A Low Complexity Frequency-domain Approach to SIMO System Identification

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Abstract— **With a rapidly changing channel in mobile communications, there is a scarcity of data samples, posing a challenge to reliable system identification. To address the problem, this paper presents a low complexity frequency-domain approach to blind single-input multiple-output (SIMO) system identification. The proposed approach is straightforward in concept and takes advantage of the computational power of FFT (Fast Fourier Transform). As a result, the new method is very efficient and effective for short data sequences, for which second-order statistics (SOS) based subspace methods suffer performance deterioration. Therefore, the proposed approach is a desirable alternative to SOS-based subspace methods to achieve good performance when data sequence is inevitably short in certain practical applications.**

I. Introduction

Multiple finite impulse response (FIR) channels driven by a common source or oversampling the received signal of an unknown linear time-invariant (LTI) channel [1] constitutes a single-input multiple-output (SIMO) system. Blind identification of SIMO systems, where the goal is to estimate the system (channel) coefficients based only on system outputs as the system input is unknown or inaccessible, has been an active and important research area due to its practical applications in wireless communications, for example, in the case of advanced receivers. Since the pioneering work [2], [3], the study of second-order statistics (SOS) based blind system identification has developed rapidly. Among the existing SOS-based schemes, subspace methods have gained popularity with the representative work [4] (see an overview of subspace methods [5], [6] and the references therein.)

The subspace methods exploit the orthogonality between the noise subspace, which is obtained based on the covariance matrix of system outputs, and the signal subspace, which is spanned by the columns of a filtering matrix formed by system impulse response. With subspace methods, closed-form identification solutions are obtained, making them computationally attractive. However, one prominent shortcoming of subspace algorithms is unsatisfactory performance when data sample size is relatively small. In this paper, we aim to overcome this drawback. Specifically, we address the problem of identifying the coefficient vector of a SIMO FIR system when output data sequence is very short.

A novel low complexity frequency-domain approach for blind SIMO system identification is developed. The new approach takes advantage of the computational efficiency of FFT (Fast Fourier Transform) and is straightforward in concept. More importantly, in contrast to SOS-based subspace methods, the proposed algorithm works well for short data sequences. This is particularly useful in a fast changing environment in mobile communications where only a few data samples are available.

This paper is organised as follows. Section II describes the SIMO system model under consideration and presents the problem statement. Section III proposes an FFT-based frequency-domain approach to blind SIMO system identification. In Section IV, a simulation example is given to compare the performance of the proposed approach (with only five output samples used) and the subspace method (with ten times more (50) output samples). Concluding remarks appear in Section V.

II. The SIMO System Model

This paper considers the following discrete-time LTI single-input L-output FIR system:

$$x_m(n) = s(n) * h_m(n) + w_m(n), \quad m = 1, 2, \cdots, L \qquad (1)$$

where $x_m(n) \in \mathcal{C}$ is the m-th channel output at time n with $\mathcal{C}$ denoting the set of complex numbers, $s(n) \in \mathcal{C}$ is the common input signal, $h_m(n) \in \mathcal{C}$ is the unknown FIR of the channel m, and $w_m(n) \in \mathcal{C}$ the additive noise at the channel m. The symbol $*$ in (1) denotes convolution. It is assumed that the maximum order of the L channels is M.

Suppose that the number of available output samples of each channel is N_s and N_s is small. Then the minimum length required of the input sequence $s(n)$ to generate N_s output samples is $N_s - M$ [7], where $N_s \geq M + 1$. Assume that the linear complexity of $s(n)$ is $N_s - M$, where we adopt the definition of linear complexity given in [5], i.e., the linear complexity of a sequence $\{y_k\}_{k=0}^{n}$ is defined as the smallest value of c for which there exists $\{\lambda_j\}$ such that

$$y_i = -\sum_{j=1}^{c} \lambda_j y_{i-j}, \quad i = c, \cdots, n$$

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$$\boldsymbol{\mathcal{H}}_m = \begin{bmatrix} h_m(0) & & & & & & \\ h_m(1) & h_m(0) & & & \mathbf{0} & & \\ & \ddots & & & & & \\ h_m(M) & \cdots & h_m(0) & & & & \\ & h_m(M) & \cdots & h_m(0) & & & \\ & & h_m(M) & \cdots & h_m(0) & & \\ & & & \ddots & & & \\ & \mathbf{0} & & & & h_m(M) & h_m(M-1) \\ & & & & & & h_m(M) \end{bmatrix} \in \mathcal{C}^{N_s \times (N_s - M)}$$

$$\boldsymbol{F}_m = \begin{bmatrix} X_m(0) & X_m(0) & X_m(0) & \cdots & X_m(0) \\ X_m(1) & X_m(1)V & X_m(1)V^2 & \cdots & X_m(1)V^M \\ \vdots & & \vdots & & \vdots \\ X_m(N-1) & X_m(N-1)V & X_m(N-1)V^{2(N-1)} & \cdots & X_m(N-1)V^{M(N-1)} \end{bmatrix} \in \mathcal{C}^{N \times (M+1)} \text{ with } V = e^{-j2\pi/N}$$

The SIMO system in (1) can be written in the vector-matrix form as

$$\mathbf{x}_m = \boldsymbol{\mathcal{H}}_m \boldsymbol{s} + \mathbf{w}_m$$

where the matrix $\boldsymbol{\mathcal{H}}_m$ is shown on the top of the page and

$$\begin{aligned} \mathbf{x}_m &= [x_m(0) \;\; x_m(1) \;\; \cdots \;\; x_m(N_s-1)]^T \\ \boldsymbol{s} &= [s(0) \;\; s(1) \;\; \cdots \;\; s(N_s-M-1)]^T \\ \mathbf{w}_m &= [w_m(0) \;\; w_m(1) \;\; \cdots \;\; w_m(N_s-1)]^T \end{aligned}$$

With the SIMO model (1), the blind system identification problem considered here can be described as: *given the outputs* $\{\mathbf{x}_m, \; m = 1, \cdots, L\}$*, determine the system coefficients* $\{h_m(n), \; m = 1, \cdots, L, \; n = 0, 1, \cdots, M\}$.

III. FFT-based Frequency-domain SIMO System Identification

It is easy to show that in the absence of noise, the outputs of the SIMO system in (1) satisfy

$$x_i(n) * h_j(n) = x_j(n) * h_i(n) \tag{2}$$

where $1 \leq i, j \leq L, \; i \neq j$. This so-called cross-relation property has been used in the time-domain formulation of blind channel identification, e.g., [8], [9], however, it has not yet been applied in the frequency domain. In this section, we utilise this property and derive an FFT-based frequency-domain approach to SIMO system identification.

Since the length of the output sequence $x_m(n)$, $m = 1, \cdots, L$, is N_s, the duration of the convolved signal in (2) is N_s+M. Taking the N-point Discrete Fourier Transform (DFT) on both sides of (2), where $N \geq N_s + M$, we have

$$X_i(k)H_j(k) = X_j(k)H_i(k), \quad k = 0, 1, \cdots, N-1 \tag{3}$$

where $1 \leq i, j \leq L, \; i \neq j$, and $X_m(k)$ and $H_m(k)$ represent the frequency-domain samples of $x_m(n)$ and $h_m(n)$, respectively, with $m = i, j$. As the DFT sequence $H_m(k)$ is defined as

$$H_m(k) = \sum_{n=0}^{N-1} h_m(n) e^{-j2\pi kn/N} = \sum_{n=0}^{M} h_m(n) e^{-j2\pi kn/N}$$

where zero padding is applied to $h_m(n)$ for $M < n \leq N-1$, equation (3) can be rewritten as

$$[-\boldsymbol{F}_j \;\;\; \boldsymbol{F}_i] \begin{bmatrix} \mathbf{h}_i \\ \mathbf{h}_j \end{bmatrix} = \mathbf{0} \tag{4}$$

where the matrix $\boldsymbol{F}_m$, $1 \leq m = i, j \leq L$, is shown on the top of the page and

$$\mathbf{h}_m = [h_m(0) \;\; h_m(1) \;\; \cdots \;\; h_m(M)]^T, \; 1 \leq m = i, j \leq L$$

Define the matrix $\boldsymbol{F}_{i,j} \in \mathcal{C}^{N \times L(M+1)}$

$$\boldsymbol{F}_{i,j} = [\mathbf{0} \; \cdots \; \mathbf{0} \; -\boldsymbol{F}_j \; \mathbf{0} \; \cdots \; \mathbf{0} \; \boldsymbol{F}_i \; \mathbf{0} \; \cdots \; \mathbf{0}]$$

where the i-th entry is $-\boldsymbol{F}_j$ and the j-th entry is $\boldsymbol{F}_i$, $1 \leq i, j \leq L$, $i \neq j$. When all the L channels are taken into consideration, in the absence of noise, from equation (4), we can form the following equation:

$$\boldsymbol{F}\mathbf{h} = \mathbf{0} \tag{5}$$

where

$$\boldsymbol{F} = \left[\boldsymbol{F}_{1,2}^T \; \boldsymbol{F}_{1,3}^T \; \cdots \; \boldsymbol{F}_{1,L}^T \; \boldsymbol{F}_{2,3}^T \; \cdots \; \boldsymbol{F}_{2,L}^T \; \cdots \; \boldsymbol{F}_{L-1,L}^T\right]^T$$

$$\mathbf{h} = \left[\mathbf{h}_1^T \;\; \mathbf{h}_2^T \;\; \cdots \;\; \mathbf{h}_L^T\right]^T \in \mathcal{C}^{L(M+1)}$$

The dimension of the matrix $\boldsymbol{F}$ is $\frac{NL(L-1)}{2} \times L(M+1)$. Hence the system coefficient vector $\mathbf{h}$ can be identified by solving the linear equations described by (5). Clearly, to expedite computation, the size N of DFT taken on the output data should be chosen such that powerful FFT techniques are applied.

Theorem: *The solution* $\hat{\mathbf{h}}$ *to the equation*

$$\boldsymbol{F}\hat{\mathbf{h}} = \mathbf{0} \tag{6}$$

$$\boldsymbol{H}_m = \begin{bmatrix} H_m(0) & H_m(0) & H_m(0) & \cdots & H_m(0) \\ H_m(1) & H_m(1)V & H_m(1)V^2 & \cdots & H_m(1)V^M \\ \vdots & & \vdots & & \vdots \\ H_m(N-1) & H_m(N-1)V & H_m(N-1)V^{2(N-1)} & \cdots & H_m(N-1)V^{M(N-1)} \end{bmatrix} \in \mathcal{C}^{N\times(M+1)}$$

uniquely (up to a constant scalar) determines the system coefficients $\{h_m(n),\ m=1,\cdots,L,\ n=0,1,\cdots,M\}$ *from the noiseless outputs* $\{\mathbf{x}_m,\ m=1,\cdots,L\}$*, if the polynomials* $H_m(z)=\sum_{n=0}^{M} h_m(n)z^{-n}$ *do not share any common zero.*

Proof: Since the matrix $\boldsymbol{F}_m$ can also be expressed by

$$\boldsymbol{F}_m = \boldsymbol{S}_N \boldsymbol{H}_m$$

where the matrix $\boldsymbol{S}_N$ is composed of the N-point DFT $S(0), S(1), \cdots, S(N-1)$ of the input $s(n)$, that is,

$$\boldsymbol{S}_N = \begin{bmatrix} S(0) & & & \\ & S(1) & & \mathbf{0} \\ & & \ddots & \\ \mathbf{0} & & & S(N-1) \end{bmatrix} \in \mathcal{C}^{N\times N}$$

and the matrix $\boldsymbol{H}_m$ consists of the N-point DFT $H_m(0), \cdots, H_m(N-1)$ of the m-th channel coefficients $h_m(n)$, $1 \le m \le L$. The matrix $\boldsymbol{H}_m$ is given on the top of the page. Also,

$$\begin{aligned} \boldsymbol{F}_{i,j} &= \boldsymbol{S}_N[\mathbf{0}\ \cdots\ \mathbf{0}\ -\boldsymbol{H}_j\ \mathbf{0}\ \cdots\ \mathbf{0}\ \boldsymbol{H}_i\ \mathbf{0}\ \cdots\ \mathbf{0}] \\ &= \boldsymbol{S}_N \boldsymbol{H}_{i,j} \end{aligned}$$

The equation $\boldsymbol{F}\hat{\mathbf{h}} = \mathbf{0}$ is therefore equivalent to

$$\tilde{\boldsymbol{S}} \left[\boldsymbol{H}_{1,2}^T\ \boldsymbol{H}_{1,3}^T\ \cdots\ \boldsymbol{H}_{1,L}^T\ \boldsymbol{H}_{2,3}^T\ \cdots\ \boldsymbol{H}_{2,L}^T\ \cdots\ \boldsymbol{H}_{L-1,L}^T\right]^T \hat{\mathbf{h}} = \mathbf{0} \quad (7)$$

where

$$\tilde{\boldsymbol{S}} = \begin{bmatrix} \boldsymbol{S}_N & & & \\ & \boldsymbol{S}_N & & \mathbf{0} \\ & & \ddots & \\ \mathbf{0} & & & \boldsymbol{S}_N \end{bmatrix} \in \mathcal{C}^{\frac{NL(L-1)}{2}\times\frac{NL(L-1)}{2}}$$

As the input sequence $s(n)$ has linear complexity $N_s - M$, its DFT $S(k) \neq 0,\ k=0,1,\cdots,N-1$ (easy to prove by contradiction), which gives rise to the matrices $\boldsymbol{S}_N$ and $\tilde{\boldsymbol{S}}$ being nonsingular. It follows from equation (7) that

$$\left[\boldsymbol{H}_{1,2}^T\ \boldsymbol{H}_{1,3}^T\ \cdots\ \boldsymbol{H}_{1,L}^T\ \boldsymbol{H}_{2,3}^T\ \cdots\ \boldsymbol{H}_{2,L}^T\ \cdots\ \boldsymbol{H}_{L-1,L}^T\right]^T \hat{\mathbf{h}} = \mathbf{0}$$

which yields

$$[-\boldsymbol{H}_j\ \ \boldsymbol{H}_i] \begin{bmatrix} \hat{\mathbf{h}}_i \\ \hat{\mathbf{h}}_j \end{bmatrix} = \mathbf{0},\ 1 \le i,j \le L,\ i \neq j \quad (8)$$

By expanding the left side of (8), we have

$$\begin{bmatrix} -H_j(0)\hat{H}_i(0) + H_i(0)\hat{H}_j(0) \\ -H_j(1)\hat{H}_i(1) + H_i(1)\hat{H}_j(1) \\ \vdots \\ -H_j(N-1)\hat{H}_i(N-1) + H_i(N-1)\hat{H}_j(N-1) \end{bmatrix} = \mathbf{0}$$

Thus, $H_i(k)\hat{H}_j(k) = H_j(k)\hat{H}_i(k)$, $k=0,1,\cdots,N-1$, which leads to, for any $1 \le i,j \le L$,

$$H_i(z)\hat{H}_j(z) = H_j(z)\hat{H}_i(z) \quad (9)$$

Since polynomials $H_m(z)$, $m=1,\cdots,L$, do not share any common zero, and suppose channel $i=q$ has the maximum order M, with (9), it is easy to show that for channel q, $H_q(z)$ and $\hat{H}_q(z)$ have same zeros, i.e., $\hat{H}_q(z) = \alpha H_q(z)$, where α is a nonzero constant. Substituting this into (9) gives $\hat{H}_j(z) = \alpha H_j(z)$. That is, $\hat{\mathbf{h}}_m = \alpha \mathbf{h}_m$, $m=1,\cdots,L$. □

When noise is present in the SIMO system (1), the estimate $\hat{\mathbf{h}}$ of the system coefficient vector $\mathbf{h}$ is obtained by

$$\hat{\mathbf{h}} = \arg\min_{\|\mathbf{h}\|=1} \|\boldsymbol{F}\mathbf{h}\|^2 \quad (10)$$

Remarks:

(i) Many different methods can solve the minimisation problem in (10); for example, the optimal solution $\hat{\mathbf{h}}$ is given by the right singular vector associated with the smallest singular value of $\boldsymbol{F}$.

(ii) The FFT-based frequency-domain approach proposed in this section is straightforward in concept. It is worth indicating that the power of the proposed approach lies in, first, the efficiency of FFT in computing the key matrix $\boldsymbol{F}_m$, and second, handling a short data sequence of outputs. Because of the intrinsic structure of the identification matrix $\boldsymbol{F}$, when the length of system output sequence is small, for which the subspace methods perform poorly due to insufficient data statistics, not only is the proposed approach computationally efficient, but it also achieves satisfactory performance. It is shown by simulation example in the next section that for a 4-channel SIMO system, when the available output duration (of each channel) is as short as $N_s = 5$, the proposed method still yields good identification of system coefficients.

IV. Simulation

Computer simulations were run to compare the performance of the proposed approach with that of the SOS-based subspace method for situations where output data length is small. We used the single-input 4-output FIR system in [4] with channel order $M = 4$. The impulse response of each channel is given in the table below.

	$h_1(n)$	$h_2(n)$	$h_3(n)$	$h_4(n)$
$n=0$	-0.049+0.359i	0.443-0.0364i	-0.211-0.322i	0.417+0.030i
$n=1$	0.482-0.569i	1.0	-0.199+0.918i	1.0
$n=2$	-0.556+0.587i	0.921-0.194i	1.0	0.873+0.145i
$n=3$	1.0	0.189-0.208i	-0.284-0.524i	0.285+0.309i
$n=4$	-0.171+0.061i	-0.087-0.054i	0.136-0.19i	-0.049+0.161i

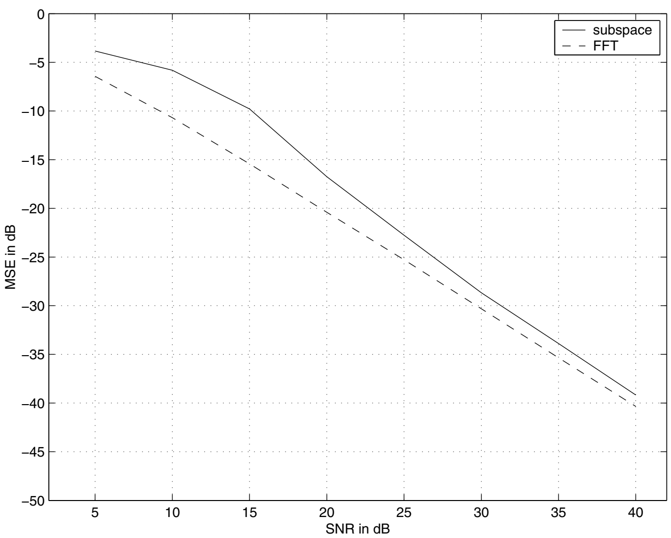

Fig. 1. Performance comparison of the subspace method [4] (with 50 output samples) and the FFT method (with 5 output samples)

White Gaussian noise was added to the outputs. We computed the mean-square-error (MSE) to be the performance measure:

$$\text{MSE (dB)} = 10\log_{10}\left(\frac{1}{T}\sum_{i=1}^{T}\|\hat{\mathbf{h}}_i - \mathbf{h}\|^2\right)$$

where T is the number of Monte Carlo runs ($T = 100$ was used), $\mathbf{h}$ is the true coefficient vector (with unit-norm), and $\hat{\mathbf{h}}_i$ is the estimated coefficient vector (with unit norm) from the i-th run.

To demonstrate the effectiveness of the proposed FFT approach when dealing with short data sequences, we used a total of five output samples in the simulation, while ten times more (50) output samples were used for the subspace method [4]. Fig. 1 compares the MSE (in dB) of the proposed frequency-domain approach and the subspace method for a range of SNR values. It can be seen that for the SNR between 5dB and 40dB, the proposed approach with ten times less output samples consistently outperforms the subspace method. Clearly, for such a short data sequence, the computational cost of the proposed approach is very low.

V. Conclusion

This paper presents a new low complexity frequency-domain approach to blind SIMO system identification. It resorts to the computational power of FFT, and for short data sequences, the proposed approach is both efficient in computation and effective in performance as opposed to SOS-based subspace methods, which are known to exhibit serious performance degradation with small data sample size. Therefore, the new approach is a desirable alternative to SOS-based subspace methods to achieve good performance when data sequence is inevitably short in certain fast varying environments.

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