot) \), is non-decreasing it provides the solution for the complete problem. Otherwise, the optimal solution will entail some pooling. If \( \pi(\cdot) \) satisfies Inada condition at zero, all agents will participate in the contract. However, exclusion region will be non-empty if \( \pi'(0) < \inf_t v(t) \).

If the principal is extremely risk averse, i.e. \( R(\cdot) \) tends to infinity for all wealth levels, then \( \pi'(e(t)) \) should approach \( t \) for all \( t \in [t_1, t_2] \) to keep the right-hand side of (11) finite; that is, all agents will provide efficient effort. We will prove this result more rigorously in Proposition 2. To proceed further, let us introduce two definitions below.

**Definition 1** Individual \( i \) with Bernoulli utility \( V_i(\cdot) \) is more risk averse than individual \( j \) with Bernoulli utility function \( V_j(\cdot) \) if there exist a strictly increasing concave function \( \phi(\cdot) \) such that \( V_i = \phi(V_j) \).

Requirement that \( \phi(\cdot) \) is strictly increasing ensures that the individuals exhibit the same ordinal preferences. If function \( V_k(\cdot) \) and \( \phi(\cdot) \) are twice differentiable and their first derivatives are strictly positive, it is easy to show that

\[
R_i(\cdot) = R_j(\cdot) + R_{\phi}(\cdot), \tag{13}
\]

where \( R_k(\cdot) \) and \( R_{\phi}(\cdot) \) are the coefficients of absolute risk aversion corresponding to \( V_k(\cdot) \) (\( k \in \{i, j\} \)) and \( \phi(\cdot) \), respectively. It is immediate from (13) that individual \( i \) is more risk averse than individual \( j \) if and only if \( R_i(z) \geq R_j(z) \) for any wealth level \( z \).

**Definition 2** Individual \( i \) with Bernoulli utility \( V_i(\cdot) \) is uniformly more risk averse than individual \( j \) with Bernoulli utility function \( V_j(\cdot) \) if for any wealth levels \( z_1 \) and \( z_2 \)

\[
R_i(z_1) \geq R_j(z_2). \tag{14}
\]

Note that if both utilities are of CARA form then notions of more risk averse and uniformly more risk averse coincide. Now we are ready to formulate
our first proposition.

**Proposition 1** Assume that principal $i$ is uniformly more risk averse than principal $j$ and let $e_k(t)$ be the solution to the system of (11) and (9) for $k \in \{i, j\}$. Assume also that equilibria are fully separating under both principals; i.e., $e'_k(t) \leq 0$. Then agent $t$ will exert more effort if he works for principal $i$; i.e., $e_i(t) \geq e_j(t)$ for all $t$.

**Proof.** Equation (11) can be written as

$$e'_k(t) = \frac{2f(t) - (\pi'(e_k(t)) - t)f'(t)}{[\pi''(e_k(t)) - R_k(\pi(e_k(t)) - s_k - te_k(t)](\pi'(e_k(t)) - t)^2f(t)}.$$  \hspace{1cm} (15)

Consider function

$$h(t, e, R) = \frac{2f(t) - (\pi'(e) - t)f'(t)}{[\pi''(e) - R(\pi'(e) - t)^2f(t)}.$$  

Assumption $e'_k(t) \leq 0$ ensures $h(t, e, R) \leq 0$. It is easy to verify that

$$\frac{\partial h(t, e, R)}{\partial R} = \frac{(\pi'(e) - t)^2h(t, e, R)}{[\pi''(e) - R(\pi'(e) - t)^2f(t)} \geq 0.$$  

From (14) $h(t, e, R_i(\cdot)) \geq h(t, e, R_j(\cdot))$. On the other hand, (9) implies $e_i(t_1) = e_j(t_1)$. Thus, the system of (15) and (9) satisfies the conditions of Lemma 1 in the Appendix, which ensures $e_i(t) \geq e_j(t)$ for all $t$. Q.E.D.

Proposition 1 implies that as long as equilibrium effort is fully separating, an increase in the principal’s risk aversion will induce agents to work harder. The harder the agent works, the less efficiency distortion is. If principals have CARA utility we can reach more conclusive results.

**Proposition 2** Assume principal has a constant coefficient of absolute risk aversion, $R$, and the equilibrium is fully separating for all $R$. Then,

(i) equilibrium effort $e(t, R)$ increases in $R$ and it is efficient when $R$ tends to infinity;\(^8\)

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\(^8\)Equilibrium effort $e(t)$ is implicitly determined by risk aversion parameter $R$ when principal’s utility is of CARA. So we explicitly express the effect of $R$ by $e(t, R)$ when it is relevant. The same treatment is also applied to equilibrium wage, etc. For simplify, we slightly abuse notations and $e'(t, R)$ below should be considered as $\partial e(t, R)/\partial t$. 

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(ii) both total wage \( w(e, R) \) and marginal wage \( w'(e, R) \) increase in \( R \), and total wage is equal to profit \( \pi(e) \) when \( R \) tends to infinity.

**Proof.** The conclusion that \( e(t, R) \) increases in \( R \) restate Proposition 1 for principals with CARA utility. To show the second conclusion in part (i), rewrite (11) as

\[
e(t)\left(\frac{\pi''(e)}{R} - (\pi'(e) - t)^2\right)f(t) = \frac{2f(t) - (\pi'(e) - t)f'(t)}{R}.
\]

Define

\[G(t, e) \equiv 2f(t) - (\pi'(e) - t)f'(t).\]

Since \((t, e) \in [t_1, t_2] \times [0, e(t_1)] = C\), where \(C \subset R^2\) is a compact set, there exists \(K > 0\) such that \(0 < G(t, e) < K\) for all \((t, e) \in C\). Taking into account that \(e'(t) \leq 0\) under the assumptions of the proposition one obtains

\[
\lim_{R \to +\infty} \pi'(e(t, R)) = t.
\]

Given the agent’s surplus, the total wage can be written as \( w(e) = \inf_t (s(t) + et) \) (see, for example, Basov 2005). Thus, the partial derivative of \( w(e, R) \) with respect to \( R \) satisfies the following condition

\[
w_R(e, R) = s_R = \int_t^{t_2} e_R(\tau, R)d\tau > 0,
\]

where the first equality follows from the envelope theorem, the second equality is derived from (4) using initial condition (6), and the last inequality follows from part (i). For marginal wage, the first- and second-order conditions for the type-\(t\) agent who faces wage schedule \( w(e, R) \) are

\[
w_e(e, R) = t \\
w_{ee}(e, R) \leq 0.
\]

Totally differentiating the first equation in (17) with respect to \( R \) and then using the second inequality in (17) and the result of part (i), one obtains \( w_{eR} = -w_{ee}(e, R)e_R \geq 0 \). Finally, since part (i) shows \( \pi'(e(t, +\infty)) = t \) and (3) implies \( w'(e) = t \), there is \( w'(e) = \pi'(e) \) when \( R \) tends to infinity. Recalling \( w(0) = \pi(0) = 0 \), we have \( w(e) = \pi(e) \) when \( R \) tends to infinity. Q.E.D.

The intuition behind Propositions 1 and 2 is the following. A risk-neutral
principal is only concerned with the expected net profits. However, if she is risk averse, the volatility of net profits concerns her too. Thus, she is willing to sacrifice some net profits and pay the agent more to reduce the volatility. Moreover, a risk averse principal assigns a greater weight to a lower realization of net profit than a higher realization so that the wage increase to a less efficient agent (with a greater $t$) is greater. The increased wage payment stimulates the agent to work harder, which in turn improves efficiency, with a greater efficiency improvement for a less efficient agent. When $R \to +\infty$, the principal tries to avoid all profit uncertainty. This requires her to choose a wage schedule, which ensures that the net profit yielded from meeting any agent is constant. Clearly, this strategy makes the agent the residual claimant on the firm’s profits. As a residual claimant, the agent exerts efficient effort.

One condition for Propositions 1 and 2 is full separation in participation region in equilibrium. The situation of full separation is not atypical. For instance, it can be easily seen from (15) that a sufficient condition for fully separating equilibrium is $f'(t) \leq 0$; i.e., the pdf of marginal effort cost does not increase. The proposition below shows that there is a lower risk aversion bound such that if equilibrium is fully separating when the principal’s risk aversion is equal to this bound then there is no pooling in equilibrium which involves more a risk-averse principal.

**Proposition 3** Assume that principals $i$ and $j$ have CARA utility and principal $i$ is more risk averse than principal $j$. Let $e_k(t)$ for $k \in \{i, j\}$ denote the equilibrium effort of agent working for principal $k$. If $e_j(t)$ is fully separating then so is $e_i(t)$.

**Proof.** Let $e_k^*(\cdot)$ denote the solution to the relaxed problem (5)-(6) for $k \in \{i, j\}$. Then, (9) and (10) imply that

$$\pi'(e_k^*(t)) \geq t. \quad (18)$$

Assume that in contrary to the claim of the proposition the pooling region is non-empty for $e_i(t)$; i.e., there exists $t = t^*$ such that

$$e_i''(t^*) > 0. \quad (19)$$

Let $\gamma \in (0, 1)$ and define $R(\gamma) \equiv \gamma R_i + (1 - \gamma) R_j$. Let $e(t; \gamma)$ be the solution to (11) when the principal’s constant coefficient of absolute risk aversion is equal
to \( R(\gamma) \) and define \( \gamma^* \) by

\[
\gamma^* \equiv \inf \{ \gamma : \exists t \in (t_1, t_2) : e'(\gamma) > 0 \}. \tag{20}
\]

Note that \( \gamma^* \) is well defined, since the set under the sign of infimum is bounded and non-empty due to (19). Definition (20) implies that

\[
e'(\gamma^*) = 0 \tag{21}
\]

and equilibrium is fully separating for \( \gamma \in [0, \gamma^*] \). Now, to prove the proposition it is sufficient to prove that \( e_j'(\gamma^*) > 0 \), which contradicts the assumption that \( e_j(t) \) is fully separating. By (15), condition (21) implies that

\[
2f(\bar{t}) - (\pi'(e(\bar{t}; \gamma^*))) - t f'(\bar{t}) = 0.
\]

Applying Proposition 1, we have \( e(\bar{t}; \gamma^*) > e_j(\bar{t}) \), which together with the concavity of \( \pi(\cdot) \) yields

\[
2f(\bar{t}) - (\pi'(e(\bar{t}; \gamma^*))) - t f'(\bar{t}) < 0.
\]

Hence, applying (15) we have \( e_j'(\bar{t}) > 0 \), which completes the proof. Q.E.D.

Proposition 3 indicates that risk aversion on the principal side reduces the chance of pooling equilibrium. The reason for this result again is the intention of a risk-averse principal to smooth net profits. The more risk averse is the principal, the higher powered incentives is she willing to provide to motivate less efficient agents work increasingly harder. This leads to a flatter \( e(t) \) curve since \( e(t_1) = t_1 \) independent of the principal’s risk attitude.\(^9\) As a result, bunching in equilibrium is less likely. From Proposition 3, it is obvious that the condition ensuring separating equilibrium under risk neutral also guarantees separating equilibrium under risk aversion as stated by the following corollary.

**Corollary 1** If \( f'(t) \leq \frac{2f^2(t)}{\pi'(t)} \) and the principal’s utility is of CARA type, then equilibrium is fully separating.

**Proof.** Condition \( f'(t) \leq \frac{2f^2(t)}{\pi'(t)} \) is equivalent to the requirement that \( v(t) \) in (12) satisfies \( v'(t) \geq 0 \); i.e., the virtual type is non-decreasing. It guarantees that there is no pooling in equilibrium for a risk-neutral principal. Therefore, by Proposition 3 there is no pooling in equilibrium if the principal is risk averse. Q.E.D.

\(^9\)See Figure 1 of the Example below.
It is worth noticing that the condition in the corollary does not impose a significant restriction on the distribution of types because the log-concavity of $F(t)$ or the monotonicity of hazard rate $\frac{f(t)}{F(t)}$, a widely adopted assumption in the literature (e.g. Biais, Martimort and Rochet (2000)), requires $f'(t) \leq \frac{f^2(t)}{F(t)}$.

To illustrate the main results of our previous analysis, we explicitly solve an example of our model to obtain closed form solutions for the equilibrium effort and wage schedule.

Example. Assume the production function is $\pi(e) = e - \frac{e^2}{2}$, agent’s type $t$ is distributed uniformly on $[0, 1]$, and the principal has a constant coefficient of absolute risk aversion, $R$. Then (11) is reduced to

$$e'(t) = -\frac{2}{1 + R(1 - e - t)^2}$$

with initial condition $e(0) = 1$. Apparently, any solution of equation (22) satisfies $e'(t) < 0$. Therefore, there is no pooling at positive effort levels and (22) completely determines the solution to the problem. If the principal is risk neutral, i.e., $R = 0$, (22) collapses to $e'(t) = -2$, which leads to the solution $e(t) = 1 - 2t$. Thus, only agents with $t \in [0, 0.5)$ participate and provide positive effort. On the other hand, when $R$ tends to infinity, (22) implies $e(t) = 1 - t$; i.e., all agents, except the least efficient one ($t = 1$), are active and provide positive effort to the principal. It is also obvious that all active agents, except for the most efficient one ($t = 0$), exert less effort under a contract signed with a risk-neutral principal. For a finite and positive $R$, it is not hard to find that the solution to (22) is

$$e(t) = 1 - \frac{1}{\sqrt{R}} \ln \frac{1 - \sqrt{R}(t + e(t) - 1)}{1 + \sqrt{R}(t + e(t) - 1)}.$$  

We illustrate it as $e(t, R)$ in Figure 1 below, which is located between effort line $e(t, 0)$ (when the principal is risk neutral) and effort line $e(t, \infty)$ (when $R$ tends to infinity). Agents who choose the outside option are those whose type $t$ falling in $(t^*(R), 1]$. Hence, setting $e(t) = 0$ in the above equation yields

$$t^*(R) = 1 + \frac{1 - \exp(\sqrt{R})}{\sqrt{R}(1 + \exp(\sqrt{R}))}.$$  

Apparently, $t^*(R)$ increases in $R$ with $t^*(0) = 0.5$ and $\lim_{R \to \infty} t^*(R) = 1$; i.e. as the principal becomes more risk averse the chance that the agent takes the
principal’s offer approaches one. In the meantime, $e(t, R)$ in Figure 1 moves from $e(t, 0)$ to $e(t, \infty)$, where all agents exert the first-best effort. As pointed earlier, $e(t, R)$ moves around the pivotal point $(0, 1)$ and becomes flatter as it moves.

Recalling the first-order condition for the agent’s optimization $w'(e) = t$ and its initial condition that $w(0) = 0$, and applying the result that $e(t) = 1 - 2t$ when $R = 0$, we have the wage contract designed by a risk-neutral principal that $w(e) = \frac{e}{2} - \frac{e^2}{4}$. Since production function is $\pi(e) = e - \frac{e^2}{2}$, it can be concluded that a risk-neutral principal and her agent equally share the output. On the other hand, $e(t) = 1 - t$ when $R$ tends to infinity. Therefore, the solution to $w'(e) = t$ is $w(e) = e - \frac{e^2}{2} = \pi(e)$, which means that the agent captures all profits, leaving the principal nothing. For a principal with more moderate risk aversion, the profit share is between these polar cases. To see this point, we solve (23) for $t$ that

$$t = 1 - e + \frac{1 - \exp(\sqrt{R}(1 - e))}{\sqrt{R} 1 + \exp(\sqrt{R}(1 - e))}$$
Thus, the solution to \( w'(e) = t \) with initial condition \( w(0) = 0 \) gives

\[
w(e) = \pi(e) + \frac{2}{R} \ln \frac{\exp\left(\frac{\sqrt{R(e-1)}}{2}\right) + \exp\left(\frac{\sqrt{R(1-e)}}{2}\right)}{\exp\left(\frac{\sqrt{R}}{2}\right) + \exp\left(-\frac{\sqrt{R}}{2}\right)}.
\]

Using the l'Hospital’s rule, it is obvious that the second term on the right-hand side of the above equation is equal to \( \frac{e^2}{4} - \frac{e}{2} \) and 0, respectively, when \( R \) tends to zero and infinity. Thus, \( w(e) \) increases from \( \frac{\pi(e)}{2} \) and \( \pi(e) \), as predicted by Proposition 2.

## 3 Agents with random outside option

This section considers a set-up similar to the previous section but extends the analysis by assuming that \( N(\cdot) \) is a genuine cumulative distribution function; i.e. the agent’s individual preferences for non-monetary job characteristics relative to the outside option are allowed to be heterogeneous. To simplify the notation we will assume that \( k = 1 \) in the single principal case of Subsection 3.1 but will restore it in Subsection 3.2 in the common agency case.

### 3.1 A single risk-averse principal

We first consider the case of a single risk-averse principal. The agent’s surplus is still defined by (3). Therefore, the probability that an agent accepts the contract and provides a positive effort is \( N(s) \). Therefore, the principal solves

\[
\max_{t_1} \int_{t_1}^{t_2} V(\pi(e(t)) - s(t) - e(t)t)N(s(t))f(t)dt
\]

subject to (4). Note, the boundary condition \( s(t_2) = 0 \) in (6) no longer holds because the value of the outside option is uncertain to the principal. It is an optimal control problem with both ends being free. The Hamiltonian for this problem is

\[
H = V(\pi - s - et)N(s)f(t) - \lambda(t)e(t)
\]

and the Pontryagin maximum principle implies

\[
\begin{align*}
\lambda'(t) &= V'(\pi - s - et)N(s)f(t) - V(\pi - s - et)N'(s)f(t), \\
\lambda(t) &= V'(\pi - s - et)(\pi'(e) - t)N(s)f(t), \\
\lambda(t_1) &= \lambda(t_2) = 0.
\end{align*}
\]
Proceeding in the same way as in the previous section, one obtains

\[
\frac{1}{N} \frac{d}{dt} (N(\pi' - t)f) - f = R(\pi' - t)^2 e'f - \frac{NV}{N'V} f.
\]  
(27)

If \(x\) takes only one value \(x_0\) with certainty, then \(N(s) = 1\) for \(s \geq x_0\) and equation (27) is reduced to (11). The transversality conditions in (26) lead to

\[
N(s(t_i))(\pi'(e(t_i)) - t_i) = 0 \quad \text{for } i = 1, 2.
\]  
(28)

It is reasonable to only consider the case where agents participate in equilibrium with a positive probability. Then, \(N(s(t_1)) > 0\) and \(\pi'(e(t_1)) = t_1\), which is the conventional “no distortion at the top” property. On the other hand, the boundary condition at \(t_2\) means either the type-\(t_2\) agent participates with zero probability or the provision of his effort is also efficient. This result of either no-service or no-distortion at the bottom does not depend on the risk aversion of the principal and has been observed by Rochet and Stole (2002) too.

It is obvious that the first term on the right-hand side of (27) becomes dominant when \(R(\cdot)\) is sufficiently large. Therefore, it leads to an efficient effort when \(R\) tends to infinity. The formal proof of this conclusion is similar to Proposition 2 and is omitted. So we only state it as a corollary below.

**Corollary 2** With random participation, equilibrium effort approaches the efficient level if the coefficient of principal’s absolute risk aversion is constant and tends to infinity.

The possibility of both most efficient and least efficient agents exerting an efficient effort leads us to ask under what condition all active agents will exert an efficient effort when they face a wage schedule proposed by a principal with finite risk aversion. It is well-known that efficient provision of effort by an active agent requires the marginal profit of the principal equal to the marginal cost of the agent, \(\pi'(e(t)) = t\). Since agent \(t\) optimally provides his effort until the marginal wage equal to his marginal cost of effort (i.e., \(w'(e) = t\)), the efficiency condition requires the wage schedule designed by the principal to be a two-part schedule that

\[
w(e) = \pi(e) - A_M,
\]  
(29)

where \(A_M\) is a constant that can be interpreted as the net profits to the principal. However, the introduction of random participation imposes a new condition for the efficiency of effort provision. To explore this condition, let
us substitute $\pi' = t$ into (27) to yield $\frac{N'}{V'} = \frac{V'}{V}$. Thus, if the principal proposes wage schedule (29), the distribution of the value of the outside option must satisfy

$$\frac{N'(x)}{N(x)} = \frac{V'(A_M)}{V(A_M)},$$

(30)

which means that $N(x)$ must be an exponential function that $N(x) = B \exp(\alpha x)$ on $[x_1, x_2]$, where $\alpha = \frac{V'(A_M)}{V(A_M)}$. Since $N(x_1) = 0$ and $N(x_2) = 1$, there are $x_1 = -\infty$ and $B = \exp(-\alpha x_2)$. Therefore, the value of the outside option must be distributed on $(-\infty, x_2]$ according to cdf $N(x) = \exp(\alpha(x - x_2))$ for the equilibrium to be semi-efficient.\(^{10}\) We formally state this result in the following Proposition.

**Proposition 4** The necessary condition for the existence of semi-efficient equilibrium is that outside option is distributed on $(-\infty, x_2]$ according to the following cumulative distribution function

$$N(x) = \exp(\alpha(x - x_2)).$$

(31)

When the distribution of the value of the outside option follows (31), the equilibrium is semi-efficient if the principal imposes a two-part wage schedule (29) with

$$A_M = \arg(V'(\cdot) = \alpha V(\cdot)).$$

(32)

**Proof.** It remains to show (32), but it follows from (30). Q.E.D.

It might be interesting to compare the results in Corollary 2 and Proposition 4 to the findings of Rochet and Stole (2002). They have discovered that the introduction of random participation can improve the allocative efficiency in the sense that random participation lifts the quality supplied to each type of consumers and moves it closer to the efficient level. The findings in Corollary 2 and Proposition 4 take us one step further and indicate that effort provision actually is likely to be close to the efficient level, especially when the principal is sufficiently risk averse.

However, why should the distribution of horizontal agent heterogeneity be exponential for efficient effort supply to principals with finite risk aversion? Apparently, the best way to design a semi-efficient contract is to make all ac-

---

\(^{10}\)The semi-efficient equilibrium is defined as that each active agent’s effort level in equilibrium is efficient but an agent’s participation decision can be inefficient.
tive agents a residual claimant of profits. This imposes a restriction on the horizontal heterogeneity of agents; that is, the semi-elasticity of the distribution of \( N(\cdot) \), defined as \( N' / N \), must be constant. Only exponential distribution can meet this restriction.

It is worth to notice that while wage schedule (29) implies that the agent provides efficient effort subject to participation, the participation decision is not efficient as long as \( A_M \) is positive. The following proposition shows that an increase in risk aversion of the principal leads to a decrease in \( A_M \), i.e. risk aversion of the principal leads to more efficient participation decision on the side of the agent.

**Proposition 5** Assume that principal \( i \) is more risk averse than principal \( j \) and the sufficient condition of Proposition 4 holds. Define \( A^k_M \) as the solution to (32) for principal \( k \in \{i,j\} \). Then \( A^i_M \leq A^j_M \).

**Proof.** According to Definition 1 there exists an increasing, concave and differentiable function \( \varphi(\cdot) \) such that \( V_i = \varphi(V_j) \). Recalling our normalization \( V_k(0) = 0 \), one obtains \( \varphi(0) = 0 \). The concavity of \( \varphi(\cdot) \) and \( V_j(0) = \varphi(0) = 0 \) yield \( \varphi(V_j(A^i_M)) \geq \varphi(V_j(A^j_M))V_j(A^i_M) \). Applying this inequality and (32), we have

\[
\frac{V_j'(A^i_M)}{V_j(A^i_M)} = \frac{V_i'(A^i_M)}{V_i(A^i_M)} = \frac{\varphi'(V_j(A^i_M))V_j'(A^i_M)}{\varphi(V_j(A^i_M))} \leq \frac{V_j'(A^j_M)}{V_j(A^j_M)}.
\]

Since \( \frac{V_j'(\cdot)}{V_j(\cdot)} \) is a decreasing function due to the concavity of \( V_j(\cdot) \), the above inequality implies \( A^i_M \leq A^j_M \). Q.E.D.

If the principal’s utility is of CARA form, i.e.

\[
V(z) = \frac{1 - \exp(-Rz)}{R},
\]

then (32) can be solved explicitly to obtain:

\[
A_M = \frac{1}{R} \ln(1 + \frac{R}{\alpha}).
\]

As predicted by Proposition 5, \( A_M \) in (34) decreases monotonically from \( 1/\alpha \) to zero as \( R \) increases from zero to infinity, indicating that increase in the principal’s risk aversion leads to efficiency gains and, as \( R \) tends to infinity, the optimal participation converges to the efficient one.
3.2 Common agency

This subsection extends the analysis in the previous subsection by allowing the agent to be hired by one of two principals; i.e., we have a common agency problem. Both principals are assumed to have the same production function, \( \pi \), and the same utility function as in the single principal case. Let us consider a model in the spirit of Hotelling model: the two principals are located at the ends of a linear city with a length equal to one, while the agent is randomly located along the Hotelling line. Thus, we can interpret \( x^i \) in (1) as the distance of an agent from principal \( i \), where superscript \( i \in \{ L, R \} \) indexes the principals, and \( k \) can be interpreted as marginal transportation cost. Assume that the principal’s profit and the agent’s cost of effort satisfy the following condition,

\[
\max_e (\pi(e) - te)) > \frac{k}{2} + \kappa, \quad \forall t \tag{35}
\]

where

\[
\kappa \equiv \arg \left( \frac{V'(\cdot)}{V(\cdot)} = \frac{N'(\frac{1}{2})}{2kN(\frac{1}{2})} \right). \tag{36}
\]

Assumption (35) simply says that the total surplus, ignoring transportation cost, is sufficiently large for positive gains of trade to exist, irrespective to the location of the agent on the Hotelling line. It guarantees the existence of symmetric equilibrium at which the agent provides positive effort with probability one.

Following (3), the agent’s surplus function from the relationship with principal \( i \) is \( s^i(t) = \max_e (w^i(e) - te) \). Given the surplus functions, the probability that the type-\( t \) agent takes the contract from the left end principal is:

\[
Q^L(s^L, s^R) = N(\min \left\{ \frac{s^L(t)}{k}, \frac{1}{2} + \frac{s^L(t) - s^R(t)}{2k} \right\}).
\]

Therefore, given surplus \( s^R(t) \), the left principal determines her wage strategy by maximizing her expected utility

\[
\max_{t_2} \int_{t_1}^{t_2} V(\pi(e) - s^L(t) - te))Q^L(s^L, s^R) f(t) dt.
\]

Clearly, the only difference between this program and (24) is \( N(s) \) in (24) has been replaced by \( Q^L(s^L, s^R) \). Then, given \( s^R \), the first-order condition for the optimal solution to the program is the same as (26) with proper notation changes.
Proposition 6 Let condition (35) be satisfied. Then the necessary and sufficient conditions for the existence of a symmetric equilibrium with positive effort supply are that each principal adopts the following wage schedule:

\[ w(e) = \pi(e) - \kappa. \] (37)

Net profit \( \kappa \) declines as the principals become more risk averse.

Proof. Necessity: Without loss of generality we assume that the agent works for the left principal. Since the provision of effort is efficient, there is \( \lambda(t) = 0 \) for all \( t \in [t_1, t_2] \) from the second corresponding equation of (26). Thus, \( \lambda'(t) = 0 \) and the first corresponding equation of (26) implies

\[ V(\cdot) \frac{\partial Q^L(s^L, s^R)}{\partial s^L} = V'(\cdot)Q^L(s^L, s^R). \] (38)

On the other hand, the first-order condition of agent’s optimization requires \( w'(e) = t \) and efficiency leads to \( \pi'(e) = t \). Therefore, the wage schedule must be in a form of \( w(e) = \pi(e) - A_D \). To determine \( A_D \), we notice that

\[ Q^L(s^L, s^R) = N\left(\frac{1}{2}\right), \quad \frac{\partial Q^L(s^L, s^R)}{\partial s^L} = \frac{1}{2k}N'(\frac{1}{2}) \] (39)

at symmetric equilibrium. Then, (38) yields \( A_D = \kappa \).

Sufficiency: There is \( \pi'(e) = t \) if both principals impose wage schedule (37). The specification (36) ensures that (38) valid. Thus, the first-order conditions for both principals are satisfied and the equilibrium is efficient. It remains to check that given tariff (37) the agent will always work for one of the principals. To show this, note that the agent’s surplus is equal to \( \max_e (w(e) - te) = \max_e (\pi(e) - \kappa - te) > \frac{k}{2} \). Since the agent travels a distance no more than \( \frac{1}{2} \), he definitely will accept the offer from one of the principals.

The monotonic decrease of \( \kappa \) in the principals’ risk aversion can be proven in the same way as the proof of Proposition 5. Hence, it is omitted. Q.E.D.

Proposition 6 states that the agent is always employed by one of the principals and is made a residual claimant of the profits. At equilibrium he chooses his employee at random with equal probability. Proposition 6 echoes the finding that duopolists impose a cost-based two-part tariff in nonlinear price competition by Armstrong and Vickers (2001), and Rochet and Stole (2002). Although the risk aversion of the principals does not affect the efficiency of the

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economy, it does change the income distribution. For CARA one can solve for, \( \kappa \) explicitly to obtain:

\[
\kappa = \frac{1}{R} \ln(1 + \frac{2kN(\frac{1}{2})}{N'(\frac{1}{2})R}).
\]

Obviously, the profit of the principals, \( \kappa \), declines in \( R \) as predicted by Proposition 6 and reaches zero when \( R \) is infinite. Therefore, the agent is better off as the principals become more risk averse.

4 Conclusions

This paper has developed a model that focuses on the effects of principals’ risk aversion on the optimal work effort supplied by an agent with an unobservable cost of effort. Although it features principals facing a single agent, extension to any finite set of agents is straightforward. While the model builds on the scenario of principals hiring an agent, the theory and its results are directly applicable to other screening problems of hidden information. We started by revisiting the standard Mussa-Rosen (1978) model and showed that an increase in risk aversion on the principal’s side monotonically pushes the provision of effort toward the efficient level. Intuitively, this happens because risk aversion makes the principal put a greater weight on avoiding profit uncertainty, particularly the worst possible scenario — a failure to make a hiring. Risk aversion also reduces the chance that the contracting results in a pooling equilibrium. We then extended the base model to allow for random participation, paralleling Rochet and Stole’s (2002) extension to the Mussa-Rosen (1978) model. We have established conditions for the optimal wage schedule leading the agent to be the residual claimant on the profits under both single principle and common agency regimes. While under the common agency the conditions are quite general and simply require that the unit transportation cost is sufficiently low to allow for full participation, the efficient wage schedule in the case of single principle requires quite a specific distribution of participation costs. In both cases the net profits of the principal(s) decrease in their risk aversion. Such a decrease improves efficiency through more efficient participation decision in the case of a single principal, however, it only affects the distribution of rents between the principals and the agent in the case of common agency. Generally, our analysis indicates that risk aversion on the principal’s side plays a positive role in improving either productive efficiency or agent’s welfare.
Appendix

In this Appendix we formulate and prove Lemma 1.

**Lemma 1** Consider two Cauchy problems for ordinary differential equations

\[ y'_i(t) = f_i(y_i, t), \ y_i(t_1) = y^*, \quad i \in \{1, 2\} \quad (40) \]

where \( f(\cdot, t) \in C^1(R) \) and \( f(y, \cdot) \) is continuously differentiable in \( (t_1, t_2) \) and continuous on \( [t_1, t_2] \). Let \( y_i(t) \) denote the solution of (40). If \( f_1(y, t) \geq f_2(y, t) \) for all \( (t, y) \in [t_1, t_2] \times R \), then \( y_1(t) \geq y_2(t) \) for \( \forall t \in (t_1, t_2) \).

Intuitively, this lemma is a version of single crossing property, which states that if two functions start at the same point and the first has a steeper slope, then the first function always remains above the second.

**Proof.** Define \( t_m \equiv \inf\{t \in (t_1, t_2) : y_1(t) \leq y_2(t)\} \). Note that set \( I = \{t \in [t_1, t_2] : y_1(t) \leq y_2(t)\} \) is bounded and not empty because \( t_1 \in I \). Expand \( y_1(t) \) and \( y_2(t) \) in an area close to \( t_1 \) to obtain

\[
\begin{align*}
y_1(t) &= y^* + f_1(y^*, t_1)(t - t_1) + o(|t - t_1|), \\
y_2(t) &= y^* + f_2(y^*, t_1)(t - t_1) + o(|t - t_1|).
\end{align*}
\]

Since \( f_1(y^*, t) \geq f_2(y^*, t) \), there exists \( \delta > 0 \) such that

\[
y_1(t) \geq y_2(t)
\]

for \( \forall t \in [t_1, t_2] \cap [t_1, t_1 + \delta] \). Since \( y_1(\cdot) \) is continuous and \( t_m \) is defined as the smallest \( t \) for which (41) is reversed, one can conclude that

\[
y_1(t_m) = y_2(t_m) \equiv y_m.
\]

Now we prove that \( t_m = t_1 \). Let us assume to the contrary that \( t_m > t_1 \) and expand \( y_1(t) \) and \( y_2(t) \) in an area close to \( t_m \) to obtain

\[
\begin{align*}
y_1(t) &= y_m + f_1(y_m, t_m)(t - t_m) + o(|t - t_m|), \\
y_2(t) &= y_m + f_2(y_m, t_m)(t - t_m) + o(|t - t_m|).
\end{align*}
\]

Since \( t - t_m < 0 \), there is \( \eta > 0 \) such that \( y_1(t) \leq y_2(t) \) for all \( t \in (t_m - \eta, t_m) \). Therefore, \( y_1(t) \leq y_2(t) \) holds for all \( t \in T = (t_1, t_2) \cap (t_m - \xi, t_m) \neq \emptyset \), where \( \xi \equiv \min\{\eta, t_m - t_1\} \). Because \( t < t_m \) if \( t \in T \), we get conclusions contradicting the definition of \( t_m \). Q.E.D.

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