UPSTREAM POWER LINE COMMUNICATION OVER DISTRIBUTION
TRANSFORMERS FOR AUTOMATIC METER READING

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<tbody>
<tr>
<td>AC</td>
<td>Alternating Current</td>
</tr>
<tr>
<td>A-D</td>
<td>Analog (to) Digital</td>
</tr>
<tr>
<td>AMI</td>
<td>Advanced Metering Infrastructure</td>
</tr>
<tr>
<td>AMR</td>
<td>Automatic Meter Reading</td>
</tr>
<tr>
<td>BPL</td>
<td>Broadband (over) Power Line</td>
</tr>
<tr>
<td>bps</td>
<td>bits per second</td>
</tr>
<tr>
<td>CPLD</td>
<td>Complex Programmable Logic Device</td>
</tr>
<tr>
<td>CT</td>
<td>Current Transformer</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current (or simply the constant offset in a signal)</td>
</tr>
<tr>
<td>DLC</td>
<td>Distribution Line Communications</td>
</tr>
<tr>
<td>DSBSC</td>
<td>Double Side Band Suppressed Carrier</td>
</tr>
<tr>
<td>EDF</td>
<td>Empirical Distribution Function</td>
</tr>
<tr>
<td>emf.</td>
<td>electromotive force</td>
</tr>
<tr>
<td>ENBW</td>
<td>Equivalent Noise Band Width</td>
</tr>
<tr>
<td>FPGA</td>
<td>Field Programmable Gate Array</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>IF</td>
<td>Intermediate Frequency</td>
</tr>
<tr>
<td>IGBT</td>
<td>Insulated Gate Bipolar Transistor</td>
</tr>
<tr>
<td>K-S</td>
<td>Kolmogorov-Smirnov</td>
</tr>
<tr>
<td>LV</td>
<td>Low Voltage (e.g. 240V)</td>
</tr>
<tr>
<td>MATLAB</td>
<td>MATrix LABoratory (registered trademark)</td>
</tr>
<tr>
<td>MLE</td>
<td>Maximum Likelihood Estimate</td>
</tr>
<tr>
<td>mmf.</td>
<td>magneto motive force</td>
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**List of Acronyms**

<table>
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<th>Acronym</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>MOV</td>
<td>Metal Oxide Varistor</td>
</tr>
<tr>
<td>MT</td>
<td>Main Terminal</td>
</tr>
<tr>
<td>MV</td>
<td>Medium Voltage (e.g. 22kV)</td>
</tr>
<tr>
<td>PCB</td>
<td>Printed Circuit Board</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Distribution Function</td>
</tr>
<tr>
<td>PLC</td>
<td>Power Line Communication</td>
</tr>
<tr>
<td>PLL</td>
<td>Phase Locked Loop</td>
</tr>
<tr>
<td>ppm</td>
<td>parts per million</td>
</tr>
<tr>
<td>Q-Q</td>
<td>Quantile-Quantile</td>
</tr>
<tr>
<td>RC</td>
<td>Resistor Capacitor</td>
</tr>
<tr>
<td>RHZ</td>
<td>Right Hand Zero</td>
</tr>
<tr>
<td>RMS</td>
<td>root mean square</td>
</tr>
<tr>
<td>SCR</td>
<td>Silicon Controlled Rectifier</td>
</tr>
<tr>
<td>SELV</td>
<td>Separated Extra Low Voltage</td>
</tr>
<tr>
<td>SPS</td>
<td>Samples Per Second</td>
</tr>
<tr>
<td>SWD</td>
<td>Sequential Waveform Distortion</td>
</tr>
<tr>
<td>TWACS®</td>
<td>Two-Way Automatic Communication system (registered trademark)</td>
</tr>
<tr>
<td>WiMAX</td>
<td>Worldwide interoperability for Microwave Access (registered trademark)</td>
</tr>
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ABSTRACT

Power line communication systems for automatic electricity meter reading which use signals capable of passing through the distribution transformer are attractive because the whole communication channel is utility owned and pre-exists. The performance of current commercial systems is limited to data rates of a few hundred bits per second (bps) which is insufficient for reading interval meters. This thesis addresses the question of whether or not this channel could be used for interval meters. Channel sounding equipment was developed to measure the upstream current transfer ratio and the design of this reliable and portable equipment is presented. It was used to sound six very different channels within La Trobe University's distribution system. The measurements are presented along with the derivation of parametric models of the channels. It is shown that all of the channels can be represented by one generic type model which is a non-minimum phase system with six zeros and five poles, one of the zeros and one of the poles being fixed and the others varying over limited frequency and damping factor ranges according to the particular channel characteristics.

Noise measurements on the medium voltage supply were conducted. These measurements were made in narrow bands across the spectrum of interest. It was shown that, irrespective of the nature of the probability distribution of the wideband noise, measurements within narrow enough bands have close to Gaussian distribution. A method for generating current signals within a meter was demonstrated and an achievable current signal level estimated. This, in combination with the channel models and the noise measurements enabled determination of the Shannon capacity limits for each of the channels. All channels exhibited Shannon limits above 10,000 bps with three
of the six channels above 30,000 bps. The implication of this finding is that the channel is suitable for remotely reading interval meters.

The thesis is a case study with the assumption is that the La Trobe distribution system is a good representation of utility owned systems but tests to confirm this assumption have not yet been performed.

Design of a complete communication system to use the channel for automatic meter reading was not attempted; however, a resonant circuit switched across the mains for meter endpoint signal generation was suggested. Operation of such a transmitter was studied in some detail and the new understanding, missing from existing literature, is presented.
STATEMENT OF AUTHORSHIP

Except where reference is made in the text of the thesis, this thesis contains no material published elsewhere or extracted in whole or in part from a thesis by which I have qualified for or been awarded another degree or diploma. No other person's work has been used without due acknowledgement in the main text of the thesis. This thesis has not been submitted for the award of any degree or diploma in any other tertiary institution.

Andrew Mackie

13/12/2010
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This work could not have been performed without the kindness and trust shown to me by the Buildings and Grounds staff. They generously gave me access to the co-generation plant and control room and to the main campus substation where our 22kV electricity supply enters the site, allowing me to install the interposing current transformers and other measurement equipment in those locations. Thanks to PRI Australia for donating two state of the art high voltage meters for me to donate to the buildings and grounds department.

Finally, thanks to you the reader for taking on the unquestionably hard and long task of having me describe and explain this research work to you.
I INTRODUCTION

1 Background

Around the world, and especially in Australia, electricity consumption has been rising for decades. Distribution businesses have had trouble upgrading their capacity to match the rise in demand [1]. Following widespread deregulation of the industry the situation has worsened and is now so bad in South Australia that the state’s utility has taken the desperate measure of appealing to the public for ideas [2]. Apart from the general upward trend in total consumption, the peak to average consumption ratio is also rising. The overall ratio has now reached a factor of 2.3 in South Australia (SA) and residential loads on their own have the even worse peak to average ratio of between 3.5 and 4 [3]. This means expensive infrastructure capacity necessary to meet peak demand spends much of the time idle or underused as evidenced by this quote from SA’s utility: "The distribution network capacity has to be almost three times larger to manage those few peak days, than is needed for the remainder of the year" [4]. The pressure on distribution infrastructure during these peaks is such that this electricity supply authority, like many others, is having to run their transformers well above the nameplate ratings (manufacturer’s recommended maximum capacities) [5]. Without new solutions the situation will only get worse; average demand is predicted to increase and the rate of increase in peak demand is predicted to exceed the rate of increase of average demand [3].

There are several important elements to a solution. Total consumption needs to be brought under control but, more importantly, the peaks in the load profile need to be smoothed. Total consumption reductions are best achieved by usage efficiency
improvements and increasing domestic customer's awareness of their load in real time. Smoothing the load profile can be brought about in many ways including embedded generation from customer owned grid connected solar modules (which generate most when demand is highest), utility controlled load management and improved price signalling to encourage customers to voluntarily reduce their load during peak periods. All of these hold some promise but the last, price signalling, is currently of most interest and was the catalyst for this research work. I published my thoughts on improving the communication of price signals to the customer [6] and described how this involves the use of interval metering. Interval metering with Automatic Meter Reading (AMR), described in detail in the next section has, for several years, been planned for universal deployment in Australia [7]. The Victorian Government is leading this deployment and has mandated Advanced Metering Infrastructure rollout to be completed by 2012 [8, 9]. Interval metering uses what the industry calls interval meters but these are also commonly referred to as “smart meters”.

2 Smart meters

Smart meters allow price signalling to the customer to, firstly, encourage suppression of demand during critical peaks, and secondly, to also allow collection of extra revenue during periods of high demand to offset the costs of capacity reserved for use during these periods. Smart meters are electronic meters, primarily for electricity although they can also be used to record and communicate water and gas consumption. They record consumption as a function of time and can be remotely read. These meters will be universal throughout Victoria when the AMI rollout is complete. They supersede accumulation meters which simply record how much product has been used since the meter was installed. This advance is very important, particularly for the electricity market, because with these meters it becomes possible to charge according to time of
use. Having the ability to raise the price when high demand is threatening to bring the network down (e.g. in Australia, when hot days result in enormous air conditioner loads), is the best way of bringing peak consumption under control. Consumers must be made aware of the fact that their consumption was very costly, when it occurred during a peak demand period, as soon as possible after the event. This will improve the likelihood of them modifying their consumption pattern. The current requirement is that readings should be available to the customer (and electricity retailer) no later than a day after consumption took place [10]. The metering time resolution must also be small enough to capture the peaks and the industry standard for electricity pricing and metering is a half hour interval [11]. These two requirements set the minimum data rate performance for the communication link for an Automatic Meter Reading (AMR) system.

3 Possibility of power line communication for smart meter AMR

The focus of this study is “over the transformer” communication for an AMR system. A back-of-the-envelope calculation shows that the AMR upstream communications link, to transmit the registers from a half hour interval meter once a day, does not require a very high data rate. The forty eight registers, each containing 10 bytes, could be sent in 80 minutes at a rate of just one bit per second. The energy used by a residential customer in a half hour with a 100Wh resolution would actually take nearer to one byte to represent rather than ten, so these figures have left a great deal of room for redundant error control information and no reduction has been allowed for data compression. This low data rate requirement suggests the possibility of having the signal from each meter pass through the distribution transformer before consolidation and re-transmission from the zone substation. However, if a large population of meters is to share the same medium, the received data rate is multiplied by at least the population size. With up to 10,000 houses supplied by a zone substation, and with each transmitting information simultaneously at
the rate of one bit per second, each substation will have an overall data rate of approximately 10,000 bits per second coming in spread over its Medium Voltage feeders.

4 The problem statement

The metering industry has a requirement for an optimum communications solution for smart meter AMR applications. A communication system using the existing path between the zone substation and the meters (that is the power lines and distribution transformers) has the advantage that no infrastructure is required between the meter transceiver and the transceiver at a central utility controlled site. This advantage certainly translates into low on-going running costs and may also give low total equipment cost. This type of communication system presently exists and the next chapter will describe some examples which use simple signalling techniques with commensurately poor data rate performance. The problem is that the performance is limited to overall data rates less than 200 bits per second, insufficient for the interval meter AMR application. An important gap in available technology, probably resulting from a gap in detailed knowledge about the characteristics and capability of complete distribution networks as channels for communication, is therefore evident. An optimum implementation would allow minimal cost automatic reading of smart meters, as well as being useful for the remotely deployed devices involved in intelligent distribution grid management. The question which this thesis addresses is therefore: can the complete distribution network, including the distribution transformer be used, from a technical standpoint, for upstream interval meter data recovery? My hypothesis is that suitable methods for signal generation exist and the channel behaviour with respect to those signals does allow information transmission at the required rate.
5 Aim and scope

The aim of this research work is to attempt to fill some of the gaps in the knowledge of how this "through transformer" channel can be used and to either disprove or find confirmatory evidence for the hypothesis. The scope is purposefully limited to upstream communication with the transmitter at a mains low voltage end point and the receiver in the zone substation, is also purposefully limited to current signals being generated at the transmitter and current signals being received on the substation medium voltage supply. A further restriction which is caused by circumstance is that this research will be in the form of a case study. We have, at La Trobe University our own electricity distribution system including a substation which distributes power from the bulk electricity supply through medium voltage feeders to fourteen distribution transformers situated around the university campus. Six channels, each through a different distribution transformer, were studied.

6 Thesis overview

Having established the motivation for this work created by the sudden acceleration of demand for AMR and placed the work in context by summarising the state of the art in the industry, chapter three will describe some preliminary work in which AMR systems described in the literature are used as a starting point for developing a channel sounding system. The literature associated with one of these systems, termed "SWD", omitted to mention some important behaviour which was discovered during this preliminary work and chapter four details a theoretical analysis of this behaviour.

Chapter five details the main research methods used in this work, specifically the design and operation of the channel sounding system. To characterise the channel, noise measurements were made alongside the transfer function attenuation and phase
measurements. The noise measurements were made in narrow frequency bands and this form of description conceals some important characteristics. This is explained and justified in chapter seven but for this task a powerful method of measuring the fit of a set of data to a normal distribution was first required. In chapter six the reasons why the Anderson-Darling test was chosen for testing the fit to normality are demonstrated.

In chapter eight the results of the channel sounding experiments are presented, they take the form of six parameterised transfer functions and a noise spectral density plot. In chapter nine the results are discussed and the channel capacity calculated. The final chapter draws the conclusion that the hypothesis, stated at the end of section 4 above, is supported by the research work.
II AN OUTLINE OF EXISTING AMR SYSTEMS

1 Introduction

This chapter describes existing practice in the field of automatic meter reading (AMR). In the first section shortcomings of common AMR systems are outlined, in particular, why many of them are not suitable for use with the new smart meters. The next section focuses on AMR systems which use Power Line Communications (PLC) and summarises pertinent points from the literature. The possibility of improvements to PLC systems formed the impetus for embarking on the present work and existing systems formed the starting point for the work. Finally, for completeness, the viable alternatives to PLC for the AMR application are briefly described.

2 The new smart meter AMR requirement

For a surprisingly long time electromechanical Ferraris disc meters were found to be perfectly satisfactory for metering electricity consumption. They have always been very cheap to make (distribution businesses still pay less than $50 for one of these meters) and with low electricity costs, and demand easily met by a robustly built distribution system, there was little incentive to upgrade to smart meters. Nowadays deregulated electricity industries all over the world are unable to lavish resources maintaining the supply and distribution network capacity well above normal demand. The business case for rolling out new smart meter networks to smooth demand has now been shown to makes sense [12] and this activity has started in jurisdictions all over the world, with Victoria in Australia amongst the leaders. These smart meters require a suitable method of communication in order for them to be read rapidly and frequently so that up-to-date price signals can be presented to the customer. This change in the metering environment
has not happened overnight, for at least the last two decades problems solvable by more intelligent metering have been recognized. In that time small numbers of smart meters, some with communication capabilities, have been deployed [13]. Some of these meters performed load profiling in which a small proportion of customers have their consumption patterns closely scrutinised. The small number of customers metered like this has been sufficient to allow prediction of the overall demand pattern. This accounts for some of the smart meters deployed to date, others have been used to meter difficult customers, for example, customers at remote locations for which manual reading is expensive and customers in rental properties with a high turnover requiring frequent meter readings. With such a small number of smart meters deployed it has proved economic to use quite crude and expensive AMR technologies. For example, in Australia in the mid-90s it was common practice to use fixed telephone line modems to automatically read some meters. Towards the end of the 1990s GSM modems were being used and now GPRS is the equivalent technology. It is obvious that providing each individual meter with its own public data carrier modem, with the associated carrier costs and infrastructure updates every decade, would be prohibitively expensive for mass roll outs of smart meters. However, other technologies have also been used for communicating with small numbers of smart meters in the past and some of these do have relevance to a large scale AMR communication system. The following paragraphs will examine such systems.

3 Pre-existing Power Line Communication (PLC) systems

3.1 Ripple

Although it is not strictly a system which could be used for transmitting readings from a meter back to the utility, it is well worth looking at what are termed “ripple control” systems. These have been around for more than fifty years and are well established
throughout the world. In Australia they have been used mainly for control of hot water services. Hot water services are an ideal controlled load, the customer does not mind when in the night his hot water is heated as long as he has a full hot tank in the morning. The utility can use these loads to control demand and distribute it geographically as they wish during off-peak periods. Using electricity to heat water is undesirable because cheaper cleaner alternatives are available, but it has been standard procedure in Australia for decades. The utilities have direct control over a large number of customer’s hot water services via meters with separate controlled load circuits. These circuits are switched remotely by the utility using ripple control signals. Ripple control signals are amplitude shift keyed audio frequency signals (in Australia between 167Hz and 1050Hz) [14] injected on to the Medium Voltage (MV) side of transformers in a zone substation. These small (of the order of 1%) audio frequency variations in the mains voltage get broadcast to all distribution transformers downstream of the feeder on which injection took place. The frequency is low enough that the signal passes over the distribution transformers and onto the Low Voltage (LV) wiring and so is received at all customers' houses connected to those distribution transformers. Reception is simply a matter of band pass filtering the mains voltage signal followed by detection and decoding. In the distant past all ripple signals were generated by rotating machines which limited them to frequencies related to the mains fundamental frequency, 50 or 60Hz and, interestingly, the receivers were also electro-mechanical originally. Nowadays semiconductor devices are used to switch a reactive load across the supply at the transmit end and the source impedance of the supply transformer results in that transformer's output voltage being modulated. The generation and reception of ripple control signals is very well understood, has been established in practice on real networks for decades and has been found to make economic sense. Ripple signals, or similar broadcast voltage signals generated at a zone substation, therefore provide a natural solution to downstream
communication. Upstream communication however is a much more difficult proposition.

3.2 Sequential waveform distortion

Sequential Waveform Distortion (SWD) is the name given to a two way PLC communication system invented in the late 1980s. It replaced the amplitude shift keying used by ripple control signalling with phase shift keying of the audio tone. Apart from the modulation technique, the downstream transmitter was similar to a solid-state ripple transmitter but did not need to be as powerful because the receivers could be made more sensitive. The interesting part of the SWD system was the fact that it performed upstream communication. To achieve this, the meters used an inductor and capacitor in series switched across the mains by a triac as shown in Figure 2.1 below:

![SWD circuit within an Electricity Meter](image)

**Figure 2.1** SWD transmitter circuit at a meter.

This is part of the same circuit that is used for downstream communication but on a much smaller scale. Switching the triac results in modulation of the supply current to the
meter. These current signals can be detected on the MV feeder at the zone substation because the supply current passes through the distribution transformer provided the frequency content of the signal is low enough. System trials have indicated that it would be possible to achieve “extremely reliable upstream communication” [15].

3.2.1 Details of operation of an SWD transmitter

The SWD technique was first conceived by the Australian engineer Peter Foord and an initial attempt to make a commercial communication system was made by Nilsen Industrial Electronics in collaboration with the Electricity Trust of South Australia [16].

The principle of operation is simple, when the triac fires a current begins to flow and the rate of change of this current is determined by the difference between the voltage on the capacitor immediately before the switching event and the instantaneous mains voltage at that time (as well as the inductance value). With reference to Figure 2.1, if the mains voltage was larger than, but with the same polarity as, the voltage on the capacitor at the time of the switching event, current will flow into the capacitor. The energy stored in the inductor’s magnetic field will begin to increase in proportion to the current. If the mains voltage were to remain fixed in value after the switching event then the energy stored in the inductor would increase and reach a peak when the current was at its maximum. At this moment there is no voltage across the inductor and the capacitor voltage has increased to become equal to the mains voltage. The capacitor now contains all the energy it started with plus the energy associated with the increase in its voltage. As current continues to flow the capacitor continues to charge further increasing its voltage. The energy it is now absorbing is coming from the inductor because the continued increase in capacitor voltage results in a reverse voltage building across the inductor which collapses its magnetic field. When the inductor field has reduced to zero all of the additional energy which was extracted from the mains and temporarily stored in the
inductor is now residing in the capacitor. With the capacitor charge at its peak, the voltage across the inductor is as high as it was at the switching event (but with the opposite polarity). The next thing that might be expected to happen is for current to flow back out of the capacitor, discharging it as some of its energy flows back into the inductor and into the mains. However, the fact that the current in the series circuit has reached zero (for the first time since the switching event) means that the triac automatically switches off and breaks the natural resonance cycle. Determining the exact shape of the pulse will be covered in a subsequent chapter, but if the assumption is made that the change in mains voltage over the duration of the pulse was negligible then the pulse shape will be close to a half sine with a width determined by the circuit’s resonant frequency.

After the triac switches off a charge is left on the capacitor. The relationship between the voltage caused by this charge and the voltage of the mains at the instant that the Triac next fires determines the polarity and the amplitude of the subsequent current pulse. If it can be arranged that the polarity of the pulses alternates in direction, then a current waveform with a strong fundamental component at half the firing frequency will result. By appropriate manipulation of the triac firing times, different fundamentals can be produced and the phase of this fundamental can be altered (with the proviso that the frequency of this fundamental is below that of the resonant frequency of the circuit).

Both the downstream communication the upstream SWD transmissions use phase shift keying, or to be more precise, differential bipolar phase shift keying. With a constant phase fundamental being produced by alternating polarity current pulses, phase shifts are accomplished by omitting a single pulse as illustrated in Figure 2.2 opposite.
The difference between the capacitor voltage at the end of a pulse and the mains voltage when the next pulse is due determines whether the pulse will be positive or negative. The fundamental frequency of transmission is well above the mains frequency so the polarity of the mains will usually still be the same sign for the pulse following the omitted pulse. This results in the pulse following the omitted pulse having the opposite polarity to that which it would have been had there been no omission. This creates a 180 degree phase change as shown in Figure 2.2 above.

For meter to zone substation communications (upstream), different meters transmit using different fundamental frequencies enabling them to be distinguished at the receiver. The receiver was a digital signal processor based device. It took its signal from a current transformer monitoring the load current on the feeder supplying the distribution transformer connected to the meters. This system had some desirable attributes: firstly, the transmitter was very simple and therefore potentially low cost, secondly, the signal passed over the distribution transformers and with each meter assigned a different frequency, did not need any form of cooperation in order to use the same communication channel. The system was trialled on the scale of a few hundred
transmitting meters. The trials apparently did show promising results but the system was never fully commercialised.

3.3 **TWACS®**

Two Way Automatic Communication System (TWACS®) [17-20] is one of the two other “over the transformer” PLC systems to which it is possible to find references in the academic literature. In contrast to SWD it has been fully developed into a commercially successful system and is widely used in the USA where there are only a few houses connected to each distribution transformer.

3.3.1 **Details of operation of a TWACS® transmitter**

Like SWD, the TWACS® system uses current pulses generated by a triac switching a load across the mains supply at the meter. However, unlike SWD the load is an inductor or resistor and this limits the times at which the signal pulses can be present to near the mains zero crossings. In the case of an inductive load, the pulse must be near the zero crossing because the current through the inductor will continue to increase until the voltage across it reverses making the pulse symmetrical around the zero crossings. In the case of a resistive load, power dissipation must be minimised in order that reasonable length transmissions can be made without the resistor overheating. In addition, pulses near the zero crossing result in a signal-to-load current ratio relatively larger (for close to unity power factor loads) than would be the case for pulses positioned elsewhere. This restriction in the position of the current pulses results in a much less flexible transmitter which can encode far fewer information bits per second than the SWD type transmitter.
3.3.2 Turtle system

The final “over the transformer” PLC system to which it is possible to find references in the academic literature is an ultra narrow bandwidth system originally designed by Hunt Technology's [21]. Hunt named the system "Turtle" presumably because it is slow and reliable. Both of these attributes are due to its use of an extremely narrow signal bandwidth of 0.01 Hz.

Like TWACS®, it is an American system. It is designed for use in AMR for remote meters and its signalling frequency is 8kHz. Apart from the very low data rate, this system has no application to meter reading in Australia. The reason for this is the fundamental difference between the American and Australian distribution systems. Like Europe, the electricity distribution systems in Australia have between 50 and 200 homes connected to each distribution transformer in contrast to America where only a few houses are ever connected to the same transformer. This means the American distribution transformers are much smaller and can pass higher frequencies with acceptable attenuation. As will be shown in this research work, 8kHz is much too high a frequency to use for over the transformer signalling when the distribution transformers have a rating in the order of 1 MVA.

4 Other competing systems

To give a better view of the context surrounding the previously described PLC systems, some of the more important types of system which form alternatives for the AMR requirement will be described. A more extensive, but America-centric, survey of the various systems available is given by Wiebe [13].
Chapter II  AMR systems

4.1  Mesh Radio

Mesh radio is a relatively new technology which uses unlicensed bands at frequencies of hundreds of Megahertz. Each transmitter has an output power of the order of milliwatts and so has a very short range. The system uses a mesh topology so that individual nodes, meters in the case of the AMR application, are within range of each other although they are generally out of range of the destination for the transmissions. The meter at each node has a radio frequency transmitter and receiver and a processor equipped with an intelligent algorithm to route messages it receives from neighbouring meters through to other neighbouring meters. The algorithm is such that the whole mesh self governs, messages being transferred from one meter to another following a path which is reasonably direct between the originating meter and one or more “meter collectors”. The meter collectors are the destination for all messages which they pass on through another trunk communication channel, for example an optic fibre link from zone substations, back to the data management center. With every meter being able to act as a repeater, relaying messages from its neighbours, the system is very robust. The failure of one meter would result in the messages it normally relays being transferred by another meter in the close vicinity. The flow of messages self adjusts to avoid congestion in any one area while maintaining reasonably direct routes for each set of measurement data as it is passes from the originating meter through to the meter collector. This network of nodes forming a mesh relies on a sufficient density of nodes so that there is always enough redundancy to cope with changing radio-link conditions with occasional failure of a node. It is also very easily expandable as a city grows or as a meter rollout proceeds. It is perfectly suited to an urban environment where every single house has a meter with an associated node in radio range of several others but it is not suited to areas of low housing density. It has other disadvantages; firstly that the nodes are sophisticated devices and must be manufactured in large quantity to be economical. Secondly, being a
radio based technology, it is vulnerable to disruption by jamming and signals can be shielded, either by unfortunate meter placement or by purposeful malicious actions.

4.2 **DLC**

In contrast to mesh radio, distribution line communications or carrier (DLC) is very closely related to power line communications (PLC) - indeed many in the industry do not distinguish between them. In this study I follow the lead of the Victorian government in their "advanced metering infrastructure project" and make a very clear distinction between DLC, which uses signals not intended to pass over the distribution transformer, and PLC which uses signals which must pass over the distribution transformer. DLC systems use “data concentrators” to pick off the messages sent from the meter from the low voltage power lines at the distribution transformer and transfer them on to another communication channel, generally a mobile data carrier service like GPRS. This makes it possible for the meters to transmit signals on the power lines at much higher frequencies than those used for PLC as it has been defined above. These higher frequencies allow the use of voltage rather than current signalling since the impedance of the channel at the point of injection of the signals is reasonably high at the frequencies used. They also enable high data rates to be accomplished using narrowband transmissions. All of this makes for a low-cost transmitter with the only disadvantage being that every single distribution transformer needs a data concentrator. In the America this type of system is not of much use because there are only a couple of houses connected to each distribution transformer. In Australia, with 50 and 200 houses connected to each distribution transformer DLC is thought to be economically viable. However, there are a total of 38,000 distribution transformers in Victoria alone. This means a large number of data concentrators are required to implement such a system. Their cost is not prohibitive in such quantity (of the order of $300), however, each of
these concentrators needs to be installed near the distribution transformer, possibly in a hostile environment, and more importantly, there is a high chance that they will have to be revisited and replaced because of rapid obsolescence in mobile telecommunications systems.

4.3 WiMAX

WiMAX is a wireless standard for providing a broadband network capable of covering long distances. Some distributors, for example, SPAusnet and Energy Australia, are currently leasing small segments of the WiMAX spectrum to provide AMR and smart grid functionality [22, 23]. For AMR, WiMAX transceivers are required in each meter and the distributor must obtain base station access.

5 Conclusion

Of the many pre-existing and possible AMR systems, Power Line Communications (PLC) has the advantages that it is long established and is one of the cheapest. The three PLC systems mentioned in the literature have all been developed into commercial products. One of them, Hunt Technology’s Turtle, has a data rate far too low for use by smart meters. The TWACS® and SWD systems have marginal data rate performance for the new smart meter requirements. However, the PLC channel would still be the logical choice for new smart meter communication system if it could be shown to be capable of sufficient data throughput.
III PRELIMINARY INVESTIGATIONS

1 Introduction

A description of the early experiments and associated equipment, which were aimed at developing a working channel sounding system, is given. The result was a crude but working current transmitter and sensitive receiver capable of demonstrating that sounding measurements over the university’s LV/MV channel were going to be possible. The investigations started with work which is based on two communication systems described in the literature and summarised in the previous chapter. Various discoveries were made along the way concerning improvements to the transmitters and receivers and these are recorded.

1.1 Background

The complete communication channels, with which this work is concerned, start at a mains socket and ends on the MV lines at the feeder in the local zone substation. The particular channels available for experimentation were part of a university campus electrical distribution system. The whole system is owned and controlled by the university and this made access for experimental work possible. The reason for this unusual situation is that the university campus includes a gas turbine co-generation plant which supplies all of the electricity, hot water and cooling requirements in addition to exporting excess power to the area’s utility owned grid. The presence of the co-generation plant added some complication. Any signals originating at a mains socket would generally have two paths to take. The first was back to the on site co-generation plant and the other was out via the main MV feeder substation onto the utilities’ grid and back to their zone substation. The exceptions to this two signal path situation were,
firstly, when the university was “islanded” and, secondly, when the co-generation plant was shut down. Islanding, when the university is isolated from the utility’s grid, occasionally occurs, sometimes intentionally and sometimes as a result of the failure of the utility’s grid. During islanding there is only one signal path present and that is back to the co-generation plant. Limited opportunities did arise during the preliminary work to examine this path in isolation but for most of the time measurements made at the co-generation plant had a proportion of the transmitted signal energy missing due to the presence of the connection to the grid. During these preliminary investigations this signal leakage was unimportant because definitive measurements for the purposes of modelling the channel were not being made. Later on, all important measurements were made, not at the co-generation plant, but at the main MV feeder substation during those times when the co-generation plant was shut down in order that the models derived were of the single path channel back to a normal substation (as opposed to a generator).

The grid MV feeder substation, which from now on will be referred to as the Thomas Cherry substation, is shown on the diagram in Appendix III I on page 301 as is the co-generation plant building.

1.2 Safety considerations

The LV mains sockets supplied a neutral and active with nominally 230V RMS (new Australian standard) at 50Hz. In fact the supply is closer to 240V. The MV side was one of three phases with a 22kV line to line voltage between each pair. I was interested in the channel when used with current signals at the input and output. Initially, for safety reasons, I included a commercial 240V to 70V transformer as part of the transmitter which meant that all “home built” circuitry operated at around 70V. This is above the SELV (separated extra-low voltage) specification of 50V RMS but in the dry and
controlled conditions of the laboratory the risk of severe electric shock was minimal. Ideally the currents I wanted on the 240V side were up to 20A peak to make signal detection on the other side of the distribution transformer a possibility. This is because tens of Amps had previously been shown to be necessary in the development of the TWACS® system [24], which was described in the last chapter. This meant that on the low voltage side of my transformer I would have to draw a signal current of about 70A peak which dictated quite a large transformer. Due to financial restrictions a second hand hobbyist’s welder was used for this step down transformer. A welding transformer is designed with significant leakage inductance in order to limit the current and this factor finally forced the abandonment of this approach for reasons that will be described later.

2 Set up for preliminary experiment

The output of the welding transformer could be short circuited at controlled times by firing a triac connected across its secondary. The peak current in this case is limited by the leakage inductance and, to a lesser extent, by the winding resistance. This situation is illustrated in Figure 3.1 below:

![Figure 3.1](image_url)

Figure 3.1 Equivalent circuit of the welding transformer current pulse transmitter.
In the diagram the transformer is modelled as an ideal transformer T1 with the primary leakage inductance referred to the secondary and combined with the secondary leakage inductance to form X1. Copper losses are similarly combined in R1. Core losses and magnetising inductance are omitted as they can be modelled as relatively large impedances appearing in parallel with the primary and will not have any significant effect because of the relatively low mains source impedance.

The triac in the diagram of Figure 3.1 is a device commonly used in AC power control. It behaves similarly to two back-to-back Silicon Controlled Rectifiers (SCRs) but has a single gate control. Initially, with no gate current, no current flows between the two main terminals of the device (MT1 and MT2). This is because the device has been selected to have a zero-gate-current break over voltage which is higher than the peak MT2/MT1 voltage in the circuit (the break over voltage for the device used was 1200V). When a gate current is injected into the device, the break over voltage is significantly reduced and when the gate current is sufficient the voltage across the main terminals exceeds the break over voltage for that gate current and the device conducts. The drop across the main terminals becomes very small, of the order of a Volt, and once “fired” in this manner the triac continues to conduct even when the gate current is removed. The only thing which can bring the device back into its non-conducting state is the main terminal current reducing to less than the “hold current” threshold which has a very small value. This happens following the next zero crossing after the device has been fired. This sequence of events is illustrated in Figure 3.2 opposite.
Figure 3.2 Triac operation.

Although the turn on time for this transmitter circuit is completely controllable by the gate control circuit, the turn off time cannot be altered. The turn off time is predetermined by the applied mains voltage waveform, which is close to sinusoidal, and the transformer leakage inductance value. The detailed theory which predicts the exact pulse shape and duration will be covered in the next chapter.

2.1 Initial transmitter

More detail concerning the first current transmitter used in the experiments will now be given.
2.1.1 Triac protection

Although the operation of a triac, described above, appears relatively straightforward, there are several complicating factors. The most important, in this application, is the fact that a triac will not cease to conduct when the current through it reduces to less than the hold current threshold if the rate of change of current at this instant is greater than a certain value, while at the same time, the rate of change of the main terminal voltage is also above a certain value. The reason for this effect is that a triac includes multiple PN junctions in very close proximity within the same device. The reverse recovery current for the junction which is switching off acts as a spurious gate current for part of the device which is about to become forward biased and is supposed to remain in its non-conducting state [25]. This effect is called commutation failure. The usual solution is the addition of an RC snubber network across the main terminals to limit the rate of change of voltage. The allowed rate of change of voltage for a particular device is determined by the rate of change of current at which it is required to commutate (switch off). In the first experiments the triac had a moderate \( \frac{di}{dt} \) at commutation and an RC snubber was adequate. In a later configuration, when a capacitor was added to form a resonant circuit, the \( \frac{di}{dt} \) was much higher as was the \( \frac{dv}{dt} \) at commutation. The snubber solution was found to be insufficient and a series saturating inductor was used to reduce the commutating \( \frac{di}{dt} \). This will be outlined in section 4.2.1 but first the rest of the initial transmit/receive system will be described.

2.1.2 Current limiting

Although previous work by others had indicated signals of up to 70 Amps on the mains would be necessary for detection of the signal on the MV side of the distribution transformer, the initial experiment was conducted using current signals of less than 10 Amps on the mains to minimise the risk of blowing a fuse and upsetting co-workers. To
limit the current an additional inductor was added in series with the triac. This inductor used a powdered iron core with a low relative permeability to avoid saturation. Details of this inductor and the rest of the transmit circuit are given in Appendix III A on page 291.

2.2 Initial transmitted current signals.

The simple triac switched circuit could produce large current pulses at twice the mains frequency. The turn on time could be selected to be a pre-determined time interval before the mains zero crossing. The initial rate of rise of current through the secondary of the transformer was controlled by the total secondary circuit inductance and by the instantaneous value of the secondary voltage at the gate firing time. Close to the mains voltage zero crossing the voltage across the inductance was zero and so the current through it reached its peak. A symmetrical current pulse was therefore produced centered on the mains zero crossing, the peak value being determined by the firing time. In this way up to 10 Amp (peak) pulses were produced on the mains which could easily be detected at some distance away on the LV supply by a current sensing coil and low noise amplifier.

2.3 Current detection sensor

In order to detect the current pulses which were being drawn from the mains socket at a remote upstream location (towards the electricity supply source), some way of measuring the current flow in the supply phase which led to the socket on which the transmitter was connected was required. The feeder for this phase would obviously be supplying many other sockets, and the challenge was to see if the transmitted current pulse could be detected on top of the total supply current to all of the equipment connected to those other sockets. A suitable location for the receiver was chosen where
the three phases (and neutral) entered the floor of the building on which the transmitter was located. This supply went to eight laboratories and fifteen offices, two sets of toilets and a pump/air-conditioning equipment room. There was no possibility of inserting anything into the pre-existing supply line, or temporarily disconnecting it, so to measure the current flowing in this 30mm diameter (including insulation) single conductor cable a current sensor had to be constructed around it to form a current transformer. The large 50Hz load current being carried by the feeder cable, combined with the fact that the spectral content of my current pulses required sensitivity to much higher frequencies, would dictate the type of core for this current transformer. This core cannot be allowed to saturate because that would mean the measurement device would be non-linear and not representing the input signal properly. A suitable spilt core with which to make a current transformer which would not saturate and which would have the desired frequency response would have been possible as is described in Appendix III B on page 292, however a simpler alternative was to use a Rogowski coil pick-up [26] connected to a low noise amplifier designed to be fed from a low impedance source. This could be constructed very cheaply [27]. The Rogowski coil is an air (or other unity relative magnetic permeability material) cored coil with the return current connection following the coil back to its start as shown in Figure 3.3.

Figure 3.3  Rogowski coil.
The coil is formed into an approximate toroid and, if constructed correctly, it responds with an output voltage which is proportional only to the rate of change of flux linking the small turns of the coil. Since the core of the coil contains no magnetic material it is immune from saturation and the very large background load current which is present on the feeder causes the sensor no problems. A current transformer relies on the core concentrating the flux so that the primary and secondary windings enclose the same flux which is being contributed to by the currents in both windings. In contrast, in the Rogowski coil case the flux produced by the current flowing in the feeder is virtually undisturbed by the presence of the coil. This means that the output in the Rogowski coil is proportional to the derivative of the current it is measuring rather than being proportional to the current. The reason for this can be seen from the integral form of Ampère’s circuital law:

$$\int Hdl = I$$

where $I$ is the enclosed current (which is flowing in the feeder in our case). The flexible Rogowski coil is wrapped around the feeder so that its ends meet. Ampère’s law tells us that the closed line integral of the magnetic field intensity, $H$, along the axis of the Rogowski coil, has a value equal to the current flow through the feeder. The corollary of this is that the total flux through the turns of the coil integrated along the length of the coil is proportional to the current which the coil is enclosing [28]. The emf, $e$, present at the output to the coil is proportional to the rate of change of the magnetic flux through the turns of the coil from Faraday’s law, that is:

$$e = \frac{d\phi}{dt}$$

This means that the output voltage is proportional to the rate of change of the current enclosed.
2.4 Other detection equipment

The output voltage from the Rogowski coil was very small and substantial amplification was needed before the signal could be displayed on an oscilloscope. The arrangement for this amplification was a discrete component amplifier [29] coupled to the coil through a microphone transformer. This was followed by a simple RC integrator to give an output proportional to the feeder current of $2\mu\text{V}$ per Amp. Details of the construction of this amplifier along with the calibration of the Rogowski coil/integrator/amplifier set up are given in Appendix III C on page 293.
3 Method, results and discussion for preliminary experiment

Having described the first practical current pulse transmitter and receiver which was made I will briefly present the experimental method and results and compare them with the commercial TWACS® system, which has already been developed using these current pulse generation principles.

3.1 Initial TWACS® (inductor only) experiments

In the TWACS® system, the presence or absence of a current pulse around the zero crossings encodes the data to be communicated. My initial experiment was concerned with the possibility of detecting the presence (or absence) of such a current pulse. In the top floor laboratory within which transmissions were made, all three phases were accessible. Each single phase socket in the laboratory was already identified with a red, blue or white dot according to the phase to which it was connected. However, the feeder’s phases, which supplied the whole top floor of offices and laboratories, could not be visually identified and so a procedure described in Appendix III D on page 295 was used to determine which feeder wire was supplying each phase. Having done this, a recording was made of the “background” load current waveform for the top floor of the building (without any current pulse transmissions being made) for the load current on the red phase. The transmitter was then set up to produce an 8 Amp peak current pulse at each zero crossing on the Red phase in the laboratory. The feeder current waveform was again recorded.
Two example results, taken when the load was light are shown in Figure 3.4:

![Figure 3.4](image)

**Figure 3.4** Recorded waveforms with transmitter off (left) and on (right).

It can be seen from the left hand trace that the load is not harmonically pure (which is to be expected since many computers, fluorescent lights and a few high power induction motors are connected to this supply), and it can be seen from the right hand trace, that the transmitted current pulses have clearly changed the waveform. The received pulses can be made even clearer by doing a subtraction of the background trace from the “transmitter on” trace. This is shown below in Figure 3.5 in which averaging has been done to smooth out the high frequency noise.

![Figure 3.5](image)

**Figure 3.5** Recovered transmitted signal

As these two recordings were taken at different times (several seconds apart) it is clear that the background current is nearly cyclo-stationary. This subtraction of a representative background waveform recorded close in time to the signal of interest is similar in effect to the way the commercial TWACS® system detects current pulses.
3.1.1 Comparison with TWACS®

The method by which the TWACS® system performs differential detection is not a simple subtraction of a single reference waveform as was used in my experiment, but continual comparison of the signal at a potential pulse transmitting zero crossing point with the signal at a close zero crossing point (within a few cycles) for which no transmission is allowed. This method relies less heavily on the assumption of cyclo stationary of the background load over the long term. The TWACS® system also uses a many-pulse-per-bit coding scheme with error correction and the initial paper specifies using 60 to 70 Amp peak currents. For these reasons it is possible to detect the TWACS® transmissions on the far (MV) side of the distribution transformer which may be supplying megawatts of power whereas my simple experiment was only detecting the transmissions on the LV side and with a background load of only a few kW. Nevertheless, this very crude version of the transmitter of a pre-existing commercial mains communication system showed that the signals could be detected remotely.

4 Set up for the first successful channel sounding experiment

Simply generating an arbitrary signal which could be detected remotely was not a very useful result. My task was to characterise the channel. For this purpose signals with a wide frequency content, or signals whose frequency content could be varied, were needed. This would allow the use of standard single frequency detection techniques, which can be made very sensitive, enabling exploration of the channel’s response in the frequency domain.
4.1 Generation of more suitable signals

A simple modification to the existing current pulse generating circuit promised to give the flexibility for sweeping the transmit signal over the range of interest. The modification was to add a series capacitor to resonate with the inductance already in the circuit. This turns the transmit circuit into the type used for SWD upstream transmissions as was described in the previous chapter. In that type of transmitter the shape of the signal pulses will be approximately half sine where the frequency of the complete sine is the resonant frequency of the inductor and capacitor. If the resonant frequency is very high with respect to 50Hz then the mains voltage will not change a great deal between the switching event caused by the firing of the triac and the time at which the resonance current first reaches zero and the triac switches off. In this case the shape of the pulse will be close to a half sine. Any difference to half sine is due to the fact that after switch on, the mains voltage, which is partly driving the process, will change slightly over the pulse length time interval. The exact effect of this changing forcing function on the behaviour of the current seems to have been ignored in previous publications on the operation of this type of circuit. The effect is very interesting and will be explored in the next chapter, but for now we will assume that it is of no consequence. With a high resonant frequency the triac can be fired at any desired intervals. The resultant current pulses will not be the same amplitude because the peak value of each pulse, and whether it is positive or negative, depends on the value of the mains voltage at the instant of firing as well as the voltage which was retained on the capacitor from the end of the last pulse. It can often be arranged that the triac’s gate firing signal is such that the resultant current pulses alternate between positive and negative. In this situation, with a constant gate firing interval of T seconds, the current pulse waveform will contain a high signal energy at the fundamental frequency 1/T Hz. Because the signal comprises alternating polarity half sine pulses with segments of zero
signal between them, and because the size of each pulse is not the same, the frequency content of the signal is quite extensive. However, with a narrow band filter at the receiver tuned to the fundamental frequency only, it is still possible to use this technique to transmit high level signals at a particular frequency with low power dissipation at the transmitter. The low power dissipation is because the voltage across the transmit circuit is in quadrature with the current through it.

4.1.1 The importance of constant phase

By omitting a single pulse trigger in the regular sequence required to produce alternating polarity current pulses, the phase of the fundamental is reversed, that is, altered by 180°. This is the technique whereby information was modulated onto the current pulse signal in the original SWD system. The technique produces a bipolar phase shift keyed signal and to avoid the use of a coherent phase reference, differential phase shift keying is implemented. In order to implement such a modulation technique it is obvious that no commutation failure events should occur because they generally result in an unintended phase reversal of the transmitted signal. Not only is it important that these spurious phase reversals should not occur in a communication system, it is even more important they should not occur in a channel sounding system. The reason for this is that, like a communication system, the channel sounding system receiver integrates the correlation between the received signal and a constant reference, ideally one which is exactly in phase, or exactly out of phase, with the received signal. It does this over a much longer period than would be the case in a communication system and there is no provision for occasional errors. This integration is made to take place over a long time to take advantage of the effect that each current pulse is adding up more rapidly than the non-coherent noise is adding up over the same time. This is the reason for the system’s sensitivity; with long integration times such a channel sounding system can measure
large attenuations by detecting the presence of a signal at the receiver which is buried deep within the background noise. If spurious random phase reversals of the transmitted signal are occurring, this integration process no longer results in a constantly growing signal. After a phase reversal, the received signal begins to destroy the built up effect within the receiver of the previously received signal which was of the opposite phase. In a communication system, with short bit times relative to the rate of random spurious phase reversal events, the problem might be tolerable because error control coding will recover the signal. In a channel sounding system, where the result of the integration is being used to measure attenuation, and the integration times will generally be much longer to maximise sensitivity, random phase reversals are disastrous.

4.2 A Practical SWD type transmit circuit

Having described the signals expected from the resonant circuit SWD type transmitter, and having emphasised the critical importance of never having random spurious phase reversal events, the circuitry and its operation for the first channel sounding system transmitter will be described.

The papers explaining the operation of SWD [15, 16] and the patent documentation for SWD and related electronically switched resonant circuit transmitters [30-33] do not offer any details on the problems of spurious phase reversal events or on the complication of circuit behaviour resulting from the mains voltage changing while the signal pulse current is flowing. In fact, the papers suggest that the fundamental of virtually any frequency below that of the resonant frequency can be produced reliably by the SWD circuit. This was not found to be the case and therefore a drastic redesign of the sounding system was required, as will be explained later. The promise of being able to transmit a high level signal with almost any fundamental frequency below the
resonant frequency of the inductor and capacitor lead to construction of an SWD type transmitter and receiver for the purpose of channel sounding. First, a 7uF (250V) capacitor was added in series with the Triac. This gives a circuit with an undamped natural frequency of 783Hz. The intention was to use this to measure the channel’s frequency response between about 100Hz and 500Hz. The diagram in Figure 3.6 shows a comparison of a current pulse produced by this “SWD” circuit with one of the same peak amplitude produced by the previously described TWACS®” circuit, both being initiated at the same time after a mains zero crossing.

![Diagram of SWD and TWACS pulses]

**Figure 3.6** Comparing TWACS® and SWD transmission pulses.

The inductor values in both cases were identical. In the TWACS® circuit case, the pulse shape, and the rate of change of current at commutation, is controlled by the instantaneous mains voltage only. The result is a relatively wide pulse with a relatively low di/dt at commutation. In contrast, the SWD pulse has its rate of change of current
controlled mostly by the resonant frequency of the inductor and capacitor and the initial capacitor charge, the mains voltage remaining relatively constant over the pulse duration. This results in a much higher di/dt at commutation for the same peak current. There are two more important differences between the operation of the two circuits which relates to their di/dt values at commutation. Firstly, the TWACS® circuit will always produce the same peak amplitude pulse when fired at the same time relative to the mains voltage whereas the SWD circuit’s peak amplitude (and therefore initial and final di/dt) depends also on the initial voltage on the capacitor at the start of the pulse. This initial voltage is the voltage that remained from the end of the previous pulse and this is determined partly by the amplitude of that pulse. The amplitude of this last pulse in turn depends on the charge that was left on the capacitor at the end of the pulse before, as well as the instantaneous mains voltage at the time of its initiation. In fact the exact amplitude of a particular SWD pulse depends on the entire history of the preceding SWD signal pulse train and its relationship with the mains voltage waveform. Even if we knew the whole history, the exact amplitude of the pulses is, in reality, difficult to predict. One reason is that the mains voltage waveform is not exactly sinusoidal, it has small variations in fundamental frequency, amplitude and harmonic content. These variations depend mostly on the distributed load which is changing in a substantially random manner. Another reason is that even a single pulse cannot have its exact shape determined purely by algebraic methods since the form of its function is transcendental. The solution to the equation describing the shape of a pulse can only be determined by an infinite number of algebraic operations, that is, numerical methods must be used. This will be demonstrated in the next chapter.
4.2.1 Commutation failure protection

Not only will the peak amplitude of an SWD circuit’s pulse be unpredictable when it is fired at a particular time relative to the mains voltage (in contrast to the TWACS® circuit), but the SWD circuit will fire pulses at all times relative to the mains zero crossings, also in contrast to the TWACS® circuit. This is because the fundamental frequency of transmission of an SWD signal is, in general, unrelated to, and asynchronous with, the mains supply frequency. Occasionally the triac will be fired when the mains is at its peak and when there is a large opposite voltage on the capacitor and this will result in a very high peak current pulse. The SWD circuit therefore has a di/dt at commutation which is generally higher than that of the TWACS® circuit (because the pulse is shorter for the same peak value), and sometimes very much higher because the peak amplitudes are variable and a proportion of them are very much higher than the average. This leads to a requirement for more stringent protection against commutation failure for the Triac in an SWD circuit. Previous SWD circuits have been designed to work directly across the mains supply and a simple snubber circuit had been found adequate to prevent commutation failure. As mentioned previously, I was initially unable to work directly with the mains supply and therefore needed much higher pulse currents through the SWD circuit in order to create an appropriate signal level on the mains supply at the transformer primary. This lead to the di/dt problem being that much harder to solve. The solution adopted involved not only an RC snubber but also a large additional series inductor designed to saturate at low currents. The core of this additional inductor, wired in series with the main inductor (partly made up of the leakage inductance of the welding transformer), was a large ferrite toroid and its exact specifications along with details of its operation are given in Appendix III E on page 295. Ferrite has a high relative permeability so a large value inductor could easily be made. This limited the initial rate of rise of current to a low value. However, a
continuous ferrite toroid also has a very low magnetic reluctance causing a high flux density at a relatively low value of mmf (magneto motive force). The ferrite inductor was purposefully designed to reach its saturating flux density at a very low current and so have no effect on the pulse shape other than near the commutation points. This technique of protecting against di/dt caused commutation failure has been used previously by others [34]. In my case it eliminated all commutation failures.

4.3 Receiver for swept signals

Having described the “SWD” type channel sounding transmitter, the initial “home made” receiver will now be briefly described. The device made comprised a pair of coherent receivers each having local oscillators of the same frequency but ninety degrees out of phase. A coherent receiver uses the homodyne technique in which the input signal is mixed with, or multiplied by, a local oscillator such that the frequency band of interest is down-shifted to around 0 Hz. This has the advantage of simplicity as with no intermediate frequency (IF) there is only the one filtering stage constituting straight forward low pass filtering. Ripple receivers have in the past used this technique for detecting ripple control signals but it is also particularly suited for sounding measurements at audio frequencies where the source signal sent down the channel can be used directly in its un-attenuated form as the in-phase local oscillator. Sweeping of the transmitter controlling oscillator to explore the channel in the frequency domain causes no problems if there is a mechanism to keep the receiver local oscillator frequency locked to it. In general there will be a phase shift through the channel and this phase shift is initially unknown. It is possible to keep the transmit and receive oscillators fixed in frequency while a phase shift between them is adjusted until the received signal is coherent with its local oscillator. However it is much simpler to use two receivers, the second with a quadrature local oscillator. With the first receiver measuring the in-phase
component of the receiver signal, and the second the quadrature component, the amplitude of the received signal can be determined no matter what phase change through the channel is present.

This initial receiving device was built using a complex programmable logic device (CPLD), a set of analog switches and some simple “glue” circuitry. The details, such as the principle of operation, circuit diagram and reference to the logic array configuration listings, are given in Appendix III F on page 296.

The local oscillator part of the device, which was just a square wave generator, was replicated exactly at the remote transmitter location and used to trigger the gate circuitry of the transmitter with each edge of the local oscillator signal producing a gate pulse. These two gate pulses per local oscillator cycle gave rise to a transmitted half cycle of one polarity followed (usually) by a half cycle of the opposite polarity. The transmitter and receiver local oscillators were originally unlocked but they were later phase locked using two GPS derived timing signals as will be covered in a subsequent chapter.

4.3.1 Receiver improvements

Having built the receiver an improvement immediately became obvious. The receiver, as it stood, multiplied the input signal by square waves at the reference frequency as described in the previously referred to Appendix (Appendix III F on page 296) and this gave it sensitivities at all odd harmonics of the reference. It could easily be made to multiply by a better representation of a single frequency by adding another analog switch to each receiver. This new switch would have its input permanently connected to signal earth (0V) and its output added in to the outputs of the other two switches. The unfiltered signals could now be made to switch between the input, the input inverted and
zero. The shape of the signal by which the receiver multiplied by could now approximate a sine using three levels (-1, 0 and 1) instead of just two which gives a square wave. The three level approximation could be made, with appropriate timing, to eliminate the third harmonic from the reference signal. This is the worst spurious sensitivity which the original receiver had with the level of sensitivity being just 9.5dB less than the sensitivity to the fundamental. It is also closest in frequency to the main sensitivity and so is not so easily dealt with by pre-filtering of the input. To eliminate the third harmonic sensitivity the reference waveform has to have amplitude zero from 0° to 30° of its cycle, from 150° to 210° and 330° to 360°. The reason this waveform has no third harmonic can be seen directly from the diagram in Figure 3.7 below which shows a sinusoid with frequency of three times the fundamental of the reference.

![Reference waveform, fundamental frequency 1/T](image1)

![Third Harmonic](image2)

**Figure 3.7** Graphical analysis for third harmonic of reference.

Over each non-zero segment of the reference waveform, the integral of the sinusoid multiplied by the reference will be zero because the area of the positive segments exactly match the area of the negative segments. The same result can be seen by doing the Fourier series analysis of the reference waveform $f(x)$ mathematically:
By selecting the $x=0$ point as shown in Figure 3.7 the cosine components are all forced to zero and the amplitudes of the sine components $b_n$ are:

$$b_n = \frac{1}{\pi} \int_{0}^{2\pi} f(x)\sin(nx)dx$$

If the time for which the reference waveform is positive and negative is defined as $2\alpha$ then the result of this integration is $b_n = \frac{4}{n\pi} \sin(n\alpha)$ [35]. Obviously if $n=3$ for the third harmonic an $\alpha$ of $60^\circ$ will give a resultant amplitude of zero. This simple modification was made and found to work as expected with a $30\text{dB}$ reduction in sensitivity to the third harmonic measured. The idea was recorded formally (see Appendix III G on page 298), with particular reference to its application to ripple receivers.

### 4.4 Lock-in amplifier

Even with the improvements to the “home made” receiver detailed in the previous section it proved to have a disappointing dynamic range (ratio of overload level to minimum detectable signal) of under $80\text{ dB}$. This would give marginal performance when used to detect and measure signals with peak currents of the order of $10$ Amps on the L.V. supply with the background load current signal being that of the whole university’s load of around $6\text{MW}$. A sinusoid with a $10\text{A}$ peak is more than $60\text{ dB}$ below the current in each phase for this load and that leaves little room for channel attenuation of the transmitted signal. The answer was to obtain a professionally made lock-in amplifier designed for measuring small signals in the presence of large out of band signals. A lock-in amplifier is a measuring receiver using the coherent receiver principle described in the last section. It will usually have two channels fed from reference signals in quadrature, an adjustable pair of low pass filters to set the detection bandwidth, and a buffered output of the in-phase reference oscillator to be used for system excitation.
Alternatively an external reference signal can be fed into it and it will phase lock its internal oscillator to this signal. A Stanford Research Systems SR830 lock-in amplifier was obtained. This has a dynamic reserve (ratio of overload level caused by out of band signals, to a signal which is measurable to full accuracy of the instrument) of 100dB. The “home made” receiver had allowed development and proving of the transmitter, current sensors and the measurement method. It had been sufficient for demonstrating the principle of the sounding technique but, to ensure success, the signal receiver from this point on was the SR830 lock-in amplifier.

5 Method, results and discussion for the first channel sounding experiment

Having described most of the equipment used for the first successful channel sounding experiment the experimental method and results will be presented. The reasons for making a more detailed study of the behaviour of the SWD signal transmitter, which is presented in the next chapter, will also be outlined.

5.1 The Channel

For reasons of accessibility, the first channel upon which experiments were conducted was from a transmitter at a mains socket in an upstairs office in the Beth Gleeson building (building BG) to the co-generation control room in the boiler house building about 735 meters away. The physical route of this channel can be seen on the diagram of the university’s MV electrical distribution system given in Appendix III I on page 301. The electrical components along this channel’s route are illustrated in Appendix III H on page 300. The path between the socket and receiver started with a current on the red phase of the 240 V supply being produced by the transmitter circuit connected to the welding transformer. This current flowed back to the secondary of a 1,500kVA transformer with a neutral return. The physical distance this current travelled was about
150m (not including the return path). This caused a current, by transformer action, on
two of the 22kV phases of the transformer’s delta connected primary. The path then
joined a buss bar connected to the utility grid (external to the university) about 20 meters
from the transformer. Some of the signal current would have flowed into the grid but
some continued on to the receiver at the co-generation plant control room. This portion
of the signal current had to flow through about 750m (not including the return path) of
underground cable to the delta wound secondary of a 10MVA transformer. This caused
a current in the 11kV (line to line) primary red phase which flowed through the
generator and returned via the neutral, as this primary and the generator are both Wye
connected. In this short connection between the 11kV/22kV step up transformer and the
7.7MW generator is a measurement and protection current transformer. The output of
the 450:1 part of this current transformer feeds a high voltage electricity meter (along
with the output of a potential transformer). The red phase current feed wire to this meter
would have been passing a signal current 1/450 th of the signal current flowing to the
generator and this wire, as it passed under the control room consol, was safely
accessible. A Rogowski coil was placed around this wire and connected to an amplifier
followed by an RC integrator and the SR830 lock-in amplifier. A single line diagram of
the channel is shown in Figure 3.8 overleaf. A helper was posted in the control room
watching an oscilloscope whose X and Y inputs were connected to the P and Q outputs
of the lock-in. Meanwhile, in the office the transmitter was being fired at a fixed
frequency from a stable oscillator. A telephone link up was made between the
experimenter (myself) in the office and the helper in the control room.
5.2 The single frequency signal detection result

This first experiment involved manual tuning of the receiver while the person at the transmitter switched the phase of the signal by 180° at random times by means of a simple CPLD based circuit which interrupted one gate pulse on input from the operator. Telephone communication from the receiver operator would indicate when he detected a phase reversal which confirmed to the transmitter operator that his signal was arriving. The signal was buried in noise and difficult to locate, the necessarily narrow bandwidth at the receiver dictated very slow adjustment of the receive frequency. As the receiver was tuned close to the transmitted carrier (the fundamental frequency of the transmitted waveform) the spot on the oscilloscope would begin to rotate rather than fly about in a random fashion. This also happened when the receiver was tuned close to harmonics and inter-harmonics present on the university load waveform. But when the receiver was close to the transmitted signal, it would jump from slowly tracing a small circle over to the opposite side of the circle every time the transmit operator reversed the phase of the signal. It had already been determined that the transmitter had some frequency bands over which random spurious phase reversals occurred. These random reversals could be
identified at the transmit end because the “home made” receiver was co-located with the transmitter. At this stage it was not entirely understood whether all these random phase reversals were still due to commutation failure or whether there was another unavoidable cause. It had been discovered, however, that there was a narrow band around 180Hz at which many tens of seconds would pass before an un-commanded phase reversal occurred. Around this frequency every time an intentional phase reversal was made at the transmit end, the helper at the receive end would report back, unprompted and a few seconds later, that the spot slowly tracing a circle on the oscilloscope display had reversed sides.

5.3 The single frequency measurement results

It was now clear that it was possible to reliably detect a signal sent from a remote mains socket. This was a long way from making measurements of attenuation and phase change at a range of frequencies but it gave sufficient confidence to negotiate for permission to make changes at the co-generation control room to improve the receiver’s sensitivity.

The Rogowski coil sensor and amplifier were replaced by a set of three interposing current transformers and low noise amplifiers. These were made to a high standard and details of the circuitry and its calibration are given in Appendix III J on page 302. This new equipment was installed permanently in the co-generation control room between the generator measurement current transformer in the 11kV line and a new high voltage meter. The current phases feeding the meter were made to go through these interposing current transformers, the signals from which were amplified and made available for accurate monitoring of the generator current and any signal currents which may also have been present. The interposing current transformers were from a high quality three
phase meter and were purpose made for accuracy at frequencies between 50Hz and several kHz and the amplifiers which followed them were constructed around the best low noise op-amps available.

Using this new calibrated equipment the previous experiment was repeated and measurements were also taken of the in-band interfering noise signals with the transmitter switched off for comparison. Details of this set of measurements along with the equipment settings are given in Appendix III K which can be found on page 303 but the important results follow.

The frequency of the fundamental of the transmitted signal was 180.11Hz, the signal peaks on the 240V line at the transmitter averaged about 2A and the line current on a single phase at the 11kV generator output was 300A (generator supplying 5.7MW) which is equivalent to 7900 Amps at 240V. This puts the RMS transmitted current at 72dB under the load current before any attenuation or signal loss to the grid is taken into account. The value measured for the fundamental of the signal received was about 0.04Amps (referred to the 240V). Assuming a value for the RMS transmitted fundamental of 1A, the measurement indicates an attenuation/loss at this frequency of about 28dB. The transmitter was then switched off and the in-band signal at the receiver dropped by 8dB. In frequency, the signal was between two large harmonics on the mains. The third harmonic (150Hz) was measured as being 58dB above the received signal and the fifth harmonic (250Hz) was 70 dB above the signal.

5.4 Discussion: The problems revealed by the measurement experiment

This first measurement experiment served its purpose in highlighting the problems which needed to be solved before data for modelling the channel could be collected.
These problems will now be briefly discussed in turn, the solutions are covered in later chapters.

The figures in the results section above give some indication of the difficulty of finding and measuring this transmitted signal amongst the interfering signals present on the line. One of the reasons the signal is so low is the inclusion of the welding transformer to avoid connecting circuitry direct to the mains. Not only does this transformer lower the circuit supply voltage and so limit the energy that can be put into the resonant circuit during each transmitted pulse, but the current seen on the mains is then reduced by the winding ratio of the transformer.

The measurements were only made at a single frequency. The reason for this is that at other frequencies the transmitter circuit would not stay stable in phase for long enough for the receiver to be set to have a large integration time and therefore a narrow bandwidth.

Although it was reasonably easy to ensure the transmitted fundamental frequency and receive local oscillator frequencies were the same to sufficient accuracy for a measurement with a few seconds duration to be taken, it was still impossible to measure any phase change along the channel. Such phase change measurements would be essential for a meaningful model to be created.

For the initial experiments the channel itself was not well defined. A large proportion of the signal would be passing out onto the grid and there was no way of measuring it with the system as it stood. This would make meaningful modelling problematic but this complex channel arrangement was still a suitable test bed for design of the equipment.
The noise measurement was very crude. It involved simply switching off the transmitter to get some idea how far below the received signal measurement the noise floor was. This measurement did not give an accurate figure for the RMS noise in a known bandwidth since the output was varying as rapidly as the bandwidth was allowing.

Finally, cross talk between phases was discovered. The transmitter for the measurement was connected to the white phase and the measurements given are for the fundamental detected on the white phase. However, a significant signal was also detected on the blue phase although none was evident on the red phase. This interesting situation can be explained by the Wye-delta transformer connection and will be covered in more detail in a later chapter.

6 Conclusion

The TWACS® circuit, as described in the literature, acted as a starting point for these investigations but, although it has been developed by its inventors into a successful commercial communication system, it was never going to be of much use as a tool for investigating the channel. The reason for this is that the signal it produces cannot be made to sweep across the frequency spectrum. Any pulse with a discontinuity, like the TWACS® signal pulse, contains some energy at all frequencies and a time domain recording of the received pulse could theoretically be used to construct a perfectly good channel model. However, the presence of very large interfering signals on this channel means that channel sounding is much better done by using a frequency sweep technique. Using this technique the receiver can be made to have sensitivity to the transmitted signal while being very insensitive to the background signals present. The work described has shown how such a frequency domain system can be made with sufficient
sensitivity. The preliminary work also highlighted where there are potential problems in developing such a channel sounding system.

The SWD circuit, the second system from the literature, had great promise because, like the TWACS® system, a large signal could be injected into the channel without significant power dissipation. As well as that, the SWD signal could be made to have a variable fundamental frequency. The initial trouble with severe conditions at the triac leading to commutation failure and therefore spurious phase reversals was entirely solved by use of a saturating inductor but this still did not eliminate all spurious phase reversals at all frequencies. This puzzle was eventually solved and the details are covered in the next chapter. The result was that the SWD circuit was only used for a single frequency measurement. It is clear that a work-around technique would have been possible by re-reversing the phase after each spurious phase change, but in fact an entirely different approach to the transmitter was adopted as will be described in a subsequent chapter. The transmitter’s other obvious weakness was the inclusion of a voltage step down stage to improve safety. Subsequently a different approach to ensuring safety was adopted and this too will be described later.

One development from this preliminary work which did survive through the whole channel modelling work was the current detection and receiver system. Use of the pair of coherent receivers with quadrature square wave local oscillators had lead to an improvement which almost eliminated sensitivity to the third harmonic of the reference frequency. This was improved upon further by obtaining a commercial lock-in amplifier with no sensitivity other than to the reference frequency itself. This was the only expensive equipment used but it was essential to the work’s success.
Another development which survived unaltered was the technique for obtaining a signal at the receive end. The interposing current transformers coupled into the high voltage metering circuit with low noise amplifiers proved entirely satisfactory. Indeed it worked so well that later on an exact replica was added at the substation where the grid connects to the university MV supply. This enabled measurements, and modelling, of a proper single channel when the co-generation plant was shut down without any complication of unknown signal leakage, as we had at this preliminary investigation stage.
IV SWD CIRCUIT BEHAVIOR

1 Introduction

In the last chapter some strange behaviour of the SWD circuit was alluded to. This behaviour suggested that the circuit was not as flexible as the literature implied and was possibly unsuitable for my intended use as a channel sounding system. In this chapter a detailed theoretical investigation into the circuit's behaviour will be presented. The observed behaviour had included different modes of operation dependant upon initial conditions, spurious phase reversals similar to those caused by commutation failure and the possibility of a constantly increasing output pulse amplitude. These discoveries lead to abandonment of the SWD circuit for the rest of the study (the replacement transmitter, a sledge hammer to crack a nut, will be described in detail in the next chapter). The theoretical investigation presented here resulted in a much better understanding of the observed behaviour. The reason for including such detail about the SWD transmitter, even though it was replaced for the purposes of this study, is that the SWD circuit would have been a much better choice if time had been available to make the workaround, mentioned in the conclusion of the last chapter, necessary to compensate for its sometimes unruly behaviour.

In this chapter two approaches will be taken to discover the reasons behind the circuit's behaviour, the first in section 3 will be the derivation of a closed form solution which predicts most of the phenomenon observed. This solution is based on a gross simplification which is necessary to obtain exact results to the equations. In the second approach of section 4, a more accurate model is used as a check. This approach follows conventional analysis of the circuit which requires numerical techniques for a solution since the resulting equations are transcendental.
2 Analytical investigation of the SWD signal transmitter

Using the SWD transmitter equipment described in the last chapter it soon became obvious that it sometimes exhibited unexpected and intriguing behaviour. The simplest was that, if it was fired at a rate close to double the mains frequency its output would reach very high levels. So high in fact that failure of the circuit would occur after a few cycles. This, and some of the other behaviours, can be explained by a very much simplified model of the circuit. To determine the reasons for still other behaviours evident in real life, a more accurate model was required. Results obtained from both of these models will be presented, compared with each other and compared with experimental results.

2.1 Mathematical modelling of the circuit

Consider the simplified SWD circuit of Figure 4.1:

![Figure 4.1 Simplified SWD circuit.](image)

It has already been explained that the SWD pulse approximates a half sine pulse. It will be shown that if the mains waveform did not change in voltage level over the period of the pulse then the pulse shape would be exactly half sine. This is because, in this case, the initial conditions and natural resonance of the circuit are the only controllers of the current. However, in reality the varying mains voltage is a forcing function present over
the current pulse time so determining the shape of the current pulse involves solving the circuit’s controlling differential equations which include the forcing function.

In general, solving these differential equations exactly, even in this simplified case, is impossible due to the solution being transcendental. To make progress with this model it is necessary to resort to a numerical technique.

The fact that the behaviour of such a seemingly simple circuit is not straightforward to predict comes as no surprise because the switch can be closed at any time resulting in the forcing function being part of a “chopped off” sine wave. Such a forcing function is mathematically less tractable than the common examples used, in for example [36].

2.2 Simplest model

Although determining all aspects of the behaviour requires a quite involved procedure using an accurate model of the circuit, it is worth first investigating how the simplest possible model behaves. It turns out that using this simple model predicts much of the behaviour and has the advantage that the solutions for the equations which describe it are obtainable exactly.

Let us assume that the circuit is such that its resonance is very much greater than 50Hz so that the mains voltage can be assumed to be a fixed DC value over the current pulse period. Further, assume that any resistance in the circuit is small enough to be ignored. The circuit is illustrated in Figure 4.2 overleaf.
Figure 4.2. Assume fixed mains voltage over duration of single current pulse. The initial current is zero because the switch starts open.

Using Kirchhoff’s Voltage Law from just after the switch is closed, the initial voltage across the inductor is: $V_{\text{Mains}} - V_{Ci}$ and this will cause an inductor current $i$ to begin to flow which, at any instant is given by: $L \frac{di}{dt} = V_{\text{Mains}} - V_{C}$. This current will of course be altering the capacitor voltage which is related to the charge $q$ on the capacitor by $V_{C} = \frac{1}{C} q$. We can now form the second order differential equation:

$$L \frac{d^2 q}{dt^2} = V_{\text{Mains}} - \frac{1}{C} q.$$  

Standard texts, for example [37], show the solution is of the form $q = A \cos(\omega t) + B \sin(\omega t) + K$ where $\omega = \sqrt{\frac{1}{LC}}$ and $A$, $B$ and $K$ are constants.

The constant $K$ is the “particular integral” $f(t) = K$ and it is constant because we are assuming the mains voltage is constant and its value is $CV_{\text{Mains}}$ (found by substitution into the differential equation). $A$ and $B$ are found by substituting known conditions into the complete general solution $q = A \cos(\omega t) + B \sin(\omega t) + CV_{\text{Mains}}$ and its derivative.

When this is done for the known conditions of no current and an initial charge on the capacitor of $CV_{Ci}$ we get the complete solution stated at the top of the next page:
\[ q = C(V_{Ci} - V_{Mains}) \cos \omega t + CV_{Mains} \] which gives the voltage on the capacitor as:

\[ V_c(t) = (V_{Ci} - V_{Mains}) \cos \omega t + V_{Mains} \]

and the current as:

\[ i_c(t) = C(V_{Mains} - V_{Ci}) \omega \sin \omega t \]

From these we can see that, in this simple case, the current pulse will be a half sine since the switch is a triac which switches off when the current reaches zero. Its length will be determined by the resonant frequency \( \omega \). Its peak amplitude is determined by the resonant frequency, capacitor value and the difference between the instantaneous mains voltage at the time of firing and the pre-existing capacitor voltage.

### 2.2.1 Circuit behaviour surprises

Even this simplified circuit with its straightforward analysis and its unrealistic assumption of constant mains voltage over the pulse period holds the answer to some of the interesting behaviour the physical circuit exhibits. To see this we must determine the exact length of the pulses so that we can determine the voltage on the capacitor at the end of the first pulse. This sets the initial conditions for the second pulse and the voltage at the end of that pulse sets the conditions for the following one and so on.

We have \( i_c(t) = C(V_{Mains} - V_{Ci}) \omega \sin \omega t \). The current starts when the triac triggers and the pulse will end when the current next reaches zero. If it is triggered at \( t=0 \) it will obviously end when \( \omega t = \pi \). If we insert the time calculated from this into the equation which gives us capacitor voltage, \( V_c(t) = (V_{Ci} - V_{Mains}) \cos \omega t + V_{Mains} \), we get the voltage on the capacitor at the end of the first pulse, \( V_{C1} = (V_{Ci} - V_{Mains}) \cos \pi + V_{Mains} \) which reduces to \( V_{C1} = 2V_{Mains} - V_{Ci} \). This will be the initial voltage on the capacitor at the start of the second pulse. In the mean time the mains voltage will have changed (we are only assuming it is constant over the duration of each pulse). Let us label the mains voltage...
with a suffix $Mains^n$ relating it to the value at the time of the $n$th pulse. We have $V_{C1} = 2V_{\text{Mains}1} - V_{C_i}$ giving the initial voltage for the second pulse and therefore the voltage on the capacitor at the end of this pulse will be $V_{C2} = 2V_{\text{Mains}2} - (2V_{\text{Mains}1} - V_{C_i}) = 2V_{\text{Mains}2} - 2V_{\text{Mains}1} + V_{C_i}$.

This gives us the initial voltage for the third pulse and so the capacitor voltage at the end of the third pulse will be $V_{C3} = 2V_{\text{Mains}3} - 2V_{\text{Mains}2} + 2V_{\text{Mains}1} - V_{C_i}$ and

$V_{Cn} = 2V_{\text{Mains}n} - 2V_{\text{Mains}(n-1)} + 2V_{\text{Mains}(n-2)}^{\ldots} + 2V_{\text{Mains}1} - V_{C_i}$ for odd $n$ and

$V_{Cn} = 2V_{\text{Mains}n} - 2V_{\text{Mains}(n-1)} + 2V_{\text{Mains}(n-2)}^{\ldots} - 2V_{\text{Mains}1} + V_{C_i}$ for even $n$.

The voltage on the capacitor is not the primary value of interest but it is important. Any capacitor will have a maximum voltage limit which cannot be exceeded. This formula clearly shows us that if we were to trigger the circuit at 100Hz when the mains frequency was 50Hz, the results will be disastrous. At each pulse the total voltage on the capacitor increases because the mains polarity reversal is countered by the equations’ successive term polarity reversals (assuming we are not triggering exactly on mains zero crossings which gives all zero terms). This interesting behaviour was first found experimentally when each time the circuit’s gate firing frequency approached 100Hz there was a flash and bang. Clearly, prior analysis of the theoretical behaviour would have been preferable. As it turns out, closed form analysis of this iterative equation with the added assumption that the mains is sinusoidal (except during current pulses), is quite straightforward.
3 Closed form analysis of the simplified SWD circuit

Let us begin with the last iterative equation for the voltage on the capacitor after an even number of triac firings (given above). If we label the general term $2V_{\text{Main}x}$ where $x= n$, $(n-1)$, $(n-2)$ etc. The even $x$ terms can be collected together from that equation which is valid for even $n$: $V_{\text{Even}x} = 2V_{\text{Main}n} + 2V_{\text{Main}(n-2)} + 2V_{\text{Main}(n-4)}$ .......

Let us assume for the moment that there is no initial charge on the capacitor and therefore $V_{Ci}$ is zero. Similarly, the odd $x$ terms are collected from that same equation which is valid for even $n$: $V_{\text{Odd}x} = 2V_{\text{Main}(n-1)} + 2V_{\text{Main}(n-3)}$ .......$+2V_{\text{Main}d}$ and so:

\[ V_{Cn} = V_{\text{Even}x} - V_{\text{Odd}x} \text{ (applies for even } n) . \]

This equation can be expressed in terms of a phase variable $\phi$ using the fact that the mains voltage is sinusoidal and the triac firing frequency is constant as the diagram in Figure 4.3 illustrates:

**Figure 4.3** Regular firing of triac at numbered positions $\phi$ radians apart.
The angle \( \phi \), separating the current pulses, is determined by the relationship between the mains frequency, 50Hz, and the transmitted signal frequency, \( S_f \) (which is generally half the gate firing frequency as will be seen later) and is given by \( \phi = \frac{50}{S_f} \pi \) radians.

Here it is assumed the first triac firing pulse arrives exactly \( \phi \) radians after the mains zero crossing. A variable \( k \) will be used to generate the odd numbered and even numbered pulses so that \( V_{even} = 2\sqrt{2} \times 240 \sum_{k=0}^{k=N} \sin 2k\phi \) gives the sum of the voltages for even values of \( x \) and \( V_{odd} = 2\sqrt{2} \times 240 \sum_{k=0}^{k=N} \sin(2k\phi + \phi) \) gives the sum of the voltages for odd values of \( x \). For example, the voltage on the capacitor after the 8th pulse is

\[
V_{C8} = 2\sqrt{2} \times 240 \left( \sum_{k=0}^{k=4} \sin 2k\phi - \sum_{k=0}^{k=3} \sin(2k\phi + \phi) \right)
\]

If \( n \) is even the voltage after the \( n \)th pulse is:

\[
V_{Cn} = 2\sqrt{2} \times 240 \left( \sum_{k=0}^{k=n/2} \sin 2k\phi - \sum_{k=0}^{k=(n/2)-1} \sin(2k\phi + \phi) \right)
\]

for odd \( n \).

So far in this section we have only been considering even \( n \). To determine the voltage on the capacitor after an odd number of pulses we must apply the same procedure to the capacitor voltage applicable for odd \( n \) given in the previous section. In this case the even \( x \) terms are subtracted and the odd \( x \) terms are added to give the following equation for the voltage after the \( n \)th pulse where \( n \) is odd:

\[
V_{Cn} = 2\sqrt{2} \times 240 \left( \sum_{k=0}^{k=(n-1)/2} \sin(2k\phi + \phi) - \sin 2k\phi \right)
\]

for odd \( n \).

These equations can be used to help show why one observed phenomenon occurred. This behaviour was the switching of the circuit between two distinct modes of operation. Those modes are alternating current pulse polarities and (seemingly) random current pulse polarities. As indicated in previous chapters, only alternating current pulse
polarities are of any use to transmit a fundamental with stable phase, so being able to determine when the circuit will be in each mode is critically important.

3.1 Examining the behaviour of the simplified SWD series equations

In order to progress we need to show that the capacitor voltage is bounded under certain conditions and we need to be able to determine those bounds. Combining intuition with examination of the last equations we can see that it is likely that at large \( n \) values many of the terms will be close to cancelling and we will only have a residual with a value of the order of the mains voltage. To make any clearer prediction we need the equation in a more tractable form and the following trigonometric series identity from [38] will give us that form:

\[
\sum_{k=0}^{n-1} \sin(a + kd) = \frac{\sin(md/2)}{\sin(d/2)} \sin\left(a + \frac{(m-1)d}{2}\right).
\]

As we have seen in the last section, we must treat the case where \( n \) is even completely differently from when \( n \) is odd. For odd \( n \): using the substitutions \( m = \left(\frac{n+1}{2}\right) \), \( a=0 \) and \( d = 2\phi \) we obtain us a closed form solution for \( \sum_{k=0}^{k=(n-1)/2} \sin 2k\phi \) and using the same substitutions for \( m \) and \( d \) but with \( a = \phi \) we obtain a closed form solution for \( \sum_{k=0}^{k=(n-1)/2} \sin(2k\phi + \phi) \) so the series equation for \( n \) odd given in the last section becomes:

\[
V_{Cn,odd} = 2\sqrt{2} \times 240 \left[ \left( \frac{\sin\left(\frac{n+1}{2}\phi\right)}{\sin\phi} \right)^2 - \left( \frac{\sin\left(\frac{n+1}{2}\phi\right)}{\sin\phi} \sin\left(\frac{n-1}{2}\phi\right) \right) \right] \text{ Volts.}
\]

If the number of gate pulses, \( n \), is even, the treatment is different because we must sum the even \( x \) terms (those with \( \sin 2k\phi \)) and subtract the odd \( x \) terms (those with \( \sin(2k\phi + \phi) \)) and also different limits are required for the two different parts.
For even \( n \), the substitutions \( m = \left(\frac{n}{2} + 1\right) \), \( a = 0 \) and \( d = 2\phi \) gives a closed form solution for \( \sum_{k=0}^{m/2} \sin 2k\phi \) and the substitutions \( m = \left(\frac{n}{2}\right) \), \( a = \phi \) and \( d = 2\phi \) gives a closed form solution for \( \sum_{k=0}^{(n/2)-1} \sin(2k\phi + \phi) \) so the series equation for even \( n \) given in the last section becomes:

\[
V_{CnEven} = 2\sqrt{2} \times 240 \left[ \frac{\sin \left(\frac{n\phi}{2}\right)}{\sin \phi} \sin \left(\frac{n\phi}{2} + \phi\right) - \left(\frac{\sin \left(\frac{n\phi}{2}\right)}{\sin \phi}\right)^2 \right] \text{ Volts.}
\]

We can now calculate the voltage on the capacitor after any number of gate pulses or cycles. We notice that at a gate firing rate of 100Hz ( \( S_f = 50 \) ), that is \( \phi = \pi \), we are in trouble because of a divide by zero but we know the voltage just ramps up as was mentioned at the end of the last section. Any other lower value of \( \phi \) gives a sensible result ( \( \phi \) is the angle between successive gate firings so a value of zero is meaningless, and any value greater than \( \pi \) means the attempted transmit frequency is lower than the mains frequency which is also not of much interest).

### 3.2 Effect of non-zero starting phase \( \theta \)

Up to this point we have assumed that the starting voltage on the capacitor, \( V_{C_i} \), was zero volts and that the gate firing sequence started in phase with the mains supply. The summing equations at the end of section 2.2.1 showed that a non-zero \( V_{C_i} \) at \( t=0 \), (with the next gate firing pulse still occurring after \( \phi \) radians) simply adds or subtracts the same voltage to the capacitor voltage for all values of \( V_{Cn} \) which is not particularly interesting. However, a starting phase, where the first gate firing pulse does not occur exactly at a mains zero crossing is much more important. Let this phase be \( \theta \), the initial mains voltage applied to the capacitor at the beginning of the first pulse (for which I
shall later designate \( n=0 \) is therefore \( V_{\text{First}} = \sqrt{2} \times 240 \sin(\theta) \) and the voltage present on the capacitor at the end of this pulse will be \( V_{\text{CFirstEnd}} = V_{\text{C0}} = 2 \sqrt{2} \times 240 \sin(\theta) \). Following this we have a continuation of the series of numbered pulses each being triggered at time \( t = n \phi + \theta \). We can use the trigonometric series identity from [38] to convert the sine sum series, which includes the voltage produced by this first pulse for which \( n=0 \), into its closed form and the resulting equations for the voltage on the capacitor after any number of pulses are given below:

\[
V_{\text{Codd}} = 2 \sqrt{2} \times 240 \left[ \sin\left(\frac{(n+1)\phi}{2}\right) \sin\left(n\phi + \theta\right) \right] - \left[ \sin\left(\frac{(n+1)\phi}{2}\right) \sin\left(\frac{(n-1)\phi}{2}\right) \right] \]  
\text{eqn. (i)}
\]

and

\[
V_{\text{Ceven}} = 2 \sqrt{2} \times 240 \left[ \sin\left(\frac{n+2}{2}\phi\right) \sin\left(n\phi + \theta\right) \right] - \left[ \sin\left(\frac{n+2}{2}\phi\right) \sin\left(\frac{n}{2}\phi + \theta\right) \right] \]  
\text{eqn. (ii)}.
\]

This pair of equations has been arranged in such a way as to highlight the interesting symmetry between them. The first square bracketed term in the equation for odd \( n \) (eqn. (i) above) is exactly the same as the second bracketed term in the equation for even \( n \) (eqn. (ii) above) but with \( n \) replaced with \( n+1 \). The second bracketed term in the equation (i) is exactly the same as the first bracketed term in equation (ii) but with \( n \) replaced by \( n-1 \).

The characteristics of equations (i) and (ii) for \( V_{\text{Cn}} \) were explored in detail, firstly by simulation using a spread sheet created in Microsoft Excel, and secondly by some further manipulation as will be detailed in the following section. One of the most important things the equations can predict is the maximum capacitor voltage at any
transmit frequency. As an example of the exploration of the equations’ behaviour, $V_{cn}$ was determined for every $n$ between 1 and 2000 for each $\phi$ value associated with every $1/10^{th}$ Hz between 150Hz and 500Hz. This was performed for different values of start phase $\theta$ but for this first example $\theta=0$. At each frequency the Maximum positive and negative $V_{cn}$ in the 2000 value sequence was determined, the positive values are plotted in Figure 4.4.

**Figure 4.4** Graph of Maximum positive capacitor voltage (simulation) for $\theta = 0$. Maximum negative capacitor voltage is a reflection of this in the horizontal axis.

At lower frequencies the maximum voltage is generally increased except at frequencies which have a relationship with 50Hz when the range reduces somewhat compared to close frequencies without such a relationship. The reason the maximum capacitor voltages predicted are not quite as large in amplitude at these frequencies is because of a sampling effect, the interaction between the gating frequency and the mains frequency produces shorter cycles at these points in the graph. For example, at 100Hz transmission frequency the gating always occurs at intervals of a quarter of a mains cycle and the
sequence of capacitor voltages is only 4 values long. Where a repeating sequence of capacitor voltage values occurs, its length is always even. This length is always related to the gating frequency (twice the transmission frequency) in the following way: The gating frequency multiplied by any combination of the integer factors of the mains frequency (1, 2, 5, 10, 25) always equals the sequence length multiplied by the mains frequency, 50Hz. 105Hz transmission frequency produces a cycle of length 42 (42×50 is ten times the gating frequency). 237.5Hz produces a cycle length of 38 (38×50 is 2×2 times the gating frequency) and so on. This explains the dips from the overall curve in the figure because short sequences synchronous with the mains waveform usually don’t pick up the extreme values of the underlying continuous function as well as do longer sequences.

Although this particular graph applies for the range of $V_{Cn}$ when the initial firing is at a mains zero crossing, $\theta = 0$, exploration of the effect of changing the starting phase $\theta$ was also performed using the MATLAB script shown in Appendix IV E on page 308. An example of this simulation is shown in Figure 4.5 below but the effect of variation of the starting phase will be analysed mathematically by further manipulation of equations A and B in the next section.

![Variation of Capacitor Voltage with Start Phase angle for 117Hz](image)

**Figure 4.5** Maximum capacitor voltage as $\theta$ varies for 117Hz transmit frequency.
3.3 Finding the maximum capacitor voltage as $\theta$ is varied

So far we have two equations, (i) and (ii) from the previous section, $V_{CnOdd}$ for odd $n$ and $V_{CnEven}$ for even $n$, both are expressible in the form:

$$V_n = \frac{A \sin(z)}{\sin(\phi)} \{\sin(x + y) - \sin(x - y)\}$$

$x$ and $z$ being different for odd and even $n$.

In the odd $n$ case $x = \frac{n}{2} \phi + \theta$ and $z = \frac{n}{2} \phi + \frac{\phi}{2}$, and in the even case $x = \frac{n}{2} \phi + \frac{\phi}{2}$ and $z = \frac{n}{2} \phi + \theta$, in both cases $y = \frac{\phi}{2}$ and $A = 2\sqrt{2} \times 240$. We can now use the identity:

$$\sin(x + y) - \sin(x - y) = 2\cos x \sin y \quad [39]$$

to change the equations into a product form which gives a better insight into how the function behaves:

The equation for odd $n$ becomes:

$$V_{CnOdd} = \frac{2A \sin(z_{Odd})}{\sin(\phi)} \cos\left(\frac{n}{2} \phi + \theta\right) \sin\frac{\phi}{2}$$

and the equation for even $n$ becomes:

$$V_{CnEven} = \frac{2A \sin(z_{Even})}{\sin(\phi)} \cos\left(\frac{n+1}{2} \phi \right) \sin\frac{\phi}{2}.$$
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$$V_{CnOdd} = 2\sqrt{2} \times 240 \sin\left(\frac{n+1}{2} \phi\right) \cos\left(\frac{n}{2} \phi + \theta\right) \sec\left(\frac{\phi}{2}\right)$$

and $$V_{CnEven} = 2\sqrt{2} \times 240 \sin\left(\frac{n}{2} \phi + \theta\right) \cos\left(\frac{n+1}{2} \phi\right) \sec\left(\frac{\phi}{2}\right).$$

It now becomes clear exactly how $\theta$ affects the maximum amplitude of the capacitor voltage. We know that the sin and cosine terms multiplied together can only ever have a maximum amplitude of 1. We can therefore plot the remaining sec term over the frequency band of interest to give the maximum possible capacitor voltage for any transmit frequency. It is shown in Figure 4.6

![Figure 4.6](image_url)

**Figure 4.6** Maximum voltage on capacitor.

If we inspect the sine and cosine terms from either formula we see that they have the same "period" with respect to $n$, for example:

$$\sin\left(\frac{n+1}{2} \phi\right) \cos\left(\frac{n}{2} \phi + \theta\right) = \cos\left(\theta + n \frac{\phi}{2}\right) \sin\left(\frac{\phi}{2} + n \frac{\phi}{2}\right).$$

It would be possible to mathematically prove that the series we have for $V_{CnOdd}$ and $V_{CnEven}$ are samples of a sine wave with period $t = \frac{4\pi}{\phi}$ where $t$ is a continuous variable by
using Whittaker’s interpolation formula \( f(t) = \sum_{n=-\infty}^{\infty} V(n) \sin \left( \frac{t - n}{\phi} \right) \) \text{[40]} to reconstruct a sine wave between the samples (to do this we should have to consider our set of samples with \( n \) positive and finite as a windowed subset of the samples for \( n = -\infty \) to \( \infty \) each with the appropriate \( V(n) \)). However, this procedure is unnecessary because in our case it is rather obvious. We can see by inspection that \( V_{CnOdd} \) and \( V_{CnEven} \) are each a constant times a product of two sampled sine waves of different phase and of period \( \frac{4\pi}{\phi} \). The result of multiplying the sin and cos sampled values at any particular \( n \) will be the same as the value of the interpolated continuous sine and cos waves multiplied together at the equivalent \( t \) value. We know that produces a single sine wave at half the period and the two different phases combine to produce a DC offset (and a phase shift). We will stay with the equation for odd \( ns \) and carry out this multiplication. We can use the same trigonometric identity as we used previously, \( \sin(x + y) - \sin(x - y) = 2\cos x \sin y \) with \( x = \theta + n\frac{\phi}{2} \) and \( y = \frac{\phi}{2} + n\frac{\phi}{2} \) to see that the result of the multiplication \( \cos x \sin y \) is

\[
\frac{1}{2} \sin\left(\theta + \phi + n\phi\right) - \frac{1}{2} \sin\left(\theta - \phi\right)
\]

which has half the period over \( n \) of \( \frac{2\pi}{\phi} \) and a DC offset of \( \frac{1}{2} \sin\left(\theta - \frac{\phi}{2}\right) \). For even \( n \) we end up with the same result except the first sine term and the offset term are added instead of the offset term being subtracted i.e. the result of the multiplication of the sine and cos terms is \( \frac{1}{2} \sin\left(\theta + \phi + n\phi\right) \pm \frac{1}{2} \sin\left(\theta - \phi\right) \) the positive sign being for \( n \) even and negative for \( n \) odd. So we now have:

\[
V_{Cn} = \sqrt{2} \times 240 \sec\left(\frac{\phi}{2}\right) \left[ \sin\left(\theta + \frac{\phi}{2} + n\phi\right) \pm \sin\left(\theta - \frac{\phi}{2}\right) \right].
\]

We finally have a single equation for the capacitor voltage which describes its sequential values. The \( \theta \) value which gives the maximum capacitor voltage for any particular
transmit frequency is also derivable from this equation by setting the factor

\[ \frac{1}{2} \sin \left( \theta + \frac{\phi}{2} + n\phi \right) \pm \frac{1}{2} \sin \left( \theta - \frac{\phi}{2} \right) \]

equal to 1. It is shown in Appendix IV I on page 304 that this maximum occurs when

\[ \theta - \frac{\phi}{2} = 2m\pi + \frac{\pi}{2} \]

where \( m = \pm 0, 1, 2, 3, \ldots \).

Figure 4.7 gives a plot of the capacitor voltage for consecutive pulses for an arbitrary \( \phi \) (arbitrary transmit frequency) and \( \theta \) for the maximum capacitor voltage for that frequency obtained using Microsoft's Excel spreadsheet program in which sequential application of the equations obtained in section 2.2.1 was performed. In this plot the operation of the equation developed in this section can be identified with the pulses alternating between the values of two offset sine waves, one with a positive offset and the other with the equivalent negative offset.

**Figure 4.7** Sequence of capacitor voltages. Randomly chosen transmit frequency but with the \( \theta \) value which results in the maximum voltage for this frequency.
Next, considering the interpolated continuous case we can re-plot the maximum capacitor voltage for $\theta = 0^\circ$ by saying the underlying two sine terms will always have a maximum of $\frac{1}{2}$ so the previous $\sec\left(\frac{\phi}{2}\right)$ graph must be reduced in amplitude by multiplying by $\frac{1}{2} + \frac{1}{2} \sin\left(\frac{\phi}{2}\right)$. We know that this maximum of the continuous case will be reached at some $n$ value in the sampled case provided there is no relationship between the transmit frequency and 50Hz. Here is the graph of that function and happily it matches Figure 4.4 (the simulation result graph) except for the dips due to sampling effects in the simulation case.

![Graph of function describing maximum voltage on capacitor with $\theta = 0^\circ$.](image)

*Figure 4.8* Graph of function describing maximum voltage on capacitor with $\theta = 0^\circ$. 
One interesting behaviour is that, generally, the capacitor voltage does not alternate positive and negative. Figure 4.9 gives an example of the sequence of capacitor voltages for the same frequency obtained in the same way as for the graph of the previous sequence shown in Figure 4.7, but with the start phase $\theta = 0$.

**Figure 4.9** Sequence of capacitor voltages for 323.1Hz, $\theta = 0^\circ$.

However, even with the capacitor voltage not following a positive-negative sequence the current pulse polarity often does. The sequence of current pulse peak amplitudes for the same frequency and start phase as in the last figure is shown in Figure 4.10. The next section investigates why this alternating current polarity is often the case and under what circumstances it is not.

**Figure 4.10** Current pulses alternate in polarity though capacitor start voltages do not.
3.4 Finding the current

The most important aspect of the circuit's behaviour is the sequence of current pulse polarities. To transmit a fundamental of constant phase the pulses must always alternate in polarity. It had been experimentally found that this was sometimes not the case although this fact is absent from the literature. The reason the circuit behaves in these two modes, alternating and not alternating current pulse polarity, can be revealed and the critical values at which the two modes separate can be determined by further analysis of the relationships established above. This will now be done and the results compared with simulations.

So far we have determined the voltage on the capacitor, which is important for selection of the capacitor rating, but not as critical as knowing the behaviour of the current. We can extend the equations developed previously to give the peak current quite simply by remembering that the current during each pulse is

\[ i_C(t) = C(V_{\text{Mains}} - V_C)\omega \sin \omega t \]

(from section 2.2.1) where \( V_{\text{Mains}} \) is the mains voltage present at the start of the pulse and \( V_C \) is the voltage on the capacitor at the start of the pulse which will, in general, be the voltage on the capacitor at the end of the last pulse. This current will have a maximum value of \( i_p = C\omega(V_{\text{Mains}} - V_C) \). We know the capacitor voltage at the start of any pulse is the capacitor voltage at the end of the last pulse, \( V_{C_{(n-1)}} \). The mains voltage for the \( n^{th} \) pulse is given by

\[ V_{\text{Mains}} = 240 \times \sqrt{2} \sin(n\phi + \theta) \]

For the first pulse occurring at angle \( \theta \) of the mains cycle, for which \( n \) is 0, the pre-existing capacitor voltage will be assumed to be zero, the peak current \( i_0 \) will therefore be given by:

\[ i_0 = 240 \times \sqrt{2}C\omega \sin(n\phi + \theta) \]

from the result in section 2.2.
For any subsequent pulse the peak current will be:

\[ i_n = 240 \times \sqrt{2} C \omega (\sin(n\phi + \theta) - V_{n-1}) \]

where \[ V_{n-1} = 240 \sqrt{2} \sec\left(\frac{\phi}{2}\right) \sin\left(\theta + \frac{\phi}{2} + (n-1)\phi\right) \pm \sin\left(\theta - \frac{\phi}{2}\right) \]

with the ± being + for \( n-1 \) even and – for \( n-1 \) odd.

### 3.5 Conditions for current's alternating behaviour

We can now write down the equation for the current pulse amplitude for any \( n \) (including \( n=0 \)) as:

\[ i_n = 240 \sqrt{2} C \omega \left( \sin(n\phi + \theta) - \sec\left(\frac{\phi}{2}\right) \sin\left(n\phi + \theta - \frac{\phi}{2}\right) \pm \sin\left(\theta - \frac{\phi}{2}\right) \right) \]

with the ± being - for \( n \) even and + for \( n \) odd.

This can be written as: \[ i_n = K_1 \left( \sin(n\phi + \theta) - K_2 \sin\left(n\phi + \theta - \frac{\phi}{2}\right) \pm K_3 \right) \]

with the ± being - for \( n \) even and + for \( n \) odd.

The current pulses will alternate in phase if all of the even \( n \) pulses are of one polarity and all of the odd \( n \) pulses are of the opposite polarity (they cannot both always be of the same polarity because that would mean an ever increasing capacitor voltage). The equation above, for \( i_n \), has the answer to when this phenomenon occurs. We need to find those conditions when the alternating addition and subtraction of the offset will cause \( i_n \) itself to alternate in polarity and that will now be considered. The equation is made up of an overall amplitude \( K_1 = 240 \sqrt{2} C \omega \), the difference between a \( \sin(n\phi + \theta) \) term and an amplitude modified sine term (reduced by factor \( K_2 = \sec\left(\frac{\phi}{2}\right) \)) of the same frequency but with a different phase shift (different by \( \frac{\phi}{2} \)) and a DC offset,
\[ K_3 = \sec\left(\frac{\phi}{2}\right) \sin\left(\theta - \frac{\phi}{2}\right). \] Both the sine terms have the same period in \( n \) of \( \frac{2\pi}{\phi} \) so a phasor representation can be used to help visualise the conditions whereby the DC shift \( \sec\left(\frac{\phi}{2}\right) \sin\left(\theta - \frac{\phi}{2}\right) \) and the ratio of the two sine waves amplitudes, \( \sec\left(\frac{\phi}{2}\right) \) gives even current pulse polarities being the same and opposite to all odd current pulse polarities.

We can imagine the phasors rotating smoothly as \( n \) increases through all positive real values but that the diagram is illuminated by a stroboscope only when \( n \) is integer. Calculations in the continuous case can be seen to be valid even though our \( n \) is integer only. First we will consider the case where the transmit frequency is high and so \( \sec\left(\frac{\phi}{2}\right) \approx 1 \) and the phasors representing the two sine terms are of almost equal length and separated in phase by the small value \( \frac{\phi}{2} \). The case where the DC offset, \( \sec\left(\frac{\phi}{2}\right) \sin\left(\theta - \frac{\phi}{2}\right) \), is zero is drawn in Figure 4.11. This, for example, occurs first at \( \theta = 5.82^\circ \) for a transmit frequency of 773Hz.
Figure 4.11 Phasors giving sine terms when there is no DC offset term.
(a) drawn when angle for first term is zero and showing relationship between both terms and (b) when terms are equal.

The resolution on the vertical axis for each of the two phasors gives the two sine terms. The blue phasor represents the first sine term $\sin(n\phi + \theta)$ and it is shown in (a) for a value of $n$ which sets its angle to zero. The red phasor represents the second sine term which at this value of $n$ has a vertical axis component of approximately $-\sin\frac{\phi}{2}$. The second diagram (b) shows the phasors symmetrical about the vertical axis at a value of $n$.
a little more than a quarter of a period in \( n \) later, that is at \( n = \frac{\pi}{2\phi} + \frac{1}{4} - \frac{\theta}{\phi} \). The extra value of \( n = \frac{1}{4} \) is the increment in \( n \) it takes the phasor to travel an angle of \( \frac{\phi}{4} \) past its quarter period point. This is because it rotates an angle of \( 2\pi \) when \( n \) increases by \( \frac{2\pi}{\phi} \) therefore it rotates an angle of \( \frac{\phi}{4} \) when \( n \) increases by \( \frac{1}{4} \). At this point the difference between the two sine terms becomes zero. The difference (first term minus second term) was positive, but it is now going to become negative. When this happens for even \( n \) or odd \( n \), the current pulse polarities cannot be alternating. Next consider what happens after a further rotation of exactly a quarter period. This situation is shown in the diagram below along with a diagram where a DC offset between the two phasors has been added.
Figure 4.12 Showing minimum DC offset which prevents sign change.

In the top diagram (a) of Figure 4.12 the phasors are in the position where the difference between the sine terms is most negative so the DC offset indicated in the bottom diagram (b) is the minimum shift which would ensure that the point of the red phasor is always below the point of the blue phasor. The diagrams show that the minimum size of
this shift is $2 \sin \frac{\phi}{4}$. This offset is obtained when $\sec \left(\frac{\phi}{2}\right) \sin \left(\theta - \frac{\phi}{2}\right) = 2 \sin \left(\frac{\phi}{4}\right)$. For small $\phi$ this is at $\theta = \phi$ and for our example frequency of 773Hz this is at $\theta = 11.6^\circ$. If we choose a start angle of less than this critical value but greater than that which gives zero DC offset ($\theta = 5.82^\circ$ for this transmit frequency) then we will have the current pulse amplitudes for even values of $n$ represented by a phasor diagram in-between the two in Figure 4.12 because the offset is subtracted and the polarity of the current pulses will therefore not be constantly alternating. We would see the result of the subtraction of the two sine terms change in polarity twice every rotation meaning the even $n$ current pulses will have polarity changes. The phasor diagram for the odd $n$ pulses will be different but it too will show that the polarity of odd current pulses changes if the DC shift is less than this critical value but greater than that for zero DC offset. This means that as $n$ increments by one from even to odd to even to odd, the current pulse polarity cannot not always alternate. For this frequency of 773Hz, a start phase of greater than 11.6° is required for the satisfactory condition of alternating polarity current pulses.

Up to this point we have been using the simplifying assumption that the transmit frequency is high enough that $\sec \frac{\phi}{2}$ can be assumed to be unity. At lower frequencies, with a larger value of $\phi$, this is not justified and the phasor diagram must be re-drawn with two phasors of different length. Consider a transmit frequency of 117Hz. For which $\sec \frac{\phi}{2} = 1.28$. The phasor diagram for no DC offset will again tell us the minimum DC offset required to ensure the sine component of one phasor minus the sine component of the other is always of the same sign and it is shown in Figure 4.13:
This relates to one of four ways of ensuring continuously alternating current pulse polarities with a DC offset and illustrates the DC offset required when the red phasor is to be continuously below the blue phasor. A similar diagram could be drawn to show the DC offset required to ensure that the red phasor was continuously above the blue phasor and two other equally valid diagrams, relating to the equation being considered, could be drawn with the red phasor leading the blue phasor. Each diagram would have a triangle with the same size edges as in the diagram shown and we can solve these triangles using the cosine rule to give the minimum offset required as $D = \sqrt{\sec^2\left(\frac{\phi}{2}\right) - 1}$. This can then be equated to the DC term in the equation, $\sec\left(\frac{\phi}{2}\right) \sin\left(\theta - \frac{\phi}{2}\right)$ to give the borderline values for the required minimum start phase:

From $\sqrt{\sec^2\left(\frac{\phi}{2}\right) - 1} = \sec\left(\frac{\phi}{2}\right) \sin\left(\theta - \frac{\phi}{2}\right)$ we get: $\sqrt{\sin^2\left(\frac{\phi}{2}\right)} = \sin\left(\theta - \frac{\phi}{2}\right)$ giving:

$\sin\left(\frac{\phi}{2}\right) = \pm \sin\left(\theta - \frac{\phi}{2}\right)$ which has solutions $\theta = \phi$, $\theta = \phi + 180^\circ$, $\theta = 180^\circ$ and $\theta = 0^\circ$. 

---

**Figure 4.13** Lower transmit frequency when phasors cannot be assumed to be equal length.
For our example frequency of 117Hz with $\phi = 76.9^\circ$, $\theta$ is 76.9°, 256.9°, 180° or 0°. These are the critical or boundary values for $\theta$, examination of the equation for $i_n$ reveals that the ranges for which continuous polarity alternation occurs is $\theta$ between 76.9° and 180°, or $\theta$ between 256.9° and 0° as illustrated in Figure 4.14. At the higher frequency example of 773Hz, the range of satisfactory starting phase value is larger at between 11.6° and 180° and 191.6° and 0°.

$$\theta$$ required for always alternating current pulses

![Diagram showing acceptable ranges for $\theta$ at 117Hz](image)

**Figure 4.14** Satisfactory start angle for transmission at 117Hz.

These theoretical results were checked by a simulation performed using Microsoft's Excel spread sheet program in which sequential application of the equations obtained in section 2.2.1, relating to a single pulse, gave the pulse polarity. The simulation was set up to check the polarity of a sequence of 4,000 pulses.

The range of $\theta$ for successfully alternating current pulses was found to confirm the results from the theory developed above as is illustrated in Table 4.1.
Table 4.1 Start phase problem range, comparison of calculation and simulation.

<table>
<thead>
<tr>
<th>Freq (Hz)</th>
<th>Problem Range by calculation (degrees for theta, ( \theta ))</th>
<th>Problem Range by simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>117</td>
<td>76.9 to 0</td>
<td>77 to -1</td>
</tr>
<tr>
<td>157</td>
<td>57.3 to 0</td>
<td>57 to -1</td>
</tr>
<tr>
<td>223</td>
<td>40.3 to 0</td>
<td>41 to -1</td>
</tr>
<tr>
<td>323.1</td>
<td>27.8 to 0</td>
<td>27 to 0</td>
</tr>
<tr>
<td>773</td>
<td>11.6 to 0</td>
<td>12 to 0</td>
</tr>
</tbody>
</table>

4 Accurate model

The previous analysis was conducted using the crude assumption that the mains voltage did not change over the period of a transmitted current pulse. The analysis revealed much about the real behaviour of the circuit, illuminated the reasons for that behaviour and had the advantage that a closed form solution was achievable. However, the assumption is clearly unrealistic, especially when the resonant frequency of the circuit is not orders of magnitude greater than the mains frequency. The effect of any resistance in the circuit was also ignored as was harmonic distortion of the mains voltage which is generally present.
To develop a model incorporating these factors we must re-construct the differential equation for a single current pulse including resistance and a more realistic forcing function:

\[
V_{\text{mains}} = L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q
\]

where \( V_{\text{mains}} \), \( i \) and \( q \) are all functions of time. This gives the second order differential equation which we need to solve:

\[
L \frac{d^2q}{dt^2} = V_{\text{mains}} - R \frac{dq}{dt} - \frac{1}{C} q
\]

We have the initial conditions of a zero current and an initial charge \( q_0 \) on the capacitor. Standard texts, for example [37], show the solution is in the form of a steady state solution plus a transient solution. Let us assume for the moment that the mains is a pure sine wave with RMS value 240V and frequency 50Hz, this gives the following steady state solution (see Appendix IV B on page 305 for details):

**Figure 4.15** Circuit model for setting up differential equations.
Chapter IV SWD circuit behaviour

\[ q_{ss} = [A \cos(2\pi50t + \delta) + B \sin(2\pi50t + \delta)] \]

where
\[
A = -\frac{(240\sqrt{2})\left(\frac{1}{C} - L(2\pi50)^2\right)}{\left(\frac{1}{C} - L(2\pi50)^2\right)^3 + \left(\frac{1}{C} - L(2\pi50)^2\right)(R(2\pi50)^2)}
\]

and
\[
B = \frac{(240\sqrt{2})\left(\frac{1}{C} - L(2\pi50)^2\right)}{\left(\frac{1}{C} - L(2\pi50)^2\right)^2 + R(2\pi50)^2}.
\]

We are most interested in the current so we differentiate the equation for \( q_{ss} \) above to give:

\[ i_{ss} = [B2\pi50\cos(2\pi50t + \delta) - A2\pi50\sin(2\pi50t + \delta)] \]

This is just the steady state solution which is a small 50Hz current flowing in the circuit.

We are much more interested in the transient so we must continue and find the complete solution. This is done by finding the complementary function by setting \( f(t) = 0 \), (standard procedure, for example [37]). This is done in Appendix IV C on page 306 and the result is:

\[ q_t = e^{\alpha t}(G \cos \beta t + H \sin \beta t) \]

where
\[
\alpha = \text{Re}\left\{-R + \sqrt{R^2 - 4\frac{L}{C}}\right\}
\]

and \( \beta = \text{Im}\left\{-R + \sqrt{R^2 - 4\frac{L}{C}}\right\}/2L \), and \( G \) and \( H \) are constants to be found from the initial conditions of zero current and capacitor voltage \( V_{c0} \). Again, we are more interested in the transient current so must differentiate the equation for \( q_t \) to give:

\[ i_t = e^{\alpha t}(H\beta \cos \beta t - G\beta \sin \beta t) - \alpha e^{\alpha t}(G \cos \beta t + H \sin \beta t). \]
Now we have the complete solution for the current given by:

\[ i = B 2 \pi 50 \cos(2 \pi 50 t + \delta) - A 2 \pi 50 \sin(2 \pi 50 t + \delta) + e^{\alpha t} \left( H \beta \cos \beta t - G \beta \sin \beta t \right) \]

\[ - \alpha e^{\alpha t} \left( G \cos \beta t + H \sin \beta t \right) \]

and for the charge:

\[ q = A \cos(2 \pi 50 t + \delta) + B \sin(2 \pi 50 t + \delta) + e^{\alpha t} \left( G \cos \beta t + H \sin \beta t \right) \]

The two constants which are still unknown are G and H and these are found by substitution of the initial conditions, zero current at the time \( t \) at the start of each pulse into the first equation for \( i \) and the initial charge \( CV_{c0} \) on the capacitor at the same time \( t \) into the second equation for \( q \). For each current pulse a different pair of constants will be found because the initial charge for each pulse will be different.

Let us find the constants for the first pulse starting at \( t = 0 \) when \( q = CV_{c0} \).

Substituting into the equation for \( q \) gives:

\[ CV_{c0} = A \cos(\delta) + B \sin(\delta) + G \]

so \( G \) is known. Also at \( t = 0 \), \( i = 0 \) and substituting into the equation for \( i \) gives:

\[ 0 = B 2 \pi 50 \cos(\delta) - A 2 \pi 50 \sin(\delta) - G \alpha + H \beta \]

so \( H \) is known.

Using the equation for \( i \) which now has no unknowns we next need to find the time \( t \) at which it first reaches zero.

4.1 Finding the current pulse polarity, amplitude and shape

The equation for \( i \) is made up of the sum of a 50Hz current caused by the mains forcing function and an exponentially decaying sine wave at a frequency of \( \beta \) rads/sec close to
the natural resonant frequency of the LC circuit. From the equations generated we need to find the point in time at which the total current first returns to zero for the first current pulse, that is, the pulse length. This will then give us the charge remaining on the capacitor and therefore the initial conditions for the following pulse. Even if the resistance is ignored, giving a solution which is simply the sum of two different frequency sine waves, finding the pulse length is not straightforward. The reason for this is that, in almost all cases, the solution is transcendental and cannot be found in closed form. The exception is where the amplitude of those two sine waves is identical but in all other cases the pulse length must be found using numerical techniques.

A MATLAB script, given in Appendix IV D on page 307, was written to implement the Newton-Raphson method for finding the roots (zero crossing) and also to plot the pulse shape to give a better understanding of the behaviour of these equations. Two plots of $i(t)$ are shown in Figure 4.16 overleaf derived directly from the equation for the current $i$ stated in the last section. In each of these two plots the circuit being simulated has a capacitor of 18 $\mu$F, an inductor of 9mH and a resistor of 0.1 $\Omega$ giving the constants $A = -3.566 \times 10^{-6}$, $B = 0.006208$, $\alpha = -5.555$ and $\beta = 2484$. For the plot in Figure 4.16 (a), starting phase $\delta = 0$ and initial capacitor voltage $V_{c0} = 0$ (giving the two constants determined by initial conditions as $G = 3.566 \times 10^{-6}$ and $H = -7.848 \times 10^{-4}$). For the plot in Figure 4.16 (b) a non-zero starting phase of 90º was used and again $V_{c0} = 0$. It can be seen that the ratio of the transient response to steady state response is very different for these two different sets of initial conditions.
Figure 4.16 Plot of current.

(a) when switch closes at mains zero crossing and (b) when closed at 90° mains phase.
Chapter IV  SWD circuit behaviour

In the top plot (a) opposite, the switch was turned on at the mains zero crossing with no charge on the capacitor. The fact that the circuit shows an immediate response is in direct contrast to the previous simplified model analysed in section 2.2. The response is not a simple 50Hz current because the switching of the 50Hz input has caused a discontinuity at \( t = 0 \). This discontinuity contains all frequencies and the circuit responds to that part of the input which is at its own resonant frequency. The current produced is made up of the sum of the transient response at the higher frequency and the steady state response at 50Hz. Both elements are about the same amplitude to start with (it is not shown but the transient becomes negligible after about half a second). In the lower graph (b) the switch is turned on when the 50Hz voltage is at its positive peak value while the capacitor voltage is again zero. This gives the resonant circuit a much bigger kick so, as expected, the transient response has a much larger amplitude than the steady state 50Hz current. Both these plots are of the equation itself which takes no account of the physical circuit’s switch re-opening when the current returns to zero. It has been plotted in this way so that the reduction in the transient response with time can be seen. In the SWD circuit with a triac, only the first half cycle will be produced because the triac will switch off when the current first reaches zero. The time at which this occurs, as determined by the Newton-Raphson approximation for the first roots of the equations above performed in the MATLAB program, is 2.25ms in the top plot and at 1.25 ms for the lower plot. The resonant frequency for this circuit is 400Hz and it has quite a high Q so we might have expected a half cycle pulse to be close to that predicted for the resonant frequency (1.25ms). This is the case when the transient current is dominant (peaking at 15 amps for the first half cycle of the lower plot) but, as was found in the physical circuits, sometimes when the pulse peak is low its width is wider than that predicted by the circuit’s resonant frequency. The MATLAB script also
calculates the voltage on the capacitor. For the top plot case where the circuit is fired at the mains zero crossing the voltage present on the capacitor at the end of the first half cycle is 251V and for the lower plot which begins at the mains peak, the finishing voltage is almost twice the mains peak voltage at 661V. These capacitor voltages are important because they then form the initial conditions for subsequent pulses. In the second case it can be seen by comparison with the simple model (where the end voltage would have been exactly twice the mains peak) that the voltage is less than the simple model predicted because the instantaneous mains voltage has been reducing during the period of the pulse. The same program which solves the above equations can be run again to find the parameters associated with a second pulse, this time using the new initial voltage on the capacitor to obtain a new pair of constants G and H. Now the transmit frequency comes in as a variable because it will determine the new value for $\delta$, the phase of the mains at the start of the new pulse. Let us take one of the same transmit frequencies as was used earlier, 117Hz, as an example so that we can compare results with those predicted by the previously developed simple model. At this frequency, the case where the first pulse was triggered at the mains zero crossing gives $\delta$ for the second pulse as 1.343 rads. Combining this with the voltage of 251V left on the capacitor at the end of the first pulse and the solution to the equations above tells us the second pulse will have a peak of 3.83 Amps, a duration of 1.28ms and we will be left with 423.9V on the capacitor. These results are significantly different to those predicted by the simple model. This is because the simple model failed to predict the fact that there would be a positive pulse next to the mains zero crossing for the obvious reason that it did not allow for any mains voltage stimulus at the first pulse position. The result is the peak current for the pulse triggered at 1.343 rads is much lower than the 10Amps predicted by the simple model and the remaining capacitor voltage at the end of the pulse is completely different from the simple model's prediction of -362V. Although there are these
discrepancies at the detailed level between the simple model and the new one based on an analysis of the more realistic differential equations, the important broad predictions of both models will later be shown to be similar.

Next let us compare the predictions for the second pulse for the situation where the first pulse was triggered at the mains peak. In this case $\delta$ for the second pulse is $\frac{\pi}{2} + 1.343$ rads and the initial charge on the capacitor is 661V. The solution to the equations above tells us the second pulse will have a peak of -28.0 Amps, a duration of 1.32ms and we will be left with -643V on the capacitor. These first two pulses are shown (along with the third) graphically below:

\[\text{Voltage on capacitor at start of second pulse} = 661\text{V}\]
\[\text{Voltage on capacitor at start of first pulse} = 0\text{V}\]
\[\text{Mains Voltage}\]

**Figure 4.17** Shape of the first three pulses predicted by the accurate model.
This is broadly what the simple model predicted. The results for the simple model are shown below for comparison.

![Graph](image)

**Figure 4.18** Current and voltage predictions from simple model.

A most important question is does the new model also predict the existence of starting phase bands which lead to alternating pulse polarities and other bands which do not? Using the simple model it was possible to predict these bands and calculate where they would be. With the new model no such direct calculation and proof is possible, a simulation must be performed by sequentially calculating the start voltages and pulse lengths for each pulse and feeding the values into the calculation for the next pulse. A MATLAB script was written to automatically perform this task for the circuit being studied with any component values (the script is not included in an Appendix due to its length and because its core is based on that in Appendix IV D, but a copy of the script is available on request). This is the way the plots of Figures 4.17 and 4.19 to 4.22 were created. At the frequency of 117Hz the simple model had predicted that, for the band of start phases between 0° and 77° continuous alternation of current pulse polarities would not occur. If we select a start phase within that band, say 45°, we would expect the new model to show the polarities do not alternate. Figure 4.19 shows the plot for that start phase (45°) and the same transmit frequency (117Hz):
Figure 4.19 Current pulses at 117Hz, starting at 45°. Polarities do not alternate.

We get a positive pulse followed by two negative pulses so we are in the range where alternation does not occur. This plot also shows why. We know from the previous work on the simple model that the capacitor's initial voltage must cross over from one side of the mains voltage to the other in order to create alternating current pulse polarities. Here we see that the small circle representing the voltage on the capacitor at the beginning of the third pulse is still above the mains voltage as it was for the previous pulse.

By an exhaustive search it was found that, at 117Hz, the range for start phases which do not result in alternation is -1° to 49°. This is much less than the simple model predicted. The reason for this is not completely understood but it may be because there are only a fixed number of different sets of conditions at the start of each current pulse. For example, there are only 117 positions for the gating pulse relative to the mains
waveform at 117Hz Transmit frequency (as detailed in section 3.1). As the sequence repeats, the conditions where a gate pulse could have upset the alternating sequence, at a start phase of 50° for example, are simply never encountered. In practice we will never have exactly 50Hz mains and exactly 117Hz transmit frequency. To overcome this disadvantage of simulation a frequency of 117.1Hz was tested. This has a repeating sequence length of 1171 pulses. This was found to increase the range of start phases which do not lead to alternating current pulse polarities slightly but a comprehensive search for the exact boundary of alternating and non-alternating sequences was not undertaken due to the processing time required.

4.2 Modes of operation

It has been noticed that sequences of current pulses where the polarity is not alternating generally have a peak amplitude significantly less than alternating sequences. These low peak amplitudes make another important phenomenon much more likely. This phenomenon we could term "delayed zero crossing". It occurs when a particularly small current pulse coincides with a high enough rate of change of main voltage to prevent the natural resonance current waveform reaching the zero crossing point at its first half cycle. These occurrences are relatively rare but have a profound effect in re-setting the sequence to one with a completely different start phase. However, because there is a capacitor voltage present after the delayed return to zero event, the new sequence is different to the case where the gating is started at that mains phase with a discharged capacitor. The delayed return to zero events sometimes cause the operating mode to switch between non-alternating and alternating current pulses. Once in the alternating mode the individual pulses are of a higher amplitude and therefore the circuit tends to stay in this mode. The effect of the delayed return to zero events shall be illustrated by an example. At a transmit frequency of 151.5Hz, with its zero crossing synchronised
with a mains zero crossing ($\theta = 0$), after the 65\textsuperscript{th} current pulse a delayed zero crossing event occurs and is shown in Figure 4.20:

![Figure 4.20 Delayed return to zero event predicted by simulation.](image)

The reason for the delayed zero crossing event is quite clear, the triac has fired when the capacitor voltage is quite close to the mains voltage causing an initially small positive pulse. The triac firing is also at a point where the mains voltage is about to exhibit a positive rate of change. This creates a current caused by the forcing function which adds to the small resonance caused current. Because the resonance current was so low, the forcing function current is enough to prevent the total current reaching a zero value at the end of the first resonance swing. The now steeply changing mains voltage means that the forcing function current continues to prevent the total current from reaching zero for several more resonance cycles.
The subsequent sequence is shown in Figure 4.21 below:

**Figure 4.21** Recovery after a delayed return to zero event.

It can be seen that the new sequence is also a non-alternating current sequence. It continues on with other occasional delayed return to zero events.

There is another phenomenon which can cause a sudden and dramatic change to a sequence. This occurs when the mains and capacitor voltages are so close in value that no significant current pulse results. This has the effect of saving a substantial capacitor voltage for a later time when the mains voltage has undergone a large change. A much larger current pulse will now be produced and this can flick the mode of operation into alternating polarity. An example of this "missing pulse" event occurs at 187Hz for the component values in the model being used and is illustrated in Figure 4.22 opposite.
A prior delayed return to zero event has caused a change in sequence and then the mains voltage matches the capacitor voltage for two consecutive pulses (the fourth and fifth gate firing positions below):

![Graph showing current, capacitor voltage, and mains voltage over time]

**Figure 4.22** Missing pulse events causing change to alternating polarity mode.

The sequence has been caused to change from non-alternating to alternating. The subsequent sequence shows an initial growing of the current pulses and then a continuous stable alternating sequence.

5 **Comparison of model predictions with experimental results**

All of the phenomena described in the analysis of the models have been found to be present in the behaviour of the physical circuits studied. The models do not predict the exact conditions under which the phenomena are observed in reality but this is not surprising as they incorporate many simplifications. There are, though, some areas of general agreement in conditions between the model phenomena predictions and
observation of the physical circuits. For example, simulation using the accurate
differential model just described predicts a band of frequencies between 187Hz and
222Hz where a starting phase of $\theta = 0$ results in a stable sequence of alternating current
pulses with no delayed return to zero events or negligible amplitude pulses. In practice it
was found that there was a narrow band at a little over 180Hz where stable operation
was encountered and outside this band non-alternating current pulses, missing current
pulses and delayed return to zero events prevented a good signal being produced. The
oscilloscope trace in Figure 4.23, taken when attempting to operate the physical circuit
described in the last chapter at a transmit frequency of 153Hz, shows the capture of a
delayed return to zero event.

![Oscilloscope Trace](image)

**Figure 4.23** Delayed return to zero event captured 2/3 of the way along the trace.

The mains voltage is shown on the blue trace and the scale for the current pulse trace is
10 Amps per division. It can be seen that before the delayed return to zero event the
behaviour was a sequence of pulses which did not alternate in polarity. In contrast,
Figure 4.24 illustrates a record of an alternating current pulse trace for a transmit
frequency of 237.5Hz shown along with the triac gating pulses. The second current pulse
is too small to be distinguished clearly but it was almost certainly negative and so fitted
into the alternating current sequence.
Chapter IV  SWD circuit behaviour

The fact that some of the current pulses were so small leaves this transmission sequence vulnerable to the events described above. At many transmit frequencies long periods of alternating current pulse operation were observed and then a spurious phase change would occur triggered by a delayed return to zero or a missing pulse event, Figure 4.25 gives an example at 262Hz with a phase change captured about ¾ of the way across the trace.

Figure 4.24 Alternating current pulse sequence.

Figure 4.25 Part of a long sequence of alternating current pulses ending in a spurious phase change ¾ of the way along this trace.
Some transmit frequencies, like the 237.5Hz from the last example, have a relationship with 50Hz (as explained in section 3.1) which gives a particularly short repeating sequence length of current pulse values, 42 in this case. This was observed in practice and sometimes, with an initial gate pulse phase in relation to the mains (θ) such that a stable alternating pulse train occurred, the signal would last many seconds without a spurious phase change. However, repeated experiments at exactly the same transmit frequency, but with a different random start phase, resulted in a non-alternating current pattern. The pattern was also seen to be dependant on the temperature of the triac. The triac has been modelled as a perfect switch in the analysis in this chapter but this is not the case in reality. It has losses and also has a minimum current required to hold it switched on. This hold current helps to prevent delayed return to zero events because the triac does not require the current to actually reach zero before switching off. If the hold current decreases, with temperature for example, the circuit becomes more susceptible to these events. If the effective resistance of the switch increases the transient current will be reduced which will also make the circuit more susceptible to delayed return to zero events.

5.1 Comparison of simple model and accurate differential equation model

The simple model yielded a straightforward answer to the question of which conditions result in alternating polarity current pulses. However the accurate differential equation based model showed that the simple model was inaccurate for the case when θ = 0, that is when the first gate pulse occurs at a mains zero crossing. It was also found that the simple model does not accurately predict current pulse levels at lower frequencies. Because of the assumption of constant mains voltage over the length of the pulse the simple model will clearly be more applicable when the resonant frequency of the circuit is high. The resonant frequency for the example above was 400Hz which is as low as
would ever be used. At this resonant frequency the pulse lengths can be up to 2.25ms so care must be taken when analysing transmit frequencies above 222Hz because the triac gate signal for one pulse could arrive before the end of the last pulse and so would not necessarily cause a new period of triac conduction. The low resonant frequency chosen for the examples above highlighted the differences between the models because it leads to significant change in the mains voltage over the duration of a current pulse. In summary, the accurate differential model gives more accurate results but has the disadvantage that it must be used iteratively by simulating each current pulse to determine the initial conditions for the next pulse, whereas the simple model can give direct predictions due to its closed form solution but is not very accurate at lower frequencies.

6 Conclusion

The SWD circuit using an inductor and capacitor is the logical transmitter configuration for generating current signals within a meter for the purpose of upstream communication. This type of circuit produces large signals due to the resonance, yet with low heat dissipation. It is important to be able to predict exactly what the signals will be for a particular gating pattern so that the receiver can be made to look for a match to that pattern within the load current signal at the substation. This chapter looked at the simplest gating pattern which uses a constant time interval between the pulses. Even with the additional simplifying assumption that the mains voltage does not change during a transmitted pulse, the circuit exhibits quite complex behaviour. The complex behaviour had been noticed experimentally with very large capacitor voltages some times being a problem, variable signal levels for seemingly the same conditions (different initial conditions), sometime alternating pulse polarity leading to a constant phase fundamental but at other times apparently random polarities and random spurious
phase reversals even when commutation failures had been eliminated. All of these phenomena have been explained and related to the theory developed in this chapter. The practical problems with using this type of transmitter for the channel sounding work were judged too time consuming to solve for the purposes of this research and a different type of transmitter, described in the next chapter, was used. However, the work in this chapter remains important for the development of a practical transmitter, whether it uses regularly spaced transmit pulses or some form of spread spectrum signal with a predetermined pattern of irregularly spaced pulses. The work could obviously be extended to give more accurate predictions by incorporating third and higher harmonics in the mains voltage forcing function for the differential model. The non-zero triac hold current could also be included as well as the effect of starting capacitor charge because a deliberately chosen quantity of starting charge could be put on the capacitor though the use of an appropriately timed preparatory pulse.
V RESEARCH METHODS

1 Introduction

Previous chapters have been leading up to how we can establish successful methods for investigating the LV/MV network as a communication channel. This chapter will detail the research methods used for the main attenuation, phase response and noise measurements. Justification for selecting the method chosen for the attenuation and phase response measurements is included in this chapter while justification for the method used to specify the channel noise is expanded on in a later chapter. A description of the channels within the La Trobe University distribution system upon which measurements were made is given and a comparison is made with the utility's meter to zone substation channels.

2 Channel characterisation

We would like to know how our channel will affect the input signal as it travels from input to output. One assumption which it is generally necessary to make is that the channel is linear. Indeed our channel was predominantly linear and how this was found to be the case is explained in section 5. Given linearity we can then encapsulate all the information about how the channel treats any signal by specifying the channel's impulse response. If the impulse response changes from moment to moment, then knowing the impulse response at one particular time would not be of any great value. The second assumption that is generally made is that this does not happen and that the channel is time-invariant. In our case this is not strictly true. The channel characteristics depend heavily on the load connected and this is always changing. However, from one five minute period to the next, the changes in load which are typically occurring have no significant effect on the response measured. Indeed, it was found that the measurements
rarely changed significantly over a period of about 20 minutes. This approximate short term time-invariance allows the impulse response to be measured and for it to sensibly represent the channel over the time during which the load patterns remain similar. One could attempt to measure the impulse response directly. The impulse response is defined as the response to an infinitely short but infinite amplitude pulse of unity area so only an approximation is ever possible. A direct measurement using such an approximation to an impulse is generally impractical as a current pulse large enough to be detected and measured at the MV receive end would certainly blow fuses on the LV side of the channel. The solution is to work in the frequency domain and probe the channel with swept sinusoidal signals. Measurement of the ratio of output to input amplitude and of the phase differences for all frequencies gives us the transfer function of the channel. This transfer function is an equivalent representation to the impulse response which is the inverse Laplace transform of the transfer function. Direct frequency domain measurements are ideally suited in this case since a narrow band receiver is able to measure highly attenuated and noise affected signals. But again, only an approximation is possible because we cannot measure the response to all frequencies. In practice this is not a problem because we only require to know the response to a limited frequency range of interest.

How the input signal is altered by passing through the channel forms only part of the channel characteristics. The signal received includes not only the input modified by the channel but also signals added by the channel itself. These signals can be lumped together and labelled noise. In the power line channel noise comes from a multitude of sources particularly at low frequencies which the distribution transformers do not block. There is impulsive noise both synchronous and asynchronous with the mains waveform, narrow band signals harmonically related to the mains frequency and at inter-harmonic
frequencies as well as wideband stochastic noise [41]. I will treat the narrow band
signals harmonically related to the mains frequency differently from all the other
sources. The reason for this is that, over the frequency band of interest, these mains
harmonic signals are dominant. The specification to which ripple receivers used in
Australia must comply, AS 1284.6 [42], contains an annex which details worst case
expected voltage harmonic levels on the mains. Below 2kHz these range between 0.3%
and 8% of the nominal 50Hz voltage level. This means that even a resistive load can
give rise to an interfering current at a mains harmonic which is 8% of the main load
current. On top of the harmonic currents due to the mains voltage waveform not being
sinusoidal there are harmonic currents produced by equipment. Various standards are in
force to limit distorting currents produced by industrial and domestic equipment, for
example AS2279.2 [43] and AS2279.1 [44], but the allowed distortions are high, more
than an Amp per piece of equipment at the fifth harmonic and in the order of two or
three Amps at the third harmonic. As more and more equipment incorporates electronic
control, which leads to non-linear voltage/current characteristics, the problem gets
worse. Measurements of these harmonics will be presented in the Results chapter but it
is clear that sections of the spectrum close to mains harmonics, particularly odd
harmonics, are of no use whatsoever for mains signalling. For this reason it is assumed
that any signalling scheme will have a mechanism for omitting these fixed frequencies
from the spectrum used and that the scheme will be able to track the harmonics as the
mains frequency changes. The levels of the mains harmonics present at the receiver do
not change rapidly and neither does their frequency because the mains frequency is quite
stable cycle to cycle. This means that the bandwidth occupied by the harmonics is very
small, negligible compared to the spectrum available between the harmonics. For this
reason they will be ignored, noise measurements only being made between the
harmonics. The noise can be measured using the same equipment as is used for the
frequency swept signal measurements. This gives results in the frequency domain and, although the exact time domain noise signal cannot be reconstructed from these measurements, some important aspects of the noise are captured and this is the research method I have adopted. It could very well be argued that this method of representing the noise is not good enough. For some signal transmission techniques the impulsive nature of the noise might be important, with a time domain representation essential to properly analyse the performance of that particular technique. My justification for using a simple power spectral density representation of the noise in narrow bands is expanded on in a following chapter, but it rests on the fact that such a representation enables the theoretical maximum channel capacity to be calculated. The reason for this is that by choosing sufficiently narrow bands the shape of the distribution of the noise within the band is always close to Gaussian as will be shown subsequently.

A diagram of the channel relating to its characterisation is shown in Figure 5.1.

![Figure 5.1 Channel characterisation model.](image-url)
3 Equipment

The biggest challenge in this work was designing a system to perform the measurements required to characterise the channel.

3.1 The transmitter

One of the main differences in the equipment used for the channel sounding experiments and that described in the chapter on preliminary work was the transmitter. The low heat dissipation of the SWD transmit circuit is essential for a practical communication system but is not necessary for a simple channel sounding system which will only be used for a short time to investigate the properties of the channel. The new transmitter used water cooled resistive elements to draw signal current and this was switched by Insulated Gate Bipolar Transistors (IGBTs). One of the problems with the SWD circuit was the use of a triac with its inherent di/dt commutation problems when switching large currents. A transistor has no such problems. Its disadvantage is that it works only with a single polarity supply but rectifying the AC supply solves this problem. It also has the very great advantage that it can be switched off at any instant whereas the triac requires the current to dip below a hold current threshold. This means that switching the resistive load on and off at any frequency was possible.
Photographs of the transmitter components (high current switch, control board and load) are shown in Figure 5.2, its circuit diagram is in Appendix V G on page 318 and the firmware configuration files can be found in Appendices V A to V C on pages 310 to 312.

Figure 5.2 Photographs of the new transmitter components, (a) high current switch, (b) resistive load and (c) FPGA/CPLD board.
3.1.1 The transmitter's transistor switch

The choice of transistor type was critical. The voltage to be switched was nominally up to about 340VDC. The required switching frequency was relatively low, in the audio range, and these criteria show that an IGBT is likely to be the best transistor type to choose [45]. A suitably high voltage rating device was found, a BUP 314 capable of withstanding 1200V $V_{CE}$. Each could can safely switch at least 25Amps and four were connected in parallel to provide a 100 Amp capability. The drive requirements for IGBTs are generally as simple as they are for MOSFETs because the gate currents are negligible but, like bipolar transistors, direct parallel connection is not always straightforward due to the possibility of unequal current sharing [46] which can, in the worst case, result in thermal runaway of one of the devices. In my circuit no special precautions to ensure even current sharing were taken. The spread of device parameters and the different gate circuit delay times were close enough that no damage to any of the transistors was encountered in several years of use of the circuit. However, it is important to point out that, should the circuit be replicated, it may not always work reliably.

3.1.1.1 Operation of the transmitter directly across the mains

The new transmitter circuit described above was initially used to replace the SWD transmit circuit connected to the output of a welding transformer. It failed spectacularly several times when used with this transformer. The problem seemed to be that the source impedance was still very inductive. The transformer’s leakage inductance that had helped form an essential part of the SWD circuit (resonating with the capacitor) was now problematic. When the set of transistors switched off, the voltages in the circuit rose to extreme levels, presumably because the rapidly collapsing magnetic field in the
inductor core generates a high voltage across the inductor winding. This voltage spike was causing breakdown of the air gap between tracks on the bridge rectifier printed circuit board. In a DC circuit, sudden interruption of an inductor current is made possible by including a normally reverse biased freewheeling diode directly across the inductor which conducts in the forward direction when the current interruption causes the inductor voltage to reverse. Unfortunately, in my circuit, the position of the inductor was on the AC side of the bridge rectifier and it was formed only by the transformer secondary flux which did not also link the primary winding. This meant the inductor terminals were not accessible because the inductor is only separate from the secondary winding in an equivalent circuit and is not separate in reality. An attempt was made to suppress the voltage spikes with a snubber and with large Metal Oxide Varistors (MOV) but the leakage inductance value was too high to make any solution like this practical. In fact the problem turned out to be fortuitous. The risk of accident involved in the generation of such high voltages and in trying to develop a circuit to rapidly dissipate the significant energy stored in the transformer’s leakage inductance at every transistor switching event outweighed the risk inherent in working with a transmitter circuit connected directly to the mains voltage. From this point on the transformer was dispensed with. The circuit was housed inside the insulated fan cooled box, pictured at the top of Figure 5.2, in such a way as to minimise the risk of electric shock. Omission of the step-down transformer had the added advantage that the current switched by the transistor circuit was not reduced by the transformer ratio and so the 100A switch could be used on a much lower load current which meant it was de-rated significantly and not in danger from the unprotected paralleling arrangement previously mentioned.
3.1.2 Principle of operation of the transmitter

Although the construction technique would not pass type testing for a commercial product, it was deemed safe enough for use in controlled laboratory experiments and it was never left connected and untended by the experimenter. The loads being switched on and off at the transmit frequency were two commercial water heaters of 10 litres capacity each and a steriliser containing a similar amount of water. The thermostats were short circuited in all units and the loads were generally run at 100 degrees centigrade with the water boiling. The nominal 240V power rating of each of the water heaters was 2,000W and that of the steriliser was 1,500W. The load is switched by a 50% duty cycle square wave with amplitude zero or 1. Using all three of these loads the average power dissipation was therefore approximately 2.75kW.

Although the load is switched on the DC side of a bridge rectifier, the effect on the mains current is exactly the same as it would be if the switch were bipolar and there was no rectifier present. For this reason, and to improve clarity, the switch will be imagined to be bipolar and the following representations of voltages across the load will be shown as a switched sinusoid although they are in fact unipolar. The current signal being produced, which is what we are interested in, is therefore proportional to the load voltage representations shown.
3.1.2.1 Signal waveform for straight forward load switching

If the transmitter's load had simply been switched on and off at the transmit frequency we would have been producing double sideband suppressed carrier (DSBSC) modulation of the mains. By switching the load current in this way we are effectively multiplying the sinusoidal mains current by a square wave of amplitude $\pm \frac{1}{2}$ with a DC offset of $\pm \frac{1}{2}$. An illustration of the waveform and frequency content we would expect was obtained using the MATLAB script in Appendix V D on page 315 and is shown in Figure 5.3. below:

![Waveform Diagram](image)

**Figure 5.3** DSBSC modulation by switching load on and off at transmit frequency of 223Hz, (a) switch control waveform, (b) mains current, (c) signal frequency content.
The upper diagram is the simple switching waveform, the middle graph is a representation of the voltage across the load and the lower graph is the power spectral density with the y axis marked to show RMS voltage values of single frequency components. The DC term produces a 50Hz output at half of the mains RMS value. With a 240V load of 5.5kW being switched, the 120V 50Hz term would account for a quarter of 5.5kW. We know the total signal power is 2.75kW so the remaining power is spread across the pairs of sidebands of which three are shown in the figure. The first pair of sidebands is centered on the transmit frequency with each sideband being 50Hz away from the signal frequency. This pair of signals is being produced by the fundamental of the square wave which we are multiplying by yielding sum and difference terms at 223±50Hz. The next pair of frequencies is being produced by the third harmonic of the signal frequency which is also part of the square wave we are multiplying by. The third harmonic produces sum and difference terms at 669±50Hz. The result of the fifth harmonic of the signal frequency modulating the mains can also be seen in Figure 5.3 centered on 1,115Hz.

The amplitudes of each pair of sidebands is determined partly by the Fourier series representation of a square wave where the \( n \)th harmonic has an amplitude given by

\[
b_n = A \frac{4}{n\pi}
\]

and only odd numbered harmonics are present. \( A \) is the peak amplitude of the square wave which is \( \frac{1}{2} \) in our case so the fundamental has amplitude \( b_1 = \frac{2}{\pi} \), the third harmonic has amplitude \( b_3 = \frac{2}{3\pi} \) and so on. The trigonometric identity

\[
\sin(x)\sin(y) = \frac{1}{2}(\cos(x - y) - \cos(x + y))
\]

[39] shows us that the amplitudes of the sidebands associated with each harmonic will be the amplitude of the harmonic
multiplied by half the mains voltage. So the first pair of signals 50Hz away from the transmit frequency will have RMS amplitudes of $\frac{240}{\pi} = 76.4V$. This means the power in each of the fundamental's sidebands for the load we have is just 557W with the remaining $2.75kW - \frac{5.5}{4}kW - 557W - 557W = 261W$ spread between all of the other higher harmonic sidebands.

This system could have been used for transmitting current signals with the receiver tuned 50Hz away from the transmitter local oscillator frequency but, apart from much more than half of the signal power being wasted due to it being outside of the receiver's bandwidth, we would have had problems as the mains frequency drifts about its nominal value. A much better scheme was developed and that was to alternate the phase of the transmitter local oscillator in step with the mains polarity.
3.1.2.2 Signal waveform for alternating phase load switching

If we change the phase of the signal controlling the load's switch by 180° every time there is a mains zero crossing as indicated in Figure 5.4 (a), then we produce the much more satisfactory signal shown in Figure 5.4 (c).

![Graph showing time and voltage](image)

**Figure 5.4** Transmitted signal produced by switching load on and off. Transmit frequency is 223Hz (a) switch control waveform, (b) mains current, (c) signal frequency content. Phase of the switching signal is reversed at every mains zero crossing.
This figure was produced using the same script as was used for the previous figure except the substitution given in Appendix V E on page 316 was made to generate the phase reversals. From the top time domain trace it can be seen that the switch happened to have been in its ON state 10ms after the start of the trace. At this point the mains waveform changes polarity so the switching waveform is also made to change polarity to an OFF state at that instant. The switch signal continues to alternate polarity at the rate determined by its period, however comparison with Figure 5.3 shows that it is now in the opposite polarity to that which it was in under the previous DSBSC scheme. This continues until the next mains zero crossing when the switching signals phase will again be forced to reverse.

The time domain trace of the signal makes it quite obvious that we will now be creating a signal at exactly the fundamental of the 223Hz fundamental. This is because the waveform is positive only when the fundamental (223Hz sine starting at \( t = 0 \)) has positive polarity and the waveform is negative only when the fundamental has negative polarity. A correlation between the waveform and the fundamental therefore will be positive. The full advantages of this new switching scheme are best shown by calculating the expected output at the switching frequency fundamental.

3.1.2.3 Output signal level with mains controlled signal phase

As before, the average of the switching waveform, which still spends exactly half the time in an ON state and half the time in the OFF state, is \( \frac{1}{2} \). This means we are still wasting a quarter of the load's power rating in a part of the signal fixed at 50Hz but this is unavoidable. Consider the component of the transmitted signal which is at the switching frequency fundamental. It can be thought of as the result of the addition of two processes, each a DSBSC modulation of one of the side bands of the fundamental
analysed in the section 3.1.2.1. To present this argument it is necessary to examine the way the new signal waveform has been created in comparison to the waveform in the last section. In section 3.1.2.1 we imagined the signal waveform as comprising a DC part and a bipolar square wave part. Each was multiplying the 50Hz mains waveform separately and the results we saw summed in the frequency domain. This time we must treat the bipolar signal to an extra process. That is, multiplication by another square wave of unity peak amplitude and no DC component. This new square wave is in phase with the mains waveform and takes its polarity at any instant. Multiplying by this new waveform creates the phase reversals of the square wave at the signal frequency as described previously. So we must now apply the effects of this new multiplication only to the frequency components created by the mixing of the bipolar signal and the mains waveform. In the frequency domain, at each frequency component of the previous scheme we apply the multiplication, except for the component created by the DC term associated with the signal frequency square wave. The effect will again be sum and difference terms for each component. Consider first the lower sideband signal fundamental term at the frequency of (233-50)Hz. When multiplied by the new square wave at 50Hz we get components at (233-50)Hz + 50Hz and at (233-50)Hz - 50Hz due to the fundamental of the 50Hz square wave. As before, the amplitude of these components is given by the Fourier series representation of the 50Hz square wave

\[ b_1 = \frac{4}{\pi}, \] 

by the original sine wave amplitude which we know was \( \frac{240}{\pi} = 76.4V \), and by the factor of a half which comes from the identities of the form

\[ \sin(x)\sin(y) = \frac{1}{2}(\cos(x - y) - \cos(x + y)). \] 

The amplitude of this first pair of new sidebands which we are now considering is therefore \( \frac{2(240)}{\pi^2} \). But, as there will be other components appearing at the same frequency due to the same process being applied to
the upper sideband signal fundamental term, we must also consider the phase. Let us define zero time, \( t = 0 \), as a point in time when each Fourier component of each waveform considered so far except for the DC term, has zero instantaneous amplitude and polarity positive just after. This applies for the mains waveform, the signal square wave and the mains frequency square wave as shown in (C) of Figure 5.4. That makes all the illustrated waveform components sine terms as in the left hand side of the identity above. The lower sideband at \((233-50)\)Hz was therefore a positive cosine term. Now we are applying DSBSC again to give a new lower sideband at \((233-50)\)Hz - 50Hz and so should use the identity 

\[
\sin(x)\sin(y) = \frac{1}{2} \left( \sin(x + y) - \sin(x - y) \right)
\]

[39]. We notice that this lower sideband component is a minus sine and that the upper sideband component at \((233-50)\)Hz + 50Hz is a positive sine. Let us now turn our attention to the upper sideband signal fundamental term at the frequency of \((233+50)\)Hz. When multiplied by the new square wave at 50Hz we get components at \((233+50)\)Hz + 50Hz and at \((233+50)\)Hz - 50Hz. We notice that the last component falls at the same frequency as one of the previously considered components and that it is at the signal frequency fundamental. The amplitude of these two components is again 

\[
\frac{2(240)}{\pi^2}.
\]

The phase of the original upper sideband at \((233+50)\)Hz was a minus cosine from the first identity used. From the identity concerning a product of a cosine and sine we see that the new lower sideband term at \((233+50)\)Hz - 50Hz will have a positive sine phase and so will add to the term at the same frequency produced by the first lower sideband considered. This gives the total signal at the signal frequency fundamental as 

\[
\frac{4(240)}{\pi^2} = 97.3V.
\]

This is considerably greater than the level of the closest signal to the signal fundamental for the previous modulation scheme. The power in this signal is now more than 900W. This is still not a very efficient transmitter with a total of 2.75kW being dissipated and only
900W being in the receiver bandwidth but it means a significant signal current of about 9A is being generated.

For completeness we should go on to look at the signal levels in the sidebands. The closest to the fundamental are signals 100Hz below and 100Hz above the fundamental. We have seen that the frequency 100Hz below the main signal has one component which is due to the lower sideband of the fundamental of the 50Hz square wave modulating the lower sideband of the mains waveform modulation of the signal i.e. (233-50)Hz - 50Hz. But there is another source of energy at this frequency. That is the lower sideband of the third harmonic of the 50Hz square wave modulating the upper sideband of the mains waveform modulation of the signal, i.e. (233+50)Hz -150Hz. The amplitude of this component at 133Hz can be calculated as \( \frac{2(240)}{3\pi^2} \) and the phase is positive sine. This adds to the previously calculated component at this frequency of amplitude \( \frac{2(240)}{\pi^2} \) and phase minus sine to give the total amplitude at 133Hz as \( (-) \frac{4(240)}{3\pi^2} = (-)32.4V \). All other components can be calculated in a similar way but they are not important in this application.
3.1.2.4 Actual signal level used

Although the calculations given above show a signal current of just over 9 Amps RMS was to be expected, the actual measured signal current at the transmitter within a small bandwidth centered on the transmitter local oscillator frequency was 8.7A RMS with all three water cooled loads connected. It is thought that this was due to their power dissipation not being exactly the same as the name plate rating when hot. Using different combinations of the three loads it was possible to transmit at several different current levels as shown in Table 5.1.

<table>
<thead>
<tr>
<th>Water heater 1</th>
<th>Water heater 2</th>
<th>Steriliser</th>
<th>Total load (kW)</th>
<th>Current in fundamental</th>
</tr>
</thead>
<tbody>
<tr>
<td>on</td>
<td>on</td>
<td>on</td>
<td>2.75</td>
<td>8.7</td>
</tr>
<tr>
<td>on</td>
<td>on</td>
<td>off</td>
<td>2</td>
<td>6.4</td>
</tr>
<tr>
<td>on</td>
<td>off</td>
<td>on</td>
<td>1.75</td>
<td>5.5</td>
</tr>
<tr>
<td>on</td>
<td>off</td>
<td>off</td>
<td>1</td>
<td>3.2</td>
</tr>
<tr>
<td>off</td>
<td>off</td>
<td>on</td>
<td>0.75</td>
<td>2.4</td>
</tr>
<tr>
<td>off</td>
<td>off</td>
<td>off</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5.1. Table of possible transmit currents for the equipment used.

The different levels could be used to crudely demonstrate channel linearity but a more sensitive technique was actually used and that is described in section 5 of this chapter. In order to maximise the accuracy of the measurements at the receive end, transmissions using anything other than the maximum current of 8.7A were rarely made.

3.1.2.5 Generation of signal by passive device

Stating that switching a resistor is, in this case, "generating" a signal might be thought to be incorrect. The reason that the signal really is originating at the switched resistor is because of the non-linearity of the transmitter. There is no power source within the transmitter but it will be represented by a signal source. The actual power source in the system is the 50Hz generator but this generator will be ignored for the purposes of
analysing the response of the system to the signal frequency. In fact, all 50Hz currents and related harmonics of 50Hz currents will be ignored in the analysis of the response of the channel to signals. This is possible if we state that no transmit signal is allowed to be harmonically related to 50Hz and that the channel is linear so no transmitted signal can ever generate any response at these disallowed frequencies. The physical situation is compared to the electrical representation which we are using in Figure 5.5.

![Figure 5.5](image)

**Figure 5.5** Comparison of the physical situation (a) with an electrical representation of the channel (b).

We shall consider only the component of the current generated at the transmitter $i_p(t)$ which is at the signal frequency fundamental and the resulting current $i_R(t)$ flowing in the MV lines at the receiver at the same frequency.
3.1.3 Load switch control circuitry and firmware

The circuitry and firmware required to switch the load at the appropriate times and to synchronise the transmitter and receiver reference signals, so that channel phase measurements could be made, will now be described.

3.1.3.1 Opto-isolation

The transmitter’s high voltage/high current IGBTs were operating with their emitters alternately connected (through the bridge rectifier diodes) to mains Active and Neutral. Although the gates are insulated, it would be very dangerous to connect them to any low voltage circuitry which may come into contact with an experimenter for the obvious reason that a single device failure would lead to all the circuitry becoming “live”. For this reason an interfacing opto-coupler circuit was employed. The board containing this interface circuitry was taken from a “CALMU” three phase meter donated by PRI Australasia. This circuit board, when it was in the meter, performed the function of isolating the meter circuitry (which was at mains potential) from communications equipment outside the meter and which was operating from an SELV supply. For this reason I could be certain that the isolation inherent in the circuit board was that required to meet the appropriate Australian Standards for production equipment. To ensure that I did not compromise the isolation by construction techniques the board was installed using the appropriate separation rules [47]. The opto-isolator and associated circuitry would add to the delay between the gate signal being generated at the control circuitry and the switching on of the load current. This switching delay, of about 30 $\mu$s, was not measured directly but it is the major component of the entire measurement system response, it was measured and calibrated out as described in Appendix V I and in the
results chapter. The current signal generated by the transmitter was monitored locally by a current transformer which again provided the necessary isolation from the mains. This current transformer was also calibrated to ensure that its transfer characteristics were fully understood.

The balance of the transmitter circuitry was made up of a CPLD/FPGA development board and some peripheral equipment to ensure that gate signals were generated with precise timing and a great deal of effort was expended in designing this element of the system. Its performance was entirely satisfactory and it is believed that the technique employed is novel so the following sections will describe it in some detail.

3.1.3.2 Gate control/receiver reference scheme

The input to the optoisolator which controls the gate is generated by a CPLD/FPGA circuit board. This board has an input from the one pulse per second output from a GPS engine. The GPS engine board is a PG-11GPS receiver from Lipac Technology and it has a connection for an external active antenna, model GLP1, which was placed outside the building. The GPS engine has an NMEA-0183 output which is a standard serial data stream for GPS modules and includes, in addition to latitude and longitude, a time stamp field in the output RS232 data sentence. The sentence was fed into the CPLD/FPGA board where the various data, delimiter and flag ASCII codes were sorted out to extract the time only. In the GPS satellite systems this time is derived from caesium atomic clocks. The time of flight of the signal from the satellites is known and compensated for so the result is an internal time accuracy of typically better than 50ns at the GPS receiver. The time as communicated over the serial data stream does not, of course, have that accuracy but it does enable each of the one pulse per second timing signals to be
uniquely identified and these timing signals do, at least nominally, conform to the stated accuracy.

3.1.3.3 Synchronisation of transmit and receive end parts of the experiments

In order to make successful measurements of the signal's phase difference between transmit and receive ends of the channel very accurate synchronisation was required and this was provided using the one pulse per second outputs from the GPS modules. But there was another synchronisation issue on a much coarser scale. That was how to ensure the receive end and transmit end started the sequence of frequencies tested on the same second. Various techniques were tried and for many months an Ethernet connection between the office housing the transmitter and the co-generator control room, at which the receiver was located, was used. Computers at both ends communicated with the FPGA boards and the equipment in the control room was effectively remotely controlled from the office. Measurement results from the lock-in amplifier were recorded on the receive end computer and retrieved over the Ethernet which was convenient but not ideal. Some planned transmit locations had no Ethernet connection available, for example it was hoped that an opportunity might arise to use the equipment to make measurements at a utility's substation where a secondary communication path for use in equipment synchronisation would not have been available. The solution was to make use of the time stamps in the GPS serial data stream.

A pre-arranged time was programmed into the receive end's FPGA shortly before commencement of a measurement run. The FPGA configuration was arranged so that the hours, minutes and seconds, decoded from the GPS time stamps, could be displayed sequentially on one of the pairs of seven segment displays on the board. A suitable start
time, a few minutes hence, was chosen and programmed in using buttons and switches and the same display. The experimenter then returned to the transmit location and programmed the same time into the transmit FPGA board. When the two FPGAs independently detected that their time stamp matched the start time which each of them had been programmed for, the experiment run was set to begin. At the exact edge of the next one pulse per second timing signal from the GPS boards the local oscillators at both ends started in exact synchronisation. Even though both local oscillators were crystal controlled, they were not accurate enough and did not have sufficient stability for the two ends to remain in synchronisation and so another synchronisation scheme was developed to keep exact phase lock over the individual measurement periods. In this scheme the one pulse per second GPS outputs were counted at both ends and used to keep the set of measurements synchronised as they stepped up in test frequency over a period of up to 20 minutes. In fact it was found that once the two ends had been synchronised by the same start time programming before the beginning of the measurements, they stayed in step for many days at least.

3.1.3.4 Fine scale synchronisation

This part of the design was critical to successful measurements and the method used is believed to be novel. It was necessary for the transmit and receive local oscillators to be in time synchronisation to a high degree of accuracy. Individual amplitude and phase measurements at each frequency could take up to one minute and a frequency difference between the two controlling clocks resulting in anything more than a few degrees difference between the receiver and transmitter local oscillators would impact the measurement accuracy unacceptably. At a signal frequency of exactly 1kHz a difference of one degree over a sixty second period is produced when compared to a local oscillator which is running at 1000.000046Hz, a difference of only 0.046 parts per million (ppm).
Both the transmit and receive boards' timing were controlled by a crystal oscillator without temperature control, while the crystals were the standard AT cut, which has excellent frequency stability with temperature, this is usually quoted at levels of tens of ppm. Clearly a system was required to keep the transmit and receive ends in lock. With the GPS one pulse per second signals we had a high accuracy reference available and the usual way of locking an oscillator to a reference would be with a phase locked loop (PLL) arrangement. The trouble with such a low reference frequency of 1Hz was that the PLL loop filter would necessarily take a long time to respond and settle. The response time would likely have been so long that it would not have been able to keep up with the crystal oscillator's drift rate. The solution was to adopt a step-wise feed forward frequency correcting technique.

A Microcontroller or Digital Signal Processor could have been used as a basis for transmit and receive control but microprocessors perform pre-programmed instructions in a serial manner. Their advantage has been that billions of identical chips can be made to do a myriad of different tasks by altering their stored programs. There is now another option in CPLDs and FPGAs. In these devices there is no program, rather the logic within the chip itself is configured by the user for the specific task. In my application these devices have the advantage that tasks can be set up to be done in parallel rather than sequentially whereas true parallel operation is impossible in a single cored microprocessor. My application demanded precise timing which is difficult to arrange in a processor which can only operate on a single instruction at a time and which would have high priority, timing critical, tasks to be done alongside many other lower priority tasks.
Chapter V Research methods

The Altera UP1 board was chosen as a development platform. It has a CPLD together with an FPGA and both of these types of devices operate by configuring logic in a user specified way. The user does this by defining the configurations in a Hardware Description Language and VHDL was the language used for this research work. Although the language used to configure CPLDs and FPGAs is the same, the internal architectures of these two device types are different. The CPLD has less logic available for the user to configure, but it is internally pre-configured in logic blocks referred to as macro cells. The CPLD then takes fewer user defined interconnections for the same function complexity as an FPGA. Because of the reduced number of user defined connections, the CPLD can have an internal non-volatile configuration memory and more predictable timing. The FPGA, on the other hand, can implement a significantly more complex process although it needs some mechanism for downloading its configuration after each power-up. Both devices were used in the receiver and transmitter designs. The CPLD was used to generate an accurate fixed frequency reference with the feed forward corrections generated from the GPS's one pulse per second output. This was the most critical timing task and it took up almost all of the 2,500 usable gates in the 128 macro cells available on the board's Altera EPM7128SLC84 CPLD device.

3.1.3.5 CPLD's configuration

The CPLD takes in the GPS one pulse per second signal as an input together with a reset signal from the FPGA. Its output is a 50kHz clock controlled by the GPS timing pulses. The first one second period between the leading edges of the timing pulses is measured using the board’s local crystal derived 25.175MHz clock. If the crystal was dead accurate this measurement would be exactly 2,5175,000 clock cycles. Any discrepancy from this figure is a measurement of the local crystal clock’s error. An
approximate 50kHz clock is derived from the crystal clock and the error measured over the first second is then used to correct the 50kHz clock in the subsequent second so that this output frequency to the FPGA is as accurate as possible. In the meantime a new error is being measured to correct the 50kHz clock in the next second.

The VHDL listing of the top block associated with the task described above is included in Appendix V A on page 310. Because this is a relatively small configuration file it will be described briefly as an example of the main technique that was used for building data processing hardware for this research work. The top block file for the configuration of the FPGA, which has eight times as many gates as the CPLD, shall be included in the Appendices but shall not be described in this way in the main text. Copies of the all of the VHDL code can be supplied if required.

The entity name of the CPLD top block is "fiftyKHZlocked". The top block is the VHDL equivalent of a "main" routine in a C program. It has one component declared called "clk_error" (listing in Appendix V B) and such components are the VHDL equivalent of subroutines. They refer to externally defined processes which, in contrast to subroutines, will be running simultaneously. Similarly to writing conventional programs, the components may be nested and the clk_error component itself declares another called "dispdot" (listing also in Appendix V B). The clk_error component is measuring the difference between the actual number of board clock cycles between the one pulse per second rising edges and that expected had the board clock frequency been exactly 25.175MHz. The difference is called "error_size" and it is connected into the top block as the first signal, an 11 bit vector. The top block contains four synchronous processes and some combinational logic. The first process ensures that, until the end of the very first GPS second period after switch on, no 50kHz reference signal is output.
because no valid error size is available. The second process sets up the adjustment values and the time at which adjustments will be made, a zero adjustment being made for most of the next processes' cycles. The third process generates the nominally 50kHz reference signal, with the small adjustments being made to the length of four equally spaced cycles every second. The fourth process generates a much lower frequency jitter reduced test tone at 360Hz derived from the 50kHz for display on an oscilloscope. This was to compare with test tones from other replica systems using boards with the same CPLD configuration but different frequency crystals and different GPS receivers.

When measurements of the discrepancy in phase between two totally separate CPLD/GPS systems were made the result showed that the phase drift between the two 360Hz signals was typically staying at a rate less 20° over a half hour period which is about 0.08ppm. The rate of change in phase difference was not noticeably greater than this when measured over smaller time intervals of 30 seconds or a minute. At the reset of the CPLDs the two 360Hz signals were forced to start with the same phase at the following one second timing pulses from the GPSs. By monitoring two separate systems while they are together in the same location and proving that the local oscillators stay in phase over long periods, it was possible to show that reliable phase difference measurements between the transmitter and receiver could be made.

3.1.3.6 FPGA's configuration

The FPGA was configured to carry out all the control and data processing tasks to automate the channel sounding experiments. As was the case for the CPLD, the FPGA configuration at the transmitter was identical to that at the receiver. Its main task was to provide the local oscillator signal. At the transmitter this signal drives the load switch and at the receiver this signal is fed into the reference input of the lock-in amplifier.
These two signals are not identical because they are taken from two different points in the signal chain as shown in the simplified schematic of Figure 5.6. The unused signal output was simply left disconnected.

![Simplified schematic of local oscillator generation.](image)

**Figure 5.6** Simplified schematic of local oscillator generation.

There is also circuitry defined in the configuration to transmit and receive through the RS232 interface of the lock-in amplifier and a second RS232 interface with a PC running Microsoft HyperTerminal to log the received data to a file. At the substation or control room the interface with the lock-in amplifier was used to prompt for measurement results to be returned at the end of each measurement period. These results went via the FPGA which did some processing to format them so that an Excel readable file could be created. The individual measurements were then passed to the PC where they were logged to the file. All of this data processing circuitry was present in the transmit end's FPGA but there it was unused. A second UART has been written into the FPGA configuration for communication with a PC running a BASIC program. This was
used at both the transmitter and receiver ends to control the sequence of events when the experiments were being run over the Ethernet. Since this was superseded by the GPS time stamp based method the details are not included here.

For the final equipment design the measurement run commenced at the instant of the rising edge of the GPS one pulse per second signal following a GPS timestamp match with the user programmed start time. This edge occurs at the same time at both the transmit and receive ends to within a tight timing tolerance, significantly less than 1 $\mu s$ (the receiver board's output specification) and typically less than 100ns. The FPGA then begins to output the first of a predetermined list of test frequencies and the main frequency list used is given in Appendix V F on page 317. It starts at 104.279Hz and contains 114 different frequencies, the highest being 3.12192kHz. The frequencies were chosen to give a good coverage of the spectrum over which attenuation is measurable. The highest concentration of test frequencies is below 1kHz because that is the area of most interest and they become quite sparse above 2kHz where the attenuation is much higher. The exact frequencies were chosen for the ease with which they could be generated and stored given the limited logic available. Areas close to the odd harmonics of 50Hz were avoided because the high level of interference here makes detection of any transmitted signal difficult or impossible. The list includes 14 repeat test frequencies at the end. These are used to check that conditions over the channel have not changed significantly between the time of the first measurement and the repeat. For most experiments this time was about 20 minutes.

The list of frequencies tested could be altered and so could the time spent taking individual measurements. A narrower bandwidth in the lock-in amplifier could be selected if the individual measurement time was increased. This increases the signal to
noise ratio but makes the likelihood of changes to the channel over the experiment's time more likely, making the whole run invalid. The main individual test time used was 10 seconds although occasionally 30 seconds was used. As mentioned previously, the main frequency list had 114 separate frequencies with a total of 128 tests to be performed. Before each measurement began the local oscillators were reset to have their start phase begin at exactly the same time as the rising edge of the GPS pulse output, the second before this being used to measure the clock error in preparation for this first second. This means there are 1 second gaps between each measurement so the complete measurement run for 10 second measurements takes 23 minutes 28 seconds. A run takes 1 hour 6 minutes and 8 seconds when made using a 30 second measurement period. Measurement runs taking more than 1 hour were too long to make significant changes in the channel unlikely, because loads change at the beginning of the day, at lunchtime and so on. For the longer individual measurement runs a reduced frequency list was used with 29 individual frequencies and three check measurements and this took only 16 minutes 32 seconds. With the short list and 10 second measurement times, a run takes less than 6 minutes. Over this short time the channel was almost always found to be stable but even over the more detailed runs lasting 23½ minutes it was rare for there to be a significant difference between the check measurements made at the end of the run and those made earlier so this is the time that was used for most experiments.

The configuration of the FPGA is volatile and every time power is removed from the board the FPGA reverts to being a set of unconnected logic elements. During development of an FPGA project the source code, which in our case is the listing in Appendix V C combined with all of the listings for nested components and a pinout file, is compiled. This produces a file which can be downloaded into the target device. To avoid this inconvenience, once the design was complete my configuration files were sent
for burning into a configuration EPROM by a specialist company. With one of these devices in the circuit the FPGA retrieved the interconnection data and configured itself on each power up which very much enhanced the portability and easy of use of the system.

3.1.3.7 Mains zero crossing detector.

To invert the phase of the generated frequency signal at every mains zero crossing a simple zero crossing detector circuit was used to give an edge which occurred just before each change in polarity of the mains and this is shown in Figure 5.7.

![Zero crossing detector circuit diagram](image)

**Figure 5.7** Zero crossing detector.

It was found necessary to take the mains supply to the step-down transformer through an RF filter to reduce conducted interference. This interference was due to the proximity of the transmitter switch. The type of filter which is often incorporated into IEC connectors was found to eliminate all interference problems. The transistor was normally held on by the full wave rectified waveform and was only off for the short period, starting just before the zero crossing, when the rectified waveform was less than its base-emitter turn on threshold. The transistor was followed by a Schmitt trigger to ensure the edge was sharp and clean. Inside the FPGA a small time delay of about 2.5ms was added to the
falling edge and this was adjusted until the edge occurred exactly on the mains zero crossing.

3.1.4 Transmitter locations

It is to be expected that the many different paths between sockets and substation in an LV/MV distribution system will lead to different channel models depending upon the physical characteristics of the path. For this research work I was interested in the extent to which models for separate paths would differ. For this reason, paths which were as different as possible were chosen. The two most significant areas of difference were path length and transformer size. For the final measurements six transmit locations were used and these are shown in the table below.

<table>
<thead>
<tr>
<th>Chan No.</th>
<th>Location (building no.)</th>
<th>Distance from substation (in straight line)</th>
<th>Transformer size</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bakery (22)</td>
<td>75m</td>
<td>500kVA</td>
</tr>
<tr>
<td>2</td>
<td>Pavilion (46)</td>
<td>525m</td>
<td>750kVA</td>
</tr>
<tr>
<td>3</td>
<td>Union Building (40)</td>
<td>375m</td>
<td>1000kVA</td>
</tr>
<tr>
<td>4</td>
<td>Boiler house (35)</td>
<td>750m</td>
<td>1000kVA</td>
</tr>
<tr>
<td>5</td>
<td>My Office, PS117(19)</td>
<td>120m</td>
<td>1500kVA</td>
</tr>
<tr>
<td>6</td>
<td>Room 127A (also 19)</td>
<td>150m</td>
<td>1500kVA</td>
</tr>
</tbody>
</table>

Table 5.2 Transmit locations.

The location of the transmit sites are shown on the campus map in Figure 5.8 opposite and an indication of the signal path to the Thomas Cherry substation in building 23 is marked.
The ratio of the length of the LV side of the channel to the length of the MV side was also different for each transmit location. For those locations far away from the substation, the pavilion, union building and boiler house, most of the path was over MV lines with the transformers in the same building as the mains socket used. For the bakery, a small kiosk transformer was mid-way between the mains socket location and the substation. For the channel from my office and room 127A the signal path was almost entirely on LV lines because the transformer was in the substation itself. The

**Figure 5.8** Campus map indicating the six channels used.  
Underlying map copyright 2010 Melway Publishing Pty Ltd. Reproduced from Melway Edition 38 with permission.
loads were also of a different nature for most of these channels. The boiler house had heavy industrial loads made up of compressors, pumps, fans and other large motorised equipment. The union building load was made up of computers, fax and printers as well as extensive fluorescent lighting as is typical for office buildings. The bakery had loads typical of commercial shop premises such as freezers and food warmers. The pavilion had very light loads and my office and room 127A were connected to a large LV network which included more than a hundred computers and servers. Any common features found in the models of the six very different channels would therefore be expected to be found in any channel on the university campus and, possibly, might also be expected on any LV/MV channel within a substation distribution area managed by a utility company.
3.2 The receiver

The receiver was made up of the previously described CPLD/FPGA board and GPS, a Stanford Research Systems SR830 lock-in amplifier and the current transformers and amplifiers to pick up the signal from the MV supply. Figure 5.9 shows the lock-in amplifier and the current transformers with their amplifiers constructed around a commercial three phase CALMU meter donated by PRI Australasia.

Figure 5.9 Receiver components, (a) lock-in amplifier, and (b) Modified CALMU meter. (CPLD/FPGA/GPS not shown).
3.2.1 Interface to MV supply

The signals from each phase were picked off the MV supply using two current transformers (CTs). The first was a large three phase CT with its primaries operating at the MV potential which is part of the network and channel itself and will be described in the next section. The currents in each of the secondaries of this large CT are monitored by a second current transformer which forms part of the receiver measurement system. This second CT is one of the CALMU meter’s three current transformers. These were slightly modified by adjustment of the number of primary windings to obtain the appropriate ratio so that the amplifiers would not overload at the university’s maximum current draw, but still keeping the signal fed to the operational amplifier input as large as possible. The existing current transformer’s load resistors were retained. The electronic workings of the CALMU meter were stripped out and discarded but the main Printed Circuit Board (PCB) outline and mounting details were recorded. A new PCB was made which contained the amplifier circuits. For simplicity and reliability the amplifiers were made from three OP27 low noise Op-AMP integrated circuits. A ±15V power supply connection was made to the device via the CALMU’s pre-existing DB9 connector. Three holes were drilled in the front of the case through which were mounted the BNC outputs. This current monitoring box was checked in the laboratory to ensure that the gain for each of its three channels was known and that the frequency response was completely flat over the spectrum of interest and that the phase response was known.
3.2.2  **Lock–in Amplifier**

The Stanford Research Systems SR830 used is a Digital Signal Processor based lock-in amplifier with the facility to externally phase lock its inbuilt local oscillator to a user provided signal, in this case provided by the FPGA board previously described. The SR830 is a two channel device in that it simultaneously multiplies the input by two quadrature phase local oscillator signals. The two results are digitally low pass filtered and can be output separately as X and Y signals. The SR830 can use these to calculate and output amplitude and phase signals. For most received signal measurements it was the amplitude and phase which was recorded although the X and Y outputs were used for displaying a phasor representation of the detected signal when the sounding system was being developed. The bandwidth of the receiver was adjustable over a wide range by selecting the digital low pass filter characteristics. Mains frequency notch filters are included in the signal path and these were always used to avoid overload within the amplifier.

Apart from detecting and measuring the received signal, the lock-in was also used for measuring the noise present within the equipment's bandwidth in the absence of a transmitted signal. In this case the phase measurement was not relevant and only the X or Y outputs were used. As will be explained in a subsequent chapter, the statistics of the X and Y outputs are identical so only the output derived from one channel was recorded. One way of determining a noise level is to take a large number of instantaneous DC measurements from which the RMS noise voltage (in the equipment's measurement bandwidth) could be calculated however, that is not the most straight forward measurement method. A simpler method for measuring in-band noise is to take a single reading from a meter with an RMS output. Suitable meters have carefully designed
circuitry to respond with true RMS readings averaged over a certain known time interval and are accurate for signals with crest factors limited to the instrument's capabilities. The SR830 also has an inbuilt mechanism for directly making in-band noise measurements. The exact method the equipment uses for calculating these results and the justification for trusting the results obtained is explained in more detail in a later chapter.

### 3.2.3 Receiver locations

There are two locations where it is safe and practical to pick my signals off the MV lines. One is the previously mentioned substation plant room where the utilities’ feeder enters the campus, situated in the Thomas Cherry substation in building 23 which is where all the channels marked on Figure 5.8 converge. The other is the co-generation control room which is in the boiler house building, the most easterly building circled in yellow in Figure 5.8. In both cases, safety and practicality are assured due to the presence of pre-existing MV current transformer installations. These current transformers are in place to provide signals, proportional to the line currents, for fault protection equipment and for metering. The transformers have a good frequency response because the fault detection needs to be quick and the metering needs to be accurate, even for current waveforms which include frequency components well above the mains frequency. It has previously been shown by others [48, 49] that the MV current transformers typically have ratio errors which deviate less than 3% from the power frequency value over the range to 20kHz and that the phase error over a similar range is less than about 10º. Because these current transformers are an essential component in order to be able to monitor the MV currents they will be considered part of the channel and so their response is included in the channel model.
Before access to the Thomas Cherry substation was granted the generator control room was used as a receiving point. As a trade for the right to install some receiving equipment here, I replaced the existing control room high voltage meter with an up-to-date GPRS readable “smart meter”. While the meter was being replaced a second device was installed in the lines from the MV current transformers to the new meter. This device comprised three “interposing” current transformers and associated low noise amplifiers housed in a CALMU meter case as described in the previous section. The advantage of using the CALMU’s case and its three current transformers with their load resistors was that there could be no argument about the quality and reliability of the connections and extra current paths (burden etc) which were going to be interposed between the MV current transformer and the MV meter in the control room. Once this new current monitoring equipment had been installed I had continuous access to the signals representing the currents on each of the three phases being supplied from the generator to the university campus.

In order to make measurements of a channel representative of a normal utility network, the receiver needed to be placed in our 22kV substation. This would allow measurement of the current on the feeder supplying the whole university from our electricity supplier’s network with the co-generation plant shut down. After some time working in the co-generator control room, access was granted to the Thomas Cherry substation. An operation similar to that performed in the control room was undertaken. Because the substation environment was much more hazardous than the control room, a specialist contractor was brought in to replace a very old (Ferraris disk) MV meter monitoring import and export for the whole university with a new “smart meter” (also donated by PRI Australasia). While this was done another CALMU based current monitoring device was interposed between the MV current transformer and the meter and connections from
this were taken out from the caged and secure 22kV switching area to a space in the plant room to which regular access was possible. Finally, with this new receiver location measurements could be made on a channel truly representative of a typical utility's meter to zone substation channel.

4 Physical and topological description of the channels

As mentioned previously, the main channel of interest is from a socket to the far side of the distribution transformer at a point upstream of the branches to several other distribution transformers as shown by the arrow in the diagram in Figure 5.10.

![Diagram of channel distribution](image)

**Figure 5.10** Channel of interest shown by arrow.

The reason for this is the distribution transformers in a normal utility's network are supplied from MV feeders originating at a zone substation and these feeders typically supply several transformers. The MV distribution network at La Trobe, however, is complicated by the presence of the co-generation plant. In addition, the measurements would only closely represent a utility’s zone substation to sockets network if our co-generation plant was disconnected and not supplying the university. This posed two

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problems. The first was that the co-generation plant is, for economic reasons, almost always run 24 hours a day through the working week so restricting the times when the university is being fed from the utility, and is presenting a normal weekday load. This offered only rare occasions in the working week when the co-generation plant was offline for the purpose of repairs or maintenance. The second problem was that the substation plant room where the utilities’ feeder enters the campus is a highly restricted area for obvious safety and security reasons. At first research was conducted by receiving at the control room and with the co-generator running. The channel being characterised included a 7.5MVA, 11kV to 22kV transformer used to step up the voltage from the generator. It also included a branch in the 22kV path going out to the utility’s network and this branch was almost always connected for the purpose of exporting excess electricity. However, although the extra transformer and the branch made the channel unrepresentative of a typical utility’s channel, it did allow work to commence on designing and proving the channel sounding system with little restriction because the generator control room was an inherently safe place in which to work. Later on measurements were made on the more representative channel over the weekends when the co-generation plant was not operating and the university was being supplied by the external utility. It is these measurements which will be concentrated on in the results chapter.

4.1 Remaining parts of an AMR channel

Upstream communication from the meter to the zone substation presents the most difficulty in an AMR system. This is because downstream communication from the substation to the meter can be handled by existing ripple systems using a voltage signal. The rest of the complete bi-directional path between the utilities' computers and the meters is also trivial because the route from the substation to the location of the
consumer consumption data base computers will always be covered by a pre-existing fibre-optic link owned by the utility. For these reasons only the upstream LV/MV channel is of interest here.

5 Chanel linearity

As discussed in section 2, the system we have as a channel must be assumed to be linear and time invariant. The linearity assumption is not correct for the large 50Hz currents flowing. The mains loads are often non-linear enough for them to create significant harmonic currents flowing even though the generator is only producing 50Hz stimulation. However, we are going to ignore the 50Hz and all harmonically related currents. As far as signal currents are concerned it is apparent that the channel does behave predominantly linearly because products of the signal frequencies with the mains harmonic frequencies were virtually undetectable. Those signals tested and the products searched for are outlined in Appendix V H on page 319. This is not as improbable as it might seem. If a transformer or motor is considered, the hysteresis curve of the iron will become progressively less linear as the magnetising force increases. For large enough 50Hz currents the device will be non-linear and therefore produce harmonic products, however, for signal currents at frequencies un-related to and much higher than 50Hz and with peak amplitudes orders of magnitude below the 50Hz currents, the same device could be considered close to linear (although perhaps not time invariant). It is less clear why non-linearity caused by switching power supplies does not seem to invalidate the linearity and time invariance assumptions for signalling frequencies except that switching would often be occurring at different frequencies in different loads with no cumulative effect other than the addition of noise. The exception to random and un-related switching is in power supplies and in triac based power control circuits, in which the rectifier diodes are switching at mains frequencies. Perhaps there are simply too few
of these circuits in comparison to the simpler loads to invalidate the assumption in practice.

Where non-linearity is obvious is in the transmitter itself since if it were not for the non-linearity created by the switching of the load we could not produce signal currents at arbitrary frequencies. However, the transmitter is not part of the channel itself and its non-linearity does not come into the transfer function representation of the channel.

6 Conclusion

The measurement methods have been described in sufficient detail to allow replication of the experiments on other LV/MV distribution systems. The university's distribution system is unusual in that it includes local generation. However, in the electricity grid as a whole, small scale distributed generation will gradually become more common place [50] so future LV/MV networks being used as communication channels will have this in common with the distribution system studied here. For the time being though, simple channels from a socket to a single point in the zone substation are most important. Happily, it was possible to set up a method for making measurements on a set of representative simple channels. The equipment developed to facilitate these measurements is self contained and portable. It does not rely on a separate channel between the transmitter and receiver for synchronisation and so the same channel sounding method could be conveniently used on any utility LV/MV network.
VI SELECTION OF A STATISTICAL ANALYSIS TECHNIQUE

1 Introduction

In the results chapter, measurements of one of the most important aspects of conditions at the receiver, the background noise, will be presented. These noise measurements will be analysed using statistical techniques to determine the goodness of fit of the distribution of the narrowband noise signal to a normal or Gaussian distribution. The importance of showing that this noise signal is normally distributed is because Shannon’s channel capacity equation then applies. Calzada and Scariano point out that “assessing normality is a very subtle and difficult task, even for expert data analysts” [51]. For this reason the present chapter is included with the aim of demonstrating why a particular goodness of fit analysis technique (Anderson-Darling) was chosen and how, when it is used with a suitable confidence level, it can show quite satisfactorily that a particular data set is from a process with an underlying normal distribution.

Three approaches to determining normality, together with an assessment of the validity of each, will be explored in some detail in this chapter. The first two approaches to be examined are the Pearson Chi Square and the Lilliefors modified Kolmogorov-Smirnov tests which are both popular choices for the assessment of a distribution’s normality. The third, the Anderson-Darling test, has been chosen because it is known to offer greater power than either of the others [52, 53]. To illustrate the relative strengths and weaknesses of the three tests an analysis of a sample population of 333 derived from a known underlying distribution is performed. This sample size was chosen because it is near the optimum for illustrating the differences between the tests for the chosen underlying distribution. The chosen distribution was close to Gaussian and smaller
sample sizes do not give enough statistically significant evidence of a deviation from Gaussian whatever test is applied. Much larger sample sizes have so much evidence within them that any test will satisfactorily reveal the non-Gaussian nature. The analysis in this chapter reveals the superiority of the Anderson-Darling test for assessing normality in the context of the present work and it is this test which will later be used to analyse experimental noise data obtained at the receiver.

2 Numerical computation with samples from known Probability Distributions

To describe signals resulting from stochastic processes (processes with a random element) practically, probability distribution functions (PDFs), which measure the probability that a sampled variable lies between specified intervals, are commonly used. It is worthwhile noting that even for large sample sizes and infinitesimal intervals the PDF does not completely define the signal because the time order of the samples has been lost and with it, for example, any temporal correlation which may exist (unless our noise has infinite bandwidth there will always be temporal correlation present in its unsampled form and so this aspect will be investigated separately). The PDF does, however, represent a practical mathematical approach to analysing some aspects of a signal’s character.

Numerical computation using several software packages will be used to assess normality (and, in a subsequent chapter, to show normality can be expected) because closed form or analytical analysis is often not possible in situations involving real noise signals. Even when it is possible it is not always the most efficient way to achieve an engineering solution. As an example of mathematical complexity in a seemingly simple situation, consider the case when two random variables with PDFs which can be described by a simple mathematical function are multiplied together. The PDF of the result can be
calculated using closed form analysis however the mathematics is often non-trivial [54]. Recently the closed form mathematics needed to multiply two triangular distributions together to produce a third has been revealed as quite extensive even though the problem is seemingly simple [55]. A numerical approach to solving the same problem with computation software like MATLAB, requires much less effort as the short MATLAB scripts 1&2 in Appendix VI A on page 321 demonstrate. However the validity of the numerical approach must be established and that is the primary aim of this discussion.

2.1 The use of modelling software to determine PDF characteristics

Software packages using numerical computation can efficiently yield answers to questions involving probability distributions. However to have confidence in those answers requires an understanding of the statistical methods which the software is employing. Several packages were used for the work described in this chapter. MATLAB was used to evaluate Chi-Square fits, “Regress+” was used to evaluate the Kolmogorov-Smirnov (K-S) test and “EasyFit” and an Excel based program were used to evaluate the Anderson-Darling test [56]. First, however, the hypothesis testing method common to all of these techniques must be described.
2.2 Hypothesis testing

In trying to determine if a set of data is likely to have come from a process with a particular underlying distribution or not it is standard practice to start with a hypothesis that the data does indeed come from the stated distribution. This is termed the null hypothesis and to check if it is acceptable, statistical evidence is searched for which would indicate the hypothesis should be rejected. If no statistically significant evidence is found the hypothesis is accepted but, of course, this does not mean the hypothesis has been proved. This rather round-about way of proceeding has become traditional, possibly because, in science, a non-mathematical theory only needs a small amount of contradictory evidence to be disproved yet no amount of corroborating evidence can prove it. Sometimes confusion around setting up null hypotheses and looking for contradictory evidence leads to the danger of misuse of the technique. This misuse is when researchers set up a null hypothesis which they hope or know is false but then go on to use the results to say something definitive about the probability of the hypothesis being false (and therefore their alternative being true). Cohen has written a well known and amusing paper on this subject [57]. In the following examination of fit measurement techniques I do start with a hypothesis I know be false. This is not misuse though because the intention is to find the different statistical techniques’ ability to detect the falsehood; I am testing the test rather than testing the hypothesis.
2.3 Generating a data set from a “near Gaussian” distribution

In the next few sections I will be testing for the fit to a Gaussian distribution using the standard Chi Square test, the K-S test and the Anderson-Darling test. I will be testing the (null) hypothesis that the data came from a Gaussian distribution. To investigate the capabilities of these tests I need a data set which has a distribution which is similar to, but not quite, Gaussian. To obtain this I use the central limit theorem result that adding independent random variables together gives a sum with a PDF which tends to Gaussian (“tends to” meaning that it becomes Gaussian in the limit - when an infinite number of variables are added together). If a limited number of variables are added, and they each come from a non-Gaussian process, the result will not have a Gaussian distribution although it may be close.

Take two independent random variables each with a uniform distribution between some limits and add them together. It is well known the result will be the convolution of the two original PDF shapes – a triangular result in this case. If we now add a third random variable so that we are convolving a rectangle with a triangle, a process illustrated in Figure 6.1, the resultant shape will be that of a bell but we also know that when \( \tau \), the “delay” parameter, is very large (negative or positive) the two component PDFs will have no non-zero components overlapping so the result will be exactly zero.
Figure 6.1 Illustrating convolution of triangle and rectangle, which produces near Gaussian PDF, gives zero for \( \tau \) very large (negative or positive).

The fact that its tails have values of zero (even in the continuous case) demonstrates that it is not a Gaussian shape. In fact it deviates from Gaussian over its whole variate’s range as Figure 6.2 demonstrates.

Figure 6.2 Comparing Gaussian curve with near Gaussian produced by adding three random variables each with uniform distribution.
A data set from such a distribution was created and its fit to Gaussian tested using various techniques. This will illustrate some ideas associated with measuring goodness of fit and hypothesis testing and will demonstrate the important features of the various methods used. The following sections describe the whole procedure.

2.3.1 Details of creation of the data set

Using MATLAB (script 2 Appendix VI A, page 321) a set of samples for three independent random variables with uniform distribution between 0 and 1 were created. One sample of each of these three variables was taken and summed to give a new random variable of value between 0 and 3 but with an, as yet, unknown underlying distribution (but with average value 1.5 exactly and standard deviation 0.5 exactly). The samples of this new random variable, the empirical data, were tested to see how well they matched Gaussian distribution. Data sets with small numbers of data points give rise to more uncertainty which shows up the advantages and disadvantages of different fitting techniques more clearly; a data set of 333 samples was used and it is recorded in Appendix VI B on page 322. Figure 6.3 shows the histogram for the data set compared to the Maximum Likelihood true Gaussian distribution with the same area, average and standard deviation:

![Figure 6.3 Histogram for sum of three uniform distribution variables.](image)
2.4 Testing “near Gaussian” data set for Normality

One of the simplest tests to check one distribution against another is a Quantile-Quantile (Q-Q) plot, this was performed followed by a Chi Square test, A K-S test and finally an Anderson-Darling test.

2.4.1 Q-Q and Probability Plots

In a Q-Q plot the quantiles of one distribution are compared to the quantiles from another. A data set’s quantiles can be plotted against a true normal distribution’s quantiles to test the model of a Gaussian distribution. The graph of the empirical data’s quantiles against the model’s quantiles would be points on a straight line at 45° (y=x) if the data distribution matched the model exactly. Because the data is stochastic the graph will not match exactly, but points will be close to the line where the fit is good. Any trend away from the 45 degree line indicates a failure of fit between the model and the data. A simpler but equivalent way of doing this is to take the empirical data’s CDF and transform the linear y probability axis to a scale which would result in a straight line for a Gaussian distribution of the data [58]. Such a Probability plot for these 333 points is shown in Figure 6.4.

Figure 6.4 Probability plot for sum of three uniform distribution variables.
This plot reveals very clearly that there is significant deviation from normality in this data set. This shape indicates tails that are thinner than normal. This is an informal graphical technique but it does reveal that, even in this relatively small sample size there is evidence that the data is not normal. We should expect to be able to find a formal technique which gives dependable and definitive answer that this data is not from a process with an underlying Gaussian distribution.

### 2.4.2 Deciding if data is likely to be from a process described by our model

Before examining the formal normality tests we need to set up criteria for selecting the most likely model and making the accept/reject model decision.

The Gaussian distribution curve shown in figure 6.3 is the one with the same average and standard deviation as the data itself. It is a fact that this particular curve is the best model (of Gaussian form) for the process producing this data set which can be arrived at from just the data set itself [59]. In other words, the parameters of this Gaussian curve (the same mean and standard deviation as the data) are those which maximise the probability that the data came from a process described by the model, any other mean and standard deviation being less likely for the underlying process given just these data points. It should be pointed out that, if we had been dealing with a PDF shape other than Gaussian for our model, the search for the model parameters using the Maximum Likelihood criteria would not have been so trivial. It should also be pointed out that what we are really interested in is, was this data from any Gaussian distribution process rather than just the Maximum Likelihood process (it could be from one with a slightly different average than the average the data itself has). For the present, let us assume we are testing against the Maximum Likelihood curve only. Now that we have selected a specific model for the data we can follow the standard scientific procedure and hypothesise that
the data came from a process which this model describes. The measurement of the fit of the data to the model gives us a value or metric, call it M. For any data set, with absolute rigour, very specific things can be said about the likelihood of obtaining that data with that metric of fit M or worse, given the hypothesis. This can be used to make a decision about whether or not to accept that the empirical data did come from a process described by the model. If, given the hypothesis that the model does describe the process, the probability of obtaining a fit metric the same or worse than we have obtained is not particularly low, then, by default, we accept the hypothesis. Conversely, if that probability is miniscule, we would be forced to reject the hypothesis. Using looser language, the former is equivalent to saying that any differences between our sample of data and the model we are proposing are not statistically significant (and the latter is equivalent to saying that we should not expect that data). Of course we must also define what we think is a “particularly low probability” or what we are considering to be “statistically significant”. This is covered in section 2.4.3.1 on selecting confidence levels. We next need to choose a formal technique and associated metric to measure the fit between the model and the data.

2.4.3 Chi Square test

We’ll begin by using a simple standard statistical technique, called the Pearson Chi Square test, to test the hypotheses that the data came from a process with an underlying Gaussian distribution. This technique is based upon separating the empirical data into a number of bins as was done for the histogram of figure 4.
The number of bins chosen is arbitrary. The definition of the Pearson Chi square statistic, or test statistic is:

\[ \chi^2 = \sum_{\text{all}} \frac{(o_k - e_k)^2}{e_k} \]

where \( k \) is the bin number and \( o_k \) is the observed bin population and \( e_k \) is the expected population (in the Pearson Chi Square test technique the goodness of fit metric is this “test statistic” or a derivative of it).

Here in table 6.1 below is the Chi Square calculation for 14 bins obtained using MATLAB Script 3 in Appendix VI A on page 321.

<table>
<thead>
<tr>
<th>Bin Number</th>
<th>expected ( k )</th>
<th>observed ( k )</th>
<th>chi square ( \chi^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.1*</td>
<td>0</td>
<td>1.1</td>
</tr>
<tr>
<td>2</td>
<td>3.6</td>
<td>4</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>8.8</td>
<td>16</td>
<td>5.9</td>
</tr>
<tr>
<td>4</td>
<td>18.3</td>
<td>25</td>
<td>2.5</td>
</tr>
<tr>
<td>5</td>
<td>32.1</td>
<td>30</td>
<td>0.1</td>
</tr>
<tr>
<td>6</td>
<td>46.2</td>
<td>29</td>
<td>6.4</td>
</tr>
<tr>
<td>7</td>
<td>55.5</td>
<td>57</td>
<td>0.0</td>
</tr>
<tr>
<td>8</td>
<td>55.5</td>
<td>55</td>
<td>0.0</td>
</tr>
<tr>
<td>9</td>
<td>46.2</td>
<td>38</td>
<td>1.5</td>
</tr>
<tr>
<td>10</td>
<td>32.1</td>
<td>46</td>
<td>6.0</td>
</tr>
<tr>
<td>11</td>
<td>18.3</td>
<td>21</td>
<td>0.4</td>
</tr>
<tr>
<td>12</td>
<td>8.8</td>
<td>8</td>
<td>0.1</td>
</tr>
<tr>
<td>13</td>
<td>3.6</td>
<td>4</td>
<td>0.0</td>
</tr>
<tr>
<td>14</td>
<td>1.1</td>
<td>0</td>
<td>1.1</td>
</tr>
</tbody>
</table>

sum: 25.1

* For the distribution of the sum of the Chi Square components to closely match a Chi Square distribution the minimum number of expected data points in each bin should be 1 [60]. This means we have got sufficient data points in this sample for these bin sizes.

**Table 6.1** Chi square for 14 bins.

The Chi Square component for each bin is a measure of fit for the number of data points within that bin to the theoretical prediction assuming the hypothesis of an underlying
Gaussian distribution is true. Due to the central limit theorem, the difference in observed to expected numbers in a particular bin will have a normal distribution were many samples sets of data to be pulled from the underlying distribution. The average difference will be zero if the hypothesis is true (and if the estimated mean and standard deviation for the curve we are fitting were accurate). We are taking the sum of the square of these differences and, because the differences have an underlying Gaussian distribution, by definition of Chi Square distributions, the result of the sum of squares will approximate a Chi Square distribution. The shape of this Chi Square distribution is dependant upon the “number of degrees of freedom”. Each bin we use gives another degree of freedom so the number of degrees of freedom is related to the number of differences squared which we have summed together. Because the numbers in all the bins must always add up to the total number of samples we had, the number of degrees of freedom is one less than the number of bins used.
We are looking at the match to a fixed normal distribution, the parameters for which we obtained from the data itself, but in fact we are interested in the fit to any Gaussian curve so we lose a further two degrees of freedom, one for the average and one for the standard deviation. This leaves us with 11 degrees of freedom and this 11 degrees of freedom defines the particular Chi Square distribution to which the sum of differences squared will conform (had the selection of 333 samples been repeatedly extracted from the underlying distribution). The PDF for this Chi Square 11 degrees of freedom distribution is shown in Figure 6.5:

![Chi square distribution for 11 degrees of freedom](image)

**Figure 6.5** Chi square distribution for 11 degrees of freedom.

### 2.4.3.1 Selecting Confidence level

Is the calculated sum of differences (25.1) for this particular data set big, or bigger than expected? Judging by the distribution above it is big; we would usually expect a smaller result. However, standard procedure in statistics is to decide on a confidence level before we look at where our particular Chi Square statistic comes in the Chi Square distribution.
The confidence level is $1-\sigma$ where the level of statistical significance is $\sigma$. What we select depends on how certain we want to be that the hypothesis is false before rejecting it. If we select a high confidence level before rejection, for example 99%, it means we are being protective of (reluctant to reject) our hypothesis. The interval is so large that, if the hypothesis were true, 99% of test statistics from repeated trials would fall within it. The chance of us rejecting a true hypothesis (type 1 error) is low, in fact the chance of this occurring with a single trial is just 1% ($\sigma$), but it also means that if the hypothesis passes the test we still cannot be very sure that it is true. When using the testing on real experimental data in later sections I will choose a much lower confidence level so being much more severe on the hypothesis.

In the Chi Square analysis for the case above, the 11 degrees of freedom specifies the specific Chi distribution and the 99% confidence level specifies the critical value on that distribution. Using the tables in [60] we determine the critical value as 24.7 so 99% of the area under the curve above is to the left of the x value 24.7. Because our test statistic is larger than the critical value we reject the hypothesis and say that the differences between our sample data and the hypothesised Gaussian distribution are statistically significant at the 1% level (if it was Gaussian then in 1% of cases we would expect as poor a fit, or worse, than the fit we have measured). To put it another way, if we had selected the data set many times from a distribution that was Gaussian then we would expect 99% of the selections to give a better fit than we have found, so based on this one selection we are rejecting the hypothesis with the reasoning that our data set is too unlikely.
2.4.3.2 Problems with the critical value obtained for Chi Square

So it looks like the Chi Square test has detected a difference between our data and data that would have come from a process with underlying Gaussian distribution. There are two things wrong with this conclusion. The first is, if the bin size had been marginally different the opposite result would have been obtained. This is illustrated in the next section and it is one of the reasons for not using Chi Square tests in this study. The second is much more subtle. The critical value of 24.7 applies for 11 degrees of freedom and we used 11 degrees of freedom because we have a composite hypothesis which we want to test against – we really want to hypothesise a fit to a normal distribution of unknown exact average and standard deviation, not just a fit to a known fixed distribution. To account for this, two extra degrees of freedom were removed and we actually tested against a known distribution with the same average and standard deviation as the data set itself. This is standard text book procedure, for example [60]. In fact, strictly speaking, the correct procedure is to test against the Maximum Likelihood distribution for the grouped or binned data, not the raw data [61]. The consequences of using the raw data to obtain the mean and standard deviation is that we can only say the correct critical value is between that for 11 degrees of freedom and that for 13 degrees of freedom in this case [62]. That is between 24.7 and 27.7. Since our test statistic falls in between these two values it means we can neither conclude the hypothesis should be rejected nor conclude that it should be accepted without going through the much more complicated procedure to identify the exact critical value which is outlined in [5,6]. It is interesting to note that good commercial statistical software (e.g. EasyFit from Mathwave [63] ) does not go through this extra procedure which means that a great deal of care must be taken when interpreting results from it.
2.4.3.3 Chi Square tests can be very sensitive to the (arbitrary) bin size choice

Here is the Chi Square goodness of fit for exactly the same data set as used above but splitting it into 15 bins rather than 14:

<table>
<thead>
<tr>
<th>Bin Number</th>
<th>expected $k$</th>
<th>observed $k$</th>
<th>chi square $\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>0</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>3.0</td>
<td>3</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>7.2</td>
<td>13</td>
<td>4.8</td>
</tr>
<tr>
<td>4</td>
<td>14.7</td>
<td>22</td>
<td>3.7</td>
</tr>
<tr>
<td>5</td>
<td>26.0</td>
<td>23</td>
<td>0.3</td>
</tr>
<tr>
<td>6</td>
<td>38.3</td>
<td>28</td>
<td>2.8</td>
</tr>
<tr>
<td>7</td>
<td>49.0</td>
<td>52</td>
<td>0.2</td>
</tr>
<tr>
<td>8</td>
<td>53.3</td>
<td>51</td>
<td>0.1</td>
</tr>
<tr>
<td>9</td>
<td>49.0</td>
<td>37</td>
<td>2.9</td>
</tr>
<tr>
<td>10</td>
<td>38.3</td>
<td>36</td>
<td>0.1</td>
</tr>
<tr>
<td>11</td>
<td>26.0</td>
<td>38</td>
<td>5.6</td>
</tr>
<tr>
<td>12</td>
<td>14.7</td>
<td>19</td>
<td>1.3</td>
</tr>
<tr>
<td>13</td>
<td>7.2</td>
<td>9</td>
<td>0.5</td>
</tr>
<tr>
<td>14</td>
<td>3.0</td>
<td>2</td>
<td>0.3</td>
</tr>
<tr>
<td>15</td>
<td>1.0</td>
<td>0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

sum: 24.6

Table 6.2 Chi Square, same data as used to obtain table 6.1 but with 15 bins instead of 14 bins.

For the 12 degrees of freedom we have in this case the 99% confidence level (significance level 1%) critical value is 26.2 so we would accept the hypothesis based on this test in contrast to the case where 14 bins were chosen. Happily, in this case, the problem with identifying the exact critical value (due to having used raw data to estimate the mean and standard deviation of the underlying distribution) does not occur. This is because the test statistic is lower than the lower bound critical value (the higher bound value for 14 degrees of freedom being 29.1).
2.4.4 K-S test

Because of the problems outlined above with the Pearson Chi Square test (sensitivity to bin size which is well known, and difficulty in determining the correct critical value, which is less well known) the Kolmogorov-Smirnov (K-S) Empirical Distribution Function (EDF) based test was investigated. “Regress+”, mathematical modelling software written by Mike McLaughlin [64], was used for this purpose. This software was used to evaluate the fit of the input file of empirical data to a given (Gaussian in this experiment) distribution.

The K-S based test is a better test than Chi Square for normality testing [65] and it is one of the more robust statistical techniques available for this problem. This test is generally more “powerful” than a Chi Square test [66, 67]. Power is a formal statistics term which means the test has a greater ability to detect a difference between two distributions when there really is a difference [52] and this is exactly what we are interested in. In this K-S test we compare the fit of the “CDF” curve for the data - which is called the Empirical Distribution Function (EDF) - with that for a known (Gaussian in our case) distribution. The absolute value of the maximum deviation between the curves is used as the goodness of fit metric. The empirical data is first scaled and shifted by subtracting the mean and then dividing by the standard deviation. The EDF of this new data is then compared with the CDF for a normal curve with mean zero and standard deviation one. The largest deviation is the K-S goodness of fit metric. This is the basis of the method which the program “Regress+” uses.
2.4.4.1 Composite hypothesis for K-S test

As with the Chi Square test, the composite hypothesis we have adds complication. When we are trying to measure the match between an empirical distribution and a hypothesised distribution, Gaussian in this case, which normal curve do we use – that is, what do we take as the mean and standard deviation for the hypothesised distribution? All we can really do is to use the mean and standard deviation of the empirical data itself. However, this completely fixes the distribution curve we are matching to, to one particular Gaussian shape. There are many others (with different means and standard deviations), one of which will be the distribution of the underlying process as opposed to the distribution for this particular set of data produced by the underlying process. In the Chi Square test this was compensated for by reducing the number of degrees of freedom by two when selecting the Chi Square distribution from which the critical value was obtained. With the K-S test the fact that we measure the match to the nearest Gaussian shape rather than the underlying process’s shape, is a problem because if many runs of the experiment were done, each run would produce an answer for how good the match is to Gaussian but each run would be using a different Gaussian shape to compare with. The answer we are looking for is how well do all the sets of data, which would be obtained if the experiment were repeated many times, match one single Gaussian shape – that being the hypothesised underlying distribution the exact parameters of which we do not know (composite hypothesis case). The standard K-S test gives a biased result favouring accepting the hypothesis for a match. This bias can be removed by modifying the K-S test. The same test statistic is used (that obtained by looking at the difference between the EDF and the CDF of the Gaussian distribution based on the mean and standard deviation of the data) but the shape of the distribution which is used to derive the critical value is altered. It is surprising that this should be possible but it is a fact that
Chapter VI  Selection of statistical analysis technique

the distribution of the K-S statistics (or any other EDF statistic like the Anderson-Darling $A^2$ as used in the next section) depends only on the distribution type being tested against (Gaussian here), the parameters estimated and the estimation method (mean and standard deviation using raw data values) and the sample size [68]. The standard method for modifying the K-S test by tabulating the distribution used to derive the critical value for the composite hypothesis case was developed by Lilliefors [69]. Good commercial statistical software packages continue to use lookup tables for the critical values (e.g. EasyFit from Mathwave [63]) but Regress+ uses an improved technique. Instead of relying on tables calculated for specific sample sizes with inherent approximations, a bespoke distribution is calculated on the fly, tailored to the exact sample size using a Monte Carlo simulation or parametric bootstrap described in detail in the program user’s guide [59].

2.4.4.2  K-S results

When the 333 data points are presented to the Regress+ program it returns the K-S statistic of 0.043 (this result was checked using the previously mentioned EasyFit program [63]). Using its bootstrap procedures, Regress+ then takes some time to calculate the distribution expected of this statistic if the underlying distribution of the input data set had been Gaussian. It reports that the K-S statistic is estimated to be within the 86$^{th}$ percentile which means that, for a 99% confidence level, the hypothesis is not rejected. It also means that had a 90% confidence level been selected the hypothesis would still not have been rejected.
2.4.5 Anderson-Darling Test

Having looked at the K-S EDF based test and found it cannot detect the difference between the data set and one which would be expected had the process which produced it been Gaussian, even for a 90% confidence level, another EDF based test, which is more powerful, will be considered. This is the Anderson-Darling (A-D) Test. Whereas the K-S test just looked at the maximum absolute difference between the EDF and the CDF of the distribution we were fitting against, the A-D test uses the $A^2$ statistic which is one of a group of test statistics called quadratic statistics. They are weighted sums of the squares of all the differences along the variates axis between the EDF and CDF of the distribution being fitted against. If no weighting is applied we have the Cramér-von Mises statistic but the Anderson-Darling $A^2$ statistic applies a weighting of

$$\frac{1}{f(x)(1-f(x))}$$

where $f(x)$ is the cumulative distribution of the distribution we are testing against (CDF), varying between close to 0 and close to 1. In this way tails are given more weight since the weighting is larger for large and small $f(x)$, and is smallest for Gaussian distribution near the average where $f(x)$ is close to 0.5. The Anderson-Darling test is that recommended for goodness-of-fit to Gaussian testing by Stephens [68]. The commercial software package, EasyFit from Mathwave [63], was used to calculate the Anderson-Darling $A^2$ test statistic for the same 333 point data set used in previous tests; this calculation was checked using a Microsoft Excel worksheet from Robust Systems and Strategy [56] which makes visible the exact calculation being performed. The Anderson-Darling $A^2$ test statistic was 0.86. Again the commercial software package was found to be taking short cuts, it reports critical values very close to those values designed for testing for fit to a completely specified distribution which are found in [53, 68, 70]. An explanatory paragraph, Appendix VI C on page 323, is
given in the program’s Help file which justifies the short cut by suggesting that the test is OK to use for comparative purposes. I have recently been caught out by using the software and quoting the critical value of 1.37 for a significance level of 0.2 in a paper [71]. The actual critical value for the composite hypothesis being tested is very different at 0.5 [72]. Luckily it makes no difference to the conclusion in this case (the test statistic was 0.39).

2.4.5.1 Procedure used for applying A-D test

In order to reach the correct conclusion for hypothesis rejection or acceptance with a specified confidence level, we will use the EasyFit program to calculate the statistic but will refer to the authoritative authors for determining the proper critical values for our chosen confidence levels. Following this procedure, and checking the tables for a test for normality with the average and standard deviation unknown in [53, 72], we arrive at a critical value of about 1 for a 99% confidence level. With the test statistic of 0.86 this means the hypothesis is accepted, as it was for the K-S test statistic at this level of significance. However, this is protective of the hypothesis, we are only rejecting that hypotheses when the data is highly unlikely given the hypothesis. Selecting a lower confidence level of 90% the critical value is about 0.65 [53, 72] and we do reject the hypothesis. At last we can say “given the hypothesis is true this data is unlikely, so the hypothesis is rejected”. This is in contrast to the K-S test case in which the (untrue) hypothesis of an underlying Gaussian distribution was still accepted for this perfectly reasonable confidence level of 90%.
3 Conclusion

In this chapter several standard methods of testing the hypothesis that data came from a process with an underlying Gaussian distribution were accessed. The Chi Square test was rejected because of some undesirable features which were demonstrated and because it is known to be of low power for normality testing. The greater power of the Anderson-Darling Normality test over the K-S test for normality was demonstrated. This greater power is the reason for selecting the Anderson-Darling test for noise measurements and analysis in this study because I will want to demonstrate convincingly that, under certain conditions, we can be sure the noise signal distribution is Gaussian. This demonstration will be performed on simulated noise data in the next chapter to show we can expect a Gaussian distribution. Then, in the subsequent results chapter, the same test will be used on experimentally measured data which will show we do actually get a Gaussian distribution.
VII NOISE

1 Introduction

In this chapter a method of representing the noise signal on the mains will be developed. Noise is different to deterministic signals in that it cannot be completely specified, so choices must be made about what is important and should be retained in the description. For reasons which will be covered, the signal is described in narrow bands across the spectrum of interest. Conveniently, we find that the signal within each band can be specified with a single random variable which has a Gaussian distribution, so a variance for each band gives a description for the whole noise signal. This form of description does not represent all the characteristics of the noise, for example, impulsive noise which has very heavy tails in its Probability Distribution Function (PDF), and is significantly different from Gaussian, will still be described by a sum of narrow band Gaussian process components. However, splitting into narrow band components is a useful form of description because Shannon’s law (which applies when the noise distribution is Gaussian) can be applied to calculate the channel capacity within each band, and this will be done in the analysis & discussion chapter. In the next paragraphs some important assumptions, and considerations which are relevant when dealing with sampled noise signals, will be introduced. An outline of the subsequent sections of this chapter will then be given.
1.1 Basic considerations when dealing with noise measurements

1.1.1 Assumption of stationarity and ergodicity

In this and following chapters the assumption will be made that the random process producing our noise signals is weakly stationary [73] and ergodic [74]. Weakly stationary means that for a limited time, the average, standard deviation, and shape of the PDF of the signal all remain unchanged. Ergodic means an ensemble of noise signals of the type we are considering could be satisfactorily represented by time segments of a single time-versus-voltage noise signal. These assumptions are necessary because some of the results used are based on theory which incorporates these assumptions. The corollary of making the assumption of stationarity is that, when taking measurements, we must be careful to limit the time over which we take samples so that, within that time, the RMS noise signal level, averaged over a shorter time, does not change. The corollary of making the assumption of ergodicity is that we must be careful that the measurement time is sufficiently long so that our single set of samples contains complete statistical information about the signal – for example, that it has at least had time to cover its entire range of likely values. So the method of representing mains noise signals developed in this chapter should only be applied to mains noise measurements taken over a time period which is not too short and not too long. Experimental evidence described in the results and analysis & discussion chapters will show that this time period is typically between seconds and several minutes for the data collected but can extend to hours for data collected within the higher frequencies bands.
1.1.2 Describing wideband noise signals without losing too much information

If we describe a noise signal simply by the statistics of the overall signal, i.e. by its PDF, we lose all time related information which was contained within the signal. To retain a great deal of the time information (although not all) we can split the signal into frequency bands and look at the statistics within each band. Another way information in the original signal can be lost is by taking discrete samples, and this will now be considered. In contrast to wideband noise signals, signals which are not random and are band limited can easily be represented by a series of samples. If the sampling is over the Nyquist rate for the signal, the gaps between the samples present no problems because there is effectively no new information in those gaps. The signal in between the samples could be exactly reconstructed by doing a discrete Fourier transform of the sampled signal, and the original signal could be reconstructed using continuous sine waves at each of the decomposed frequencies at the appropriate phases. With a wideband noise signal that has not been band limited, say a white noise signal, whatever the sample rate, there is always information missed in between samples. This problem is analogous to mapping a coastline with elements similar to those Mandelbrot made famous [75] – more and more new detail is revealed as the scale is increased yet the detail at different scales has similarities (the PDF is unchanging with time scale in the Gaussian white noise case). Apart from the wideband nature of noise signals, their other characteristic, which is different from most tractable signals, is their randomness. To fully represent a sine wave we only need specify its frequency and phase and a small number of samples does this satisfactorily. These samples allow the signal’s value to be predicted for any time value; the same applies to more complex deterministic signals. In contrast, a random noise signal can never be fully represented no matter how many samples are specified and this is true even if the noise is band limited (because it is not time limited
and not periodic). We might ask what is the best we can do to specify such a signal? A first answer might be to list as many samples per unit time as possible for as long a time as possible but usually a more practical and concise specification is required. This is why probability distribution functions are used, but, as previously mentioned, all time elements are lost by using only this type of signal description.

1.1.2.1 Mixing deterministic components into the description

One way of retaining some of the important time information in the description of the signal is to split the signal into its spectrum and specify the way the signal behaves in many narrow bands across the bandwidth of interest. By taking a purely random signal and looking at what that signal does over a restricted bandwidth we are introducing a new degree of freedom, frequency, which is deterministic. As part of the signal description process, the deterministic frequency has to be “mixed” with the random signal to create a narrow band noise signal. Later on it will be useful if this deterministic component can be extracted again so that the statistics of the random part on its own can be examined. This process of extracting and describing the statistics of the random elements in each band is illustrated in Figure 7.1.
Figure 7.1 Splitting a noise signal into bands and the representation of components within each band by a statistical distribution of a random variables, the envelope amplitudes in this case, (b) compared with no splitting (a).

Unless we are going to chop the signal into an infinite number of small bands, it must first be limited to a maximum frequency, so let us start by considering the process of low-pass filtering. If we imagine sampling a white noise signal with an infinitely high sample rate and then band limiting it by low-pass filtering, a new deterministic element
is introduced, the cut-off frequency of the low-pass filter. This turns the signal
description from a purely random one, as defined by Chatfield [76], into a mixed one
with a random part. The wide band white noise signal we started with had to have an
infinite density of samples to represent it, but the low-pass filtered white noise signal can
be fully represented with a finite number of samples per second. If we can assume the
filtering is brick-wall filtering, the sample rate need only be twice the low-pass filter cut
off frequency. We can now specify the low-pass filtered component of the original
signal by giving the random element (the list of filtered samples) and the deterministic
element (the cut-off frequency) separately. For a brick wall filter and Nyquist rate
sampling the filtered samples are random and can be concisely specified by a PDF. The
entire low-pass signal can now be described by a given PDF (e.g. Gaussian RMS 5V )
along with a statement that its spectral content is limited to below a given frequency. In
the case of low-pass filtered white noise we have discarded almost the entire original
unfiltered signal; the signal we are left with (which has a finite bandwidth) is an
infinitesimally small part of the original signal. For this reason some potentially
important information can be lost in the filtering process. For noise signals that lost
information is the PDF of the original signal. It will be shown by experiments described
later in this chapter that low-pass filtering makes the output PDF tend to Gaussian no
matter what form of PDF the input had. It is interesting to note that for white noise
which was Gaussian in the first place no important information is lost in the low-pass
filtered description.
1.1.2.2  Signal description in narrow band segments

Extending the argument from low-pass to band-pass, we can split our noise signal into many segments of bandwidth B. The component of the noise within each band is described in relation to a reference frequency \( \omega \) which is usually, but not necessarily, the center frequency of the band. The signal within each band can be represented using two deterministic elements, \( \omega \) and B, together with the separated out random element. Again, if the random element is purely random, and this requires that the amplitude versus frequency shape of the segment is rectangular, it can be represented by its PDF with no important information being lost. It will be shown in this chapter that the random part of the signal within each band segment can be described by the envelope of the signal within the band. This envelope can, in turn, be described by two random orthogonal components, only one of which need be retained.
1.1.3 Outline of the rest of this chapter

In the previous chapter the Anderson-Darling test for normality was described and selected for use in this study. This test, and heuristics, will be used in section 2 to show that the output from a low-pass filter can be expected to have nearer a Gaussian distribution than its input, and that high order low turnover low-pass filtering will result in an output close to Gaussian. This applies even if the input has a distribution very far from Gaussian. In section 3 it will be shown that this tendency to Gaussian can also be expected for the outputs of a narrow band filtering process when the outputs are a pair of low-pass signals representing the in-band signal. The groundwork will also be laid for showing that a single random variable, as opposed to two, can be used to represent noise in a narrow bandwidth. Section 4 of this chapter shows the relevance of the results developed in section 3 to measurements of noise made with the narrow band filter used in the experimental work; specifically, why only one of the two random orthogonal components of the in-band noise (P and Q) need be retained. Finally, in section 5, a simulation is presented which closely replicates the equipment and settings used, to make measurements of the noise on the MV lines in the experimental work. The simulation shows that we should expect a Gaussian distribution for the components of the filter’s narrow band output, irrespective of the distribution of the mains noise signal at the filter’s input.
2 Output of Low-pass filter tends to Gaussian PDF

2.1 Effect of low-pass filtering on a non-Gaussian PDF of sampled data

As a convenient first example, let us take sample data from the underlying PDF described in the last chapter (three random variables each with uniform PDFs added together). As was described, it is clearly identifiable as non-Gaussian when a large number of samples are taken, yet it has a “bell shape” curve with values far from the mean much less likely than values nearer to the average (as we would expect from the random component of the mains noise). We shall consider the effect on the PDF of low-pass filtering this noise data. The data we produced by adding three random variables is what we would expect from sampling an white noise signal in that each data point is totally independent of the last. To filter this sampled data with a digital filter we would pass it through one with coefficients which form the impulse response of the filter. For a low-pass digital filter, the impulse response must have the coefficients sum to a non-zero value (otherwise DC will not get through). It must also have more than one non-zero coefficient (otherwise the signal passes through with an identical shape). The simplest filter that meets these criteria is one with two unity value taps and a single delay element. That is, we add adjacent samples. Since these samples are independent, we can expect the PDF of the result to be nearer to Gaussian (than the original data) from the central limit theorem. To further explore this tendency to Gaussian after low-pass filtering the simulation described in the next section was performed.
2.2 Checking the hypothesised effect of low-pass filtering on a non-Gaussian PDF

A data set of 5,000 points was created by adding three random variables each with uniform distribution between 0 and 1 as described in the last chapter. The fit to Gaussian was accessed from the Anderson-Darlington $A^2$ test statistic obtained using the “Easyfit” software [63] described in the last chapter. One of the advantages of this test statistic in this application is that, provided the sample size is reasonably large (and several thousand is quite sufficient), the critical value for the test statistic does not depend appreciably on the sample size. So the critical value is still about 1 for a 99% confidence level for this 5,000 point data set (as it was for the much smaller data sets used in the previous chapter). The $A^2$ test statistic for the 5,000 point data set was found to be 2.35, which is well above the critical value and so indicates a poor fit; a crude estimation based on a formula [56] approximating the tail of the $A^2$ distribution indicates about a one in 170,000 chance of getting a data set with a fit as bad as this (or worse) if the underlying process was indeed Gaussian. This data was passed through the simple single zero rolling average filter with two unity value taps (adding adjacent samples) previously described in section 2.1 and implemented as part of the MATLAB script in Appendix VII B on page 323. Following filtering, the $A^2$ test statistic for the output was 0.98; we would accept the hypothesis of underlying Gaussian distribution at the 99% confidence level. Now the chance of a resultant data set with as bad a fit has reduced to less than 1 in 100 (from 1 in 170,000 before filtering). This is a huge improvement in fit using a filter that was just a two sample averaging filter. In following sections we will explore what happens when higher order (steeper roll-off) and lower turnover frequency low-pass filtering is applied.
2.3  *Necessity to re-sample (decimate)*

We can extend the simplest averaging filter to use three samples (a two zero filter with a lower 3dB cut off frequency), and this might be expected to produce an even better fit to Gaussian than those obtained for the experiment described in section 2.2. The three samples being added represent what we would expect from sampling a white noise signal; they are all independent. This means the central limit theorem still applies. If we repeat the experiment with this new filter, on the same data as used in section 2.2, we get an $A^2$ test statistic of 1.0 for the 5,000 point output set (using the MATLAB script in Appendix VII B). There seems to be a counter effect at work here because the fit is not further improved. This is an interesting and unexpected result which deserves more investigation. The experiment was repeated many times, with different data sets and filters summing three and four adjacent samples. Similar results were obtained and Appendix VII F on page 327 contains some examples. As the results in this Appendix show, summing three (or four) independent samples does not consistently result in a better fit than summing just two samples in a single zero filter. Yet summing just two samples always results in a much better fit than the original data.
2.3.1 **Explanation for the apparent divergence from the results which would be expected in light of the central limit theorem**

Implicit in the treatment of our data with a “filtering” operation is that the data points represent sampled points of a continuous signal which is a function of time with each sample separated from its neighbours by a fixed amount of time. When we extend the averaging filter in length we are adding up more independent variables for each output data sample. Although we might expect a better fit to Gaussian due to the central limit theorem, each sample in the output represents less “information” about the original signal than each sample in the input. Looked at in the time domain this is because each output sample is made up of the sum of several input samples and there are a myriad ways of obtaining the same sum. Therefore each output sample is being less specific about the signal close to it in time than each input sample was; redundancy has been added due to the adding of correlation between filtered samples. In the frequency domain the explanation that each output represents less information than each input sample is clearer. Because we are performing more low-pass filtering, every time we add a zero we are reducing the amount of information in the output. The reason for this is that some of the severely attenuated high frequency part of the signal has been lost due to the fixed amplitude resolution used for each data point. However, even though we have less overall information we are retaining the same number of points to represent the signal. After filtering with several zeros, and at a lower turnover frequency, we now have many more data points than are required to represent the information in the new filtered signal. These, effectively redundant, data points make the fit worse than we should expect for that number of points if each contained entirely new information. This counter effect can be isolated and can be seen clearly by the following procedure: take a particular number of points from a distribution and calculate the fit to the same
distribution; next, replicate each point to double the number of points without adding any extra “information”. Although the histogram shape for the original set and the doubled set is identical, the fit is much worse because the extra points should have been expected to add information improving the fit, but they were not real new random points. Although there is nothing intrinsically wrong with representing a signal with more data points than are necessary, it does cause a problem in this application because the goodness of fit techniques are all sensitive to the number of data points used. This problem can be thought of as a “stale data” problem and it is illustrated in Figure 7.2.

![Figure 7.2](image-url)  
**Figure 7.2** Illustration of the “stale data” problem – too many data points for the amount of information in the black line signal.

The red line represents a band limited signal which is sampled at the positions of the black crosses and the black line represents the signal after low-pass filtering with sampled values indicated by dots. In both cases the signal is represented by the same number of samples. In the post filtered data case the samples are no longer appropriately spread along the time axis for the purpose of distribution fitting. Middleton [74] pp207-209, explains that sampling a signal which is strictly band limited in frequency at this high sample rate (compared with the spectral content) results in superfluous points and means that the samples are no longer “algebraically independent”. When a periodically
sampled waveform, which is band limited to \(< B \) Hz, is sampled every \(T_0\) seconds, the smallest number of sampling points necessary to represent the original continuous waveform uniquely occurs when \(T_0 = \frac{1}{2} B\). Sampling above this rate means that if a single sample were changed, a new continuous signal, limited to bandwidth \(B\), would not be represented; it would require more than one point to be changed in order for the bandwidth to remain \(< B\). This means the points are mutually dependant [74]. Only samples taken at \(T_0 = \frac{1}{2} B\) uniquely represent the signal and retain algebraic independence between samples.

### 2.3.2 Solution - maintain an appropriately sampled representation of the signal

This stale data counter effect, which is reducing the goodness of fit statistic at the same time as filtering is increasing the goodness of fit, can be eliminated by re-sampling the output data from the filter so that there is no redundancy. This will be done in the following section (2.4). Re-sampling at the rate which retains algebraic independence, whilst still uniquely representing the signal, could be slightly problematic where the filtering is not brick wall. The rolling average filters which have been used for simplicity in the experiments reported here have gentle roll-offs. For example, the single zero, two sample averaging filter has a null at half the input sample rate and a -3dB point at a quarter of the sample rate. If we decimate this filter output by a factor of two this will retain algebraic independence of the output samples which satisfies the requirement that the output should not have added stale data. However, after decimation, the output is not limited to the lower frequency components in the input due to aliasing of the frequency components above the -3dB point. For these noise signal examples, this does not matter because the input is flat band limited white noise which has no correlation between components of the noise at higher frequencies to components at lower frequencies. The apparent aliasing distortion of the output waveform causes no problem since it is only
the statistics we are interested in, and this aliasing just serves to re-flatten the spectrum of the output of the filters with the aliased noise power above the -3dB point adding to the noise power below. The spectrum of the signal represented by the decimated outputs is again flat to a frequency of half the new sample rate.

2.3.3 Prevention of distortion of conclusions by reduced power of goodness of fit test

Before we implement the decimation solution described above, we must protect against another effect which would conspire to distort the results and obscure the effect of filtering. It is evident from the 5,000 point experiment, with results given in Appendix VII F (page 327), that with a single order filter, if data points are not discarded, there is no consistent improvement in fit when higher order filtering is used. But, if we start discarding data points to retain independence between samples we will rapidly reduce the total number of samples representing the signal to levels which will make any measurement of fit to Gaussian unreliable. For example, if the 5,000 data points are decimated by two for each pole, we will end up with only a few hundred data points left after a few poles. This small amount of data will not contain enough evidence to make a compelling argument either for a fit, or for the lack of a fit. As we performed decimation to eliminate the stale data effect, we have results of the filtering being hidden by the rapidly reducing power of the fit-to-Gaussian test itself. Increasing the data set length is the solution and it has no disadvantage so long as the process producing the data is stationary. If the hypothesis is true, and the data is Gaussian, then increasing the amount of data which is used to test for a fit will result in a better fit. If the hypothesis is untrue, then increasing the amount of data will result in a worse fit. In other words, increasing the data set length has increased the power of the test. For this reason, the next set of experiments used an input signal sample size of 100,000 data points. As before, without
the discarding of points the fit is initially improved by the addition of more zeros, in this case up to a limit of about four zeros at which point the stale data problem becomes evident again. Some results where the data set length has been increased are shown in Appendix VII G on page 328.

We are now ready to investigate the result of decimation along with filtering, without fear of the effects being concealed by the failure of the procedure we are using to measure the effects.

2.4 Application of re-sampling to the previous example

If we choose to re-sample (decimate), thus retaining the independence of the samples representing the signal, and then test the fit of the smaller output data set, we consistently get an improvement in fit by adding a zero. Using this re-sampling procedure, the result quickly converges to a very good fit. For example, the first 100,000 data point sample set used in the previously mentioned set of experiments passed through a one zero filter then was then decimated by two, resulting in an $A^2$ test statistic of 4.9 for the 50,000 point output. The other results are tabulated in Table 7.1 below and were obtained by using the MATLAB script in Appendix VII B.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Fit ($A^2$ statistic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>no filtering</td>
<td>36</td>
</tr>
<tr>
<td>one zero</td>
<td>4.9</td>
</tr>
<tr>
<td>two zeros</td>
<td>0.67</td>
</tr>
<tr>
<td>three zeros</td>
<td>0.61</td>
</tr>
<tr>
<td>four zeros</td>
<td>0.43</td>
</tr>
<tr>
<td>Five zeros</td>
<td>0.42</td>
</tr>
</tbody>
</table>

**Table 7.1** The goodness of fit to Gaussian measurement showing consistent improvement in fit when low-pass filtering is performed and the output is represented by an appropriate density of samples.
Each zero improves the fit to Gaussian and for a five zero moving average filter, decimated by six, the $A^2$ test statistic for the 16,667 point output is 0.42 indicating an excellent fit. The chance of getting data with a fit worse than this is about 1 in 3 (assuming the data came from a process with an underlying Gaussian distribution) so the hypothesis of a Gaussian distribution for the filtered data should certainly not be rejected.

### 2.5 Is re-sampling distorting the result?

It could be argued that decimating, thus reducing the number of points in a non-Gaussian data set, will improve the test statistic anyway. This is true because there is less evidence of non-normality with which to reject the normality hypothesis, but the improvement in the statistic obtained by decreasing the number of points is much smaller than that obtained when low pass filtering as well. The 16,667 point output of the filter after decimation, in the last stage of the last example, fits the Gaussian hypothesis very well indeed, but the fit for the same number of points selected from the input data set is about 6.0. Although an $A^2$ test statistic of 6.0 shows there is much less evidence for rejecting the Gaussian hypothesis than there was for the original 100,000 point set, it would still indicate a negligible chance that the data came from a Gaussian process. Re-sampling on its own does improve the test statistic, but it does not interfere with the demonstration of the powerful effect of low-pass filtering which was shown in Table 7.1.

### 2.6 Input data set was entirely random

Digital low-pass filtering of non-Gaussian wideband noise signals, with a moving average filter, results in an output which tends to Gaussian as the order of filtering is increased with a concomitant reduction in the turnover frequency. This effect has been
shown by reducing the sample rate at each filter stage output to maintain independence of the samples. This raises the question of what happens when the filter input samples are not statistically independent, in which case decimating cannot restore statistical independence. This is the case considered in the next section.

2.7  More realistic data sets

In the preceding sections a random data set was input to the filter and each value in this data set was algebraically and statistically independent of any other value in the set. This was convenient because the reason for the better fit of the output to Gaussian was obvious from the central limit theorem. However, in real life, any noise signal that we come across will inevitably already have had some spectrum limiting effect applied to it. If we sample this signal, and the sampling rate is above the Nyquist frequency required for its spectral content, each sample will have some correlation to the sample(s) before. We could test the tendency to Gaussian under filtering proposition using a noise signal with an arbitrary spectrum which has been carefully prepared to ensure that the sample rate is at exactly twice that of the highest frequency component. In this case the samples, although not statistically independent, will still be algebraically independent and the decimation at each filtering stage will maintain this algebraic independence throughout the process. However, a more useful experiment is to abandon all requirements for any independence between input samples and use an input signal which has a strong correlation between successive samples. This type of signal is easily obtained by using the same low-pass filtering as has been described previously on the sample data set before it is input to the digital filter that we are examining. If we take the same wideband 100,000 sample point data set used in the last experiment to obtain the results in the previous table, and pass it through the 2 zero filter but do not discard any of the output, we get a 100,000 point band limited low-pass noise signal in which there is significant
correlation between successive sample points. This would represent a realistic band
limited signal which has been sampled above the Nyquist rate appropriate for the
spectral content of the signal. The previous experiments were repeated using this low-
pass signal as the input to the filter under test. The results are given in Appendix VII H
on page 329 and they show that, provided the decimation step is included between each
filtering stage to ensure we do not swamp the fit test with stale data, the fit to Gaussian
is improved as more filtering is applied. This demonstrates that even though we have
lost the complete algebraic independence of samples and cannot explain the result in
terms of the central limit theorem, the same tendency to Gaussian effect as was seen
previously is still evident.
2.8 Section 2 Summary

We have seen that digital low-pass filtering of non-Gaussian noise signals results in an output which tends to Gaussian as the order of low-pass filtering increases and the bandwidth is reduced. This effect is more evident if the density of samples used is reduced as the bandwidth is reduced in order to maintain the level of algebraic independence between the samples. It is important to note that in re-sampling the filter output we are simply discarding redundant samples and not changing the signal being represented. The necessity to represent the signal without oversampling is dictated by the requirements of the goodness of fit measurement process, the process by which we are determining how well the signal fits a Gaussian distribution.

Later it will be shown that the results extend to the narrow band filtering performed in measurements on the mains noise signal. Before that however, narrow band noise signals and their analysis will be considered.
3 Narrow band signals and band-pass filtering

3.1 Comparison of low-pass and band-pass descriptions of noise signals

It was indicated in section 1.1.2.1 of the introduction to this chapter, that to describe a random white noise signal, two components are required, the random element and a deterministic component (e.g. Gaussian noise RMS amplitude 5V, in a measurement bandwidth up to 1kHz). Let us now compare this with the description used when splitting the signal into frequency bands. When we low-pass filter a noise signal it becomes more and more like a slowly varying “DC level”. The lower the cut-off frequency, the smaller and more slowly varying it is. An analogous situation arises when band-pass filtering a noise signal, the smaller the bandwidth the smaller the output signal and the closer it comes to being a sine wave. This tendency to sinusoidal output from a narrow band filter was described by Rice [77]. It has been established, in section 2, that the PDF of a low-pass filtered noise signal (re-sampled at a rate appropriate for the filtering applied) becomes more Gaussian than the original signal. The PDF of the band-pass filtered signal will not behave in the same way as the bandwidth is reduced as the signal is tending to look sinusoidal and the PDF of a sine wave looks very different from Gaussian. This sinusoid at the center frequency of the filter is an entirely deterministic component in the output of the band-pass filter. If we use a procedure to isolate it, we are left with a non-sinusoidal part which contains the random “information” in the signal. This procedure is based on Rice’s [77, 78] description of a narrow band noise signal \( N(t) \) which is a sine wave at frequency \( \omega_0 \) amplitude modulated by a stochastic component \( \tilde{A}(t) \) and with a random phase \( \varphi(t) \).

That is:

\[
N(t) = \tilde{A}(t) \sin(\omega_0 t + \varphi(t))\]
The sine wave component is isolated by considering the random envelope and phase parts of the signal, $\tilde{A}(t)$ and $\varphi(t)$, separately from the reference frequency at $\omega_0$ rads/sec.

The expression for the band-pass noise signal can be put in terms of two orthogonal sinusoidal components according to:

$$ N(t) = \tilde{A}(t)[\sin(\omega_0 t)\cos[\varphi(t)] + \cos(\omega_0 t)\sin[\varphi(t)]] $$

so

$$ N(t) = \tilde{A}_s(t)\sin[\omega_0 t] + \tilde{A}_c(t)\cos[\omega_0 t] $$

which avoids the use of a phase term, the phase term being combined with $\tilde{A}(t)$ into two new amplitude terms $\tilde{A}_s(t)$ and $\tilde{A}_c(t)$ where:

$$ \tilde{A}_s(t) = \tilde{A}(t)\cos[\varphi(t)] \quad \text{and} \quad \tilde{A}_c(t) = \tilde{A}(t)\sin[\varphi(t)] $$

These random terms $\tilde{A}_s(t)$ and $\tilde{A}_c(t)$ both behave exactly as in the low-pass filter case. The PDFs approach Gaussian as the bandwidth is reduced. This means that, for sufficiently narrow bandwidths, the RMS levels of $\tilde{A}_s(t)$ and $\tilde{A}_c(t)$ are all that are needed to describe the random components of the in-band signal without any further loss of information. All of this will be expanded on in the following sections and demonstrated experimentally in section 5.

### 3.2 Representation of a band-pass filtered random signal using vectors

If the band-pass signal is derived from a Gaussian process, the PDF of the phase, $\varphi(t)$, is uniform and that of the envelope, $\tilde{A}(t)$, is Rayleigh [77, 78]. A logical way of thinking about, and representing, a sine like signal such as $\tilde{A}(t)\sin[\omega_0 t + \varphi(t)]$ which has a known average frequency is to plot it as a vector on a (P-Q) plane. This is
performing the same process as was used to obtain $\tilde{A}_s(t)$ and $\tilde{A}_c(t)$ in the last section.

It is conventional to plot this vector on a plot with a horizontal axis labelled P and vertical axis labelled Q and to refer to it as a phasor. P stands for “in Phase” and is the specified (arbitrary) reference phase, the orthogonal axis being Q (for Quadrature). Any sinusoidal wave at the specified frequency $\omega_0$ will be a stationary point in this plane, the position depending upon its amplitude and phase relative to the arbitrary reference. In the same way in which a sinusoidal wave can be represented on this plane, the narrow band noise signal $N(t)$ can be represented; but in this case the phasor will not be stationary. If the instantaneous frequency drops below the reference frequency (that is, the random phase $\varphi(t)$ increases) the phasor rotates anti-clockwise. As the short term peak amplitude, $\tilde{A}(t)$, changes so the phasor length changes. Provided the reference frequency is in the center of the signal’s band, and the signal power is symmetrical about that reference frequency, then over the long term the phasor does not continue to make rotations in one direction or the other about the origin but quivers around the origin. The projections of the phasor onto the in-phase and quadrature axis will be referred to as $P(t)$ and $Q(t)$ such that $P(t) = \tilde{A}_c(t)$ and $Q(t) = \tilde{A}_s(t)$. In following sections these P and Q components will be used to describe the narrow band signal because of the familiarity and usefulness of this phasor representation and because they relate directly to experimental measurement results.

3.3 Removing the deterministic (sinusoidal) part from a narrow band noise signal to leave the P and Q phasor components

By resolving the vector representing the narrow band noise signal into its P and Q components we are effectively removing the deterministic part of the signal, its short term sinusoidal structure at the reference frequency. A formal description will be given
in the next section but the P(t) and Q(t) components could loosely be described as the “short term correlation” of the reference frequency sinusoidal waves, represented by the P and Q axes, with the signal. They are the two non-deterministic parts which make up the signal and are a pair of random variables. Consider an un-filtered wideband noise signal. It is specified by a single random variable, yet the part of this signal which we are interested in and which inhabits the narrow band close to the reference frequency is represented by two random variables P(t) and Q(t). The reason we now have two variables is because the non-deterministic parts of the signal are in the envelope’s amplitude, $\tilde{A}(t)$, and in the phase $\varphi(t)$ measured relative to the reference sine wave, and, as explained previously, $\tilde{A}(t)$ and $\varphi(t)$ are alternatively represented in a rectangular co-ordinate system by the P(t) and Q(t) pair.

This way of representing a narrow band random process with two orthogonal vectors is similar to the standard formal representation [79-81], but in these texts the vectors are the real (R) and imaginary (I) parts of the complex envelope. This complex form of representation will be considered in the next section. Comparison of the representation of the narrow band signal by two real variables, P and Q, with the complex form using R and I is made later in section 3.5.
3.4 *The usual formal representation of narrow band signals using R and I*

The real (R) and imaginary (I) parts of the complex envelope, which are usually used to formally represent a narrow band signal, will be defined in this section. The formal treatment of narrow band signals usually involves introduction of the “analytic signal” (also called “pre-envelope”) which is a complex signal made up of a real part, the original signal, and an imaginary part, which is the Hilbert transform of the original signal [79, 81-84]. The Hilbert transform operation:

\[
\hat{e}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{e(\tau)}{t-\tau} d\tau
\]

is defined and examined in detail in Appendix VII I on page 330 but for the purposes of this section its effect in the frequency domain can be defined by this transfer function [81]:

\[
H(j\omega) = -j \text{sgn} \omega.
\]

From this we see the Hilbert transform multiplies the imaginary part of negative frequency components by \(j\) and the imaginary part of positive frequency components by \(-j\) (leaving original real components unaltered). We assume the signal \(N(t)\) is real so its Fourier transform \(N(\omega)\) is Hermitian, that is \(N(-\omega) = N(\omega)^*\) where the star denotes the complex conjugate. If \(N(\omega) = N_R(\omega) + jN_I(\omega)\) then the Hilbert transform of the noise signal will be:

\[
\hat{N}(\omega) = -jU_+(\omega)N_R(\omega) + U_+(\omega)N_I(\omega) + jU_-(\omega)N_R(\omega) - U_-(\omega)N_I(\omega)
\]

where

\[
U_+(\omega) = \begin{cases} 
1 & \text{for } \omega > 0 \\
0 & \text{for } \omega < 0
\end{cases}
\]

and

\[
U_-(\omega) = \begin{cases} 
0 & \text{for } \omega > 0 \\
1 & \text{for } \omega < 0
\end{cases}
\]

(note: undefined at \(\omega = 0\))

This is multiplied by \(j\) and added to the original signal to give the analytic signal \(A\) overleaf:
\[ A(\omega) = N(\omega) + j\tilde{N}(\omega) \]

\[ = U_+(N_R + jN_I) + U_- (N_R + jN_I) + U_+ N_R + jU_+ N_I - U_- N_R - jU_- N_I \]

After collecting real and imaginary terms we are left with:

\[ A(\omega) = 2U_+(\omega)N_R(\omega) + 2jU_+(\omega)N_I(\omega). \]

(note: undefined at \( \omega = 0 \))

This shows the analytic signal is twice the original signal with the negative frequency components removed. So this signal in the time domain is the same as the inverse Fourier transform of twice the original signal in the frequency domain but with the integration performed only over the positive frequencies because its result is zero over negative frequencies:

\[ A(t) = \frac{1}{\pi} \int_0^\infty N(\omega)e^{j\omega t} d\omega. \]

We can legitimately use this analytic signal to represent the original signal because we have not added anything; we have simply removed a redundant negative frequency half. The negative frequencies in the original signal are redundant and add no more information because they are defined by the fact that the original signal is real and so has Hermitian symmetry.

The analytic signal can now be frequency shifted to be centered on zero Hz to give the complex envelope \( C(t) \), which is the low frequency equivalent of the analytic signal. If the original band pass signal was centered on \( \omega_0 \) then in the time domain, the complex envelope will be the inverse transform of the frequency shifted analytic signal. Thus:

\[ C(t) = R(t) + jI(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(\omega - \omega_0)e^{j\omega t} d\omega. \]
The real and imaginary components of this complex envelope, R and I, are then used to represent the original real narrow band signal, in combination they give the envelope $\tilde{A}(t)$, and phase $\varphi(t)$, of the original real signal. This treatment is elegant because removal of the negative frequency components means that the down shifted signal (the complex envelope) does not have the components of the signal above the center frequency folding over on top of components below the center frequency as would happen if we downshifted the original signal as illustrated in Figure 7.3.

![Figure 7.3](image)

**Figure 7.3** Adding of upper and lower sidebands when down shifting a real signal (magnitudes depicted in the complex frequency domain).

In the analytic signal/complex envelope case, the two frequency shifted versions of the signal, one centered on the original signal’s center frequency and the other centered on DC, can be thought of as exactly representing one another because there is no fold over problem. This is in contrast to the baseband signal in Figure 7.3 which cannot be used to fully represent the band-pass signal because if it were shifted back up to the original center frequency the sidebands would be symmetrical and this was not necessarily the case for the original signal. A time domain explanation for this is as follows: The baseband signal is a real signal, it is simply the envelope of the original signal described with a single time dependant variable. When the original signal is represented as
\[ \tilde{A}(t) \sin(\omega_0 t + \varphi(t)), \] as it was in section 3.1, the baseband signal is \( \tilde{A}(t) \). Because \( \varphi(t) \) is missing from this description, the baseband signal does not contain all the information in the original signal. On the other hand, the complex envelope is a complex signal, it contains two variables which do represent all of the information in the original signal \( \tilde{A}(t) \sin(\omega_0 t + \varphi(t)) \), exactly as the P and Q vectors described in section 3.2 did when the original signal was expressed as \( \tilde{A}_c(t) \sin(\omega_0 t) + \tilde{A}_e(t) \cos(\omega_0 t) = Q(t) \sin(\omega_0 t) + P(t) \cos(\omega_0 t) \). The \( \varphi(t) \) information is retained in the angle between the real and imaginary components of the complex envelope as it was between the P and Q components of the real signal.

3.5 **Comparison of the use of a complex baseband signal with the use of two real variables, P and Q, to represent a narrow band signal**

It has been shown that P and Q (defined in section 3.2) or R and I (defined in section 3.4) can be used to represent a narrow band signal. In the next chapter measurements which yield P and Q will be described, but in order to borrow some of the results from analysis using the complex representation, the two methods are compared here. As stated in the previous section, the formal treatment of real narrow band signals uses a single complex time dependent variable, \( C(t) \), to represent the “information” in the signal. This complex baseband signal modulates a frequency (the carrier) which results in a complex signal, \( A(t) \), identical to the original narrow band signal with the negative frequency components removed. Compare this with the case in which P and Q, the two real baseband signals are used to represent the “information” in the signal. P and Q modulate two real quadrature carriers such that when the two modulated sinusoids are added we end up with the original narrow band signal. The P and Q signals in this case, and the real (R) and imaginary (I) components of the complex baseband signal, are
analogous to one another although the mathematical route from them to the original narrow band signal is different. In addition, the R and I components are usually used to mathematically describe a pre-existing isolated narrow band signal, whereas the P and Q signals will generally result from a filtering or measurement process. That is, the P and Q signals are used to define the narrow band signal which really exists as part of a wideband signal.

In a measurement system, the P component is obtained by multiplying the real input signal by a cosine wave at the center frequency of the measurement equipment’s pass band. For each sinusoidal component in the input, a sum and difference component result at the multiplier’s output. The sum components, those with a frequency which is the sum of the input sinusoidal frequency and the cosine wave at the center frequency of the measurement equipment’s pass band, are filtered out and discarded. It is interesting to compare this process with that used for obtaining the real component of the complex baseband signal in the usual formal representation of narrow band signals. Primak, Kontorovich and Lyandres [81] obtain the real part by multiplying the Hilbert transform of the band-pass signal by $\sin \omega_0 t$ and then adding the band-pass signal multiplied by $\cos \omega_0 t$ as illustrated in Figure 7.4 below:

![Figure 7.4](image-url)  

Figure 7.4 Showing the method for obtaining the real part of the complex envelope.
When this is done (shown in detail in section 3.6.2) the sum components cancel each other out. We are left with exactly the same difference components which we have for \( P \) in the measurement case when \( P \) is obtained by multiplying the input frequency components by a cosine wave. In the complex envelope analysis case, the high frequency components get eliminated by cancelation; in the measurement equipment case the high frequency components are attenuated by a low-pass filter, and if the attenuation is sufficient, they can also be considered eliminated.

A similar comparison with \( Q \) can be carried out for the components in the Imaginary part of the complex envelope which is obtained by multiplying the Hilbert transform of the band-pass signal by \( \cos \omega_0 t \) and then adding the band-pass signal multiplied by \( \sin \omega_0 t \).

In this section the use of the Hilbert transform was introduced. Its use to generate the real and imaginary components of the complex envelope was touched upon. In the next section this use of the Hilbert transform will be demonstrated in more detail and the Hilbert transform relationship between components within the \( P \) and \( Q \) signals will also be explored.
3.6 Comparing the operation of a lock-in amplifier with narrow band signal analysis

3.6.1 Lock-in amplifier analysis

In the following chapter, chapter 8, measurements using a lock-in amplifier will be presented. This piece of equipment is a narrow band detector essentially performing a filtering and frequency downshift operation simultaneously to produce the P and Q output signals referred to in previous sections. An analysis of its operation is presented here to show how the P and Q signals are related by the Hilbert transform (which is defined in Appendix VII I).

The lock-in amplifier multiplies the input signal by a sine wave and a cosine wave (at the reference frequency) to give two wide band signals, $P_w$ and $Q_w$ which are then low-pass filtered. This is illustrated in Figure 7.5 below:

![Figure 7.5 Diagram of lock-in amplifier measurement equipment.](image-url)
We can determine how the frequency components of these signals, $P_w$ and $Q_w$, are related to frequency components at the input by using the trigonometric identities:

\[
\begin{align*}
\sin (A) \cos (B) &= \frac{\sin (A+B) + \sin (A-B)}{2} \\
\sin (A) \sin (B) &= \frac{\cos (A-B) - \cos (A+B)}{2} \\
\cos (A) \cos (B) &= \frac{\cos (A+B) + \cos (A-B)}{2}.
\end{align*}
\]

Any frequency at the input results in a sum and difference frequency after multiplication by the reference frequency. It is instructive to split the input into two sidebands and treat signals in each separately. Spectral components in each of the two sidebands can be represented by a sine wave and a cosine wave at the same frequency (this avoids the need to define the phase of the input frequency components), one pair creating a tone in the lower sideband and the other pair a tone in the upper sideband. Let us now track what outputs are caused by the four groups of input frequency components, sine below the reference, sine above the reference, cosine below the reference and cosine above the reference. The reference frequency is $\omega_0$ (rads/sec), the lower sideband frequency is $\phi_L$ (rads/sec) and the upper sideband frequency is $\phi_U$ (rads/sec). The assumption shall be made that $\phi_L > 0$ (not a severe restriction for real signals) and $\phi_U < 2\omega_0$. This means that it is always possible to isolate the baseband components from the components which have been shifted up in frequency (if this is not the case then the highest frequency part of the down shifted upper sideband can overlap the lowest frequency of the up shifted lower sideband and so a filter cannot isolate them). Let a frequency component in the lower sideband be $A_L \sin \phi_L t + B_L \cos \phi_L t$ and a frequency component in the upper sideband be $A_U \sin \phi_U t + B_U \cos \phi_U t$. 
Let the amplitude of the local oscillator be 1. The unfiltered $P_w$ is then:

$$\cos(\omega_0 t) [A_L \sin \phi_L t + B_L \cos \phi_L t + A_U \sin \phi_U t + B_U \cos \phi_U t]$$

& the unfiltered $Q_w$ is similarly:

$$\sin(\omega_0 t) [A_L \sin \phi_L t + B_L \cos \phi_L t + A_U \sin \phi_U t + B_U \cos \phi_U t]$$

Applying the trigonometric identities and separating out the components due to the two sidebands we get:

$$P_w = \frac{1}{2} A_L \left[ \sin (\omega_0 \pm \phi_L) t \pm \cos \omega_0 t \pm \cos (\omega_0 \mp \phi_L) t \right]$$

$$+ \frac{1}{2} A_U \left[ \sin (\omega_0 \pm \phi_U) t \pm \cos \omega_0 t \pm \cos (\omega_0 \mp \phi_U) t \right]$$

and

$$Q_w = \frac{1}{2} A_L \left[ \cos (\omega_0 \pm \phi_L) t \mp \cos (\omega_0 \pm \phi_U) t \right]$$

$$+ \frac{1}{2} A_U \left[ \cos (\omega_0 \pm \phi_U) t \mp \cos (\omega_0 \pm \phi_U) t \right]$$

After passing both signals through an ideal low-pass filter with a cut-off below $(\omega_0 + \phi_L)$ rads/sec and above $(\phi_U - \omega_0)$ we get:

$$P = \frac{1}{2} A_L \left[ \sin (\omega_0 - \phi_L) t \right] + \frac{1}{2} B_L \left[ \cos (\omega_0 - \phi_L) t \right]$$

$$+ \frac{1}{2} A_U \left[ \sin (\phi_U - \omega_0) t \right] + \frac{1}{2} B_U \left[ \cos (\phi_U - \omega_0) t \right]$$

and

$$Q = \frac{1}{2} A_L \left[ \cos (\omega_0 - \phi_L) t \right] + \frac{1}{2} B_L \left[ \sin (\omega_0 - \phi_L) t \right]$$

$$+ \frac{1}{2} A_U \left[ \cos (\phi_U - \omega_0) t \right] + \frac{1}{2} B_U \left[ - \sin (\phi_U - \omega_0) t \right]$$

If we just look at the components of $P$ and $Q$ due to signals in the lower sideband, call them $P_L$ and $Q_L$, we see that $Q_L = \hat{P}_L$ where the hat symbol indicates the Hilbert transformation $\hat{H}$, since $\hat{H}(\cos A) = \sin A$ and $\hat{H}(-\sin A) = \cos A$ and similarly, $P_U = \hat{Q}_U$. 

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Also, since \( \hat{H}(-\cos A) = -\sin A \) and \( \hat{H}(\sin A) = -\cos A \), we see that \( P_L = -\hat{Q}_L \) and \( Q_U = -\hat{P}_U \).

We can now write:

\[
P = P_U + P_L = \hat{Q}_U - \hat{Q}_L \quad \text{and} \quad Q = Q_U + Q_L = \hat{P}_U - \hat{P}_L.
\]

This makes it clear why \( P \) is related to \( Q \) (and vice-versa) by the Hilbert transform, yet \( P \) cannot normally be obtained from \( Q \) since the components in the \( Q \) signal due to the upper sidebands cannot be isolated from components in the signal due to the lower sidebands. The obvious exception is when one of the sidebands is empty.

### 3.6.2 Narrow band signal analysis

A similar analysis can be performed to determine the components of the complex envelope of a band limited signal. This is an instructive exercise in that it shows the real and imaginary components (\( R \) and \( I \)) of the complex envelope of a narrow band signal are exactly the same shape as the \( P \) and \( Q \) components at the output of a lock-in amplifier performing narrow band filtering. This is conditional on the frequency content at the output of the filter being the same as that of the narrow band signal being analysed, and the reference frequency, \( \omega_0 \), being the same in both cases. Using the same upper and lower sideband notation as before, the real component of the complex envelope due to multiplication by \( \cos (\omega_0 t) \), is given by:

\[
R_{\cos} = \cos (\omega_0 t) [A_L \sin \phi_L t + B_L \cos \phi_L t + A_U \sin \phi_U t + B_U \cos \phi_U t]
\]

and the real component due to Hilbert transforming the input and then multiplying by \( \sin (\omega_0 t) \), is given by:

\[
R_{\text{Hilbert}} = \sin (\omega_0 t) [B_L \sin \phi_L t - A_L \cos \phi_L t + B_U \sin \phi_U t - A_U \cos \phi_U t].
\]
When the trigonometric identities (stated in section 3.6.1) are used to perform the multiplications, we get the two elements of the real part of the complex envelope as:

$$R_{\cos} = \frac{1}{2} A_L \left[ \sin (\omega_0 + \phi_L)t - \sin (\omega_0 - \phi_L)t \right] + \frac{1}{2} B_L \left[ \cos (\omega_0 + \phi_L)t + \cos (\omega_0 - \phi_L)t \right]$$

$$+ \frac{1}{2} A_U \left[ \sin (\omega_0 + \phi_U)t + \sin (\phi_U - \omega_0)t \right] + \frac{1}{2} B_U \left[ \cos (\omega_0 + \phi_U)t + \cos (\phi_U - \omega_0)t \right]$$

and

$$R_{\text{Haine}} = \frac{1}{2} A_L \left[ -\sin (\omega_0 + \phi_L)t - \sin (\omega_0 - \phi_L)t \right] + \frac{1}{2} B_L \left[ -\cos (\omega_0 + \phi_L)t + \cos (\omega_0 - \phi_L)t \right]$$

$$+ \frac{1}{2} A_U \left[ -\sin (\omega_0 + \phi_U)t + \sin (\phi_U - \omega_0)t \right] + \frac{1}{2} B_U \left[ -\cos (\omega_0 + \phi_U)t + \cos (\phi_U - \omega_0)t \right].$$

When added together the sum frequency terms all cancel and we are left with a definition for the real part of the complex envelope which is exactly twice the P component in the analysis of the lock-in amplifier:

$$R = 2P = A_L \left[ -\sin (\omega_0 - \phi_L)t \right] + B_L \left[ \cos (\omega_0 - \phi_L)t \right]$$

$$+ A_U \left[ \sin (\phi_U - \omega_0)t \right] + B_U \left[ \cos (\phi_U - \omega_0)t \right].$$

The algebra will not be given here, but in the same way as R was shown to be 2P, the magnitude of the imaginary part, I, can be shown to be equal to 2Q.

The only difference between the P and R, and Q and I, is the factor of two. This is due to the fact that, in defining the complex baseband signal, we are effectively using two local oscillators (one sine and the other cosine) in the construction of both the R and I outputs, and each oscillator has an amplitude of 1. This is in contrast to the lock-in amplifier which only uses one local oscillator (a cosine) to create the P signal, and one (a sine) to create the Q signal. This factor of two has been described as creating a “stubborn 3 dB error” which can cause some confusion [83], but the comparison of the two situations given above makes it perfectly clear why the power, as represented by the complex
baseband signal, appears to be twice the power in the envelope of the real signal which one would normally measure. The factor of two affects both R and I, and so when they are added geometrically, and the power of the signal calculated, it appears 3dB greater than that calculated from the P and Q signals.

As before with P and Q, it follows that $I_L = \hat{R}_L$. The part of I that results from the lower sideband components alone is equal to the Hilbert transform of the R signal which is due to the lower sideband components alone (and again, unfortunately this part of the signal cannot generally be isolated). Similarly, $R_U = \hat{I}_U$, $R_L = -\hat{I}_L$ and $I_U = \hat{R}_U$, and summarising:

$$R = R_U + R_L = \hat{I}_U - \hat{I}_L \quad \text{and} \quad I = I_U + I_L = \hat{R}_U - \hat{R}_L.$$  

Since the complex baseband signal represents the information in the narrow band directly, the two equations above show that the Hilbert transform relationship between the two orthogonal components of the signal in the narrow band are intrinsic to the narrow band signal itself, and are not an artefact of the measurement process of frequency shifting and low-pass filtering performed by the lock-in amplifier.

3.7 Choice of reference frequency $\omega_0$

In working towards the conclusion that, although P and Q cannot generally be derived from each other, we do not need both to represent the information in the signal which we are interested in, it is necessary to examine the role played by the reference frequency.

We are particularly interested in when $P = \hat{Q}$ or $P = -\hat{Q}$ because in these cases P defines Q and vice versa.
In derivations of a representation of a narrow band signal by the amplitude of a vector defined by P and Q (section 3.6.1), or by the amplitude of its complex envelope defined by R and I (3.6.2), a reference frequency \( \omega_0 \) was used. Rice called this the “midband frequency” [78] but Dugundji explained that theoretically, for the purpose of representing the envelope, its choice was entirely arbitrary; it doesn’t matter what reference frequency is chosen, the same envelope results [84] and the envelope contains all of the important information in the signal. This being the case, it is worth considering whether one reference frequency is better than another but before that a set of graphs will be presented of P and Q and the envelope (which is defined by \( \sqrt{P^2 + Q^2} \) as the equations in section 3.1 can be used to show).

In these graphs the quite extraordinary result, that the choice of reference frequency is immaterial to the envelope [84], will be demonstrated and the relationship of the Hilbert transform of the Q component to the Q component itself, for different reference frequency choices, will be shown. The horizontal axis is the time axis and covers exactly one second, the input signal for derivation of the traces (using the equations from section 3.6.1 above) was a combination of two sinusoids, one at 9Hz and one at 11Hz. The lower frequency sinusoid was \( 12 \sin 2\pi 9t + 6 \cos 2\pi 9t \) and the other sinusoid was \( 4 \sin 2\pi 11t + 8 \cos 2\pi 11t \). First, the reference frequency was chosen to be in the exact center of the input frequencies at 10Hz, then it was chosen to be offset, but still within the input frequency band at 10.7Hz. In both cases there are upper and lower sidebands.
Figure 7.6 (b) and (c) below show the signals. Note that, for the two different reference frequencies, identical envelopes result although the P and Q signals are quite different. Also shown is that the negative of the Hilbert transform of Q (−Q̂), does not equal P.

Next, the situation when the reference frequency is exactly on the edge of the band of frequencies in the signal is briefly considered. It might have been expected that a match between −Q̂ and P would occur since the input signal has no upper side band component. However, calculating the Hilbert transform is problematic since the components of P and Q will include a zero frequency term for which the Hilbert transform is undefined.

Now the situation with the reference frequency above the band of signal frequencies is considered. Figure 7.7 opposite shows this with the reference frequency at 11.1Hz and 12Hz.
Figure 7.7 Reference frequency above input frequencies, (a) just above at 11.1Hz and (b) at 12Hz.
Note that now $P = -\hat{Q}$.

It is interesting that as long as the reference frequency chosen is above the highest frequency, $P = -\hat{Q}$. It is also interesting that the reference frequency can be any frequency higher than the input frequencies and the envelope is unchanged.

The same effects are evident if the reference frequency is below the lowest frequency in the signal. In this case the signal is all in the upper sideband and instead of $P = -\hat{Q}$ we find $P = \hat{Q}$ as expected from the analysis in section 3.6.1. Figure 7.8 illustrates this with the reference at 8.9Hz and 1Hz.

Figure 7.8 Reference frequency below input frequencies, (a) just below at 8.9Hz and (b) at 1Hz.
Note that now $P = \hat{Q}$. 

Chapter VII Noise
So far, in performing this analysis for the real signals P and Q we have specified only two input signals, one at 9Hz and the other at 11Hz. There would be no alteration to any of the conclusions if the input signal had contained other frequency components between these two components. We would still have a narrow band-pass signal at the input, and the P and Q outputs would still be identical to R and I, the real and imaginary parts of the complex envelope. These results, obtained by simulation, could be replicated on real equipment, given sharp enough low-pass filters to separate out the sum frequency components and difference frequency components. Use of a reference frequency below 1Hz, however, would result in a different situation. In the case of a reference just below 1Hz the difference frequency components would occupy the band just above 8Hz to just above 10Hz and this would overlap with the sum components which would occupy the band just below 10Hz to just below 12Hz. Separation would then be impossible. However, the R and I components of the complex envelope can still be determined and, for the sake of completeness, a reference frequency of zero Hz will be considered. At this frequency the definition of the real part of the complex amplitude (R), the sum of the result of multiplying the signal by cos at the reference frequency, and the result of multiplying the Hilbert transform of the signal by sine at the reference frequency, becomes simply the real part of the signal itself – since cosine zero = 1 and sine zero = 0. Similarly, the imaginary part of the complex envelope of the signal is simply the Hilbert transform of the signal. This is why $R = \hat{I}$ even when the reference frequency is zero as shown in Figure 7.9.
Conclusion for section 3.7

If we have a narrow-band signal, we can represent it using P and Q, two real baseband signals, provided the reference frequency chosen doesn’t make it impossible to separate out the difference frequency products from the sum frequency products. If the reference frequency is chosen to be outside the signal’s band width, only one baseband signal, P or Q, is necessary to fully describe all of the information in the narrow band signal. This is because the other baseband signal can be derived from the Hilbert transform of the single base-band signal which we chose to represent the narrow-band signal.

3.8 Section 3 summary

The relationship between representations of band-pass and low-pass noise signals was considered. They were shown to have similarities, so that the tendency-to-Gaussian result from section 2 can be expected to extend from low-pass to band-pass, and this will be confirmed in section 5. The difference between low-pass and band-pass filtering was shown to be the introduction of a reference (carrier) frequency in the band-pass case. Given this frequency, two variables are derived which define the behaviour of projections of the narrowband signal on two orthogonal basis vectors at the chosen
reference frequency. This reference frequency is normally chosen to be at the center of the band which the signal occupies but, as demonstrated in section 3.7, this does not have to be the case. For different reasons the reference frequency is sometimes chosen to be offset, for example, in Viswanathan’s paper on the autocorrelation of white noise [85], it was offset because the high frequency limit of the noise was allowed to tend to infinity to represent true white noise. If we choose an offset of just greater than half the bandwidth, to put the reference frequency at the band edge, then provided the band edges are steep enough to ensure negligible signal power is outside the bandwidth, the signal will be single sideband. This means that the real base-band representation of the signal will have no “mixing” of components from the upper and lower sidebands as illustrated in section 3.4. It also means that the baseband signal’s two orthogonal components form an exact Hilbert transform pair and therefore either one of them is sufficient to fully describe the band-pass signal. This applies not only to the complex envelope representation of the signal but also to the P and Q representation. If P and Q result from measurements where \( \omega_0 \) is central in the pass-band, they are not entirely independent, components within them are related to each other by the Hilbert transform as was shown in section 3.6.1. However, the P signal cannot normally be derived from the Q signal. Even though this is the case, conditional upon some assumptions being met, most important characteristics of the input noise signal which inhabits the filter bandwidth can be unambiguously represented by either the P or the Q signal on its own. The statement that either P or Q is sufficient to represent the noise signal in the bandwidth demands the solid argument which will be presented in section 4 to follow.
4 Relevance to measurements made

4.1 Lock-in amplifier fixes reference frequency at center of filter bandwidth

For the narrow band measurements made with a wideband input signal using a lock-in amplifier we do not have a choice of where we place our reference frequency $\omega_0$, relative to the measurement band required. This is because the band selection of the output is performed by low-pass filters which, for filtering of real signals, are necessarily symmetrical about zero in the complex frequency domain. This means the pass band must be symmetrical about the reference frequency and the P and Q components are a vector representation of the in-band frequencies referenced to that center frequency. If the reference frequency is moved, then the pass band moves too. As we have seen in the last section, when the reference frequency is in the center of the pass band, the P and Q outputs each contain information from the upper and lower sidebands irrevocably mixed together, and both are necessary for a complete definition of the in-band signal.

4.2 Placing the reference frequency at a band edge

In order to demonstrate that discarding P or Q in the narrow band measurement case (where the reference frequency is in the center of the band) still retains all the information about the signal’s statistics, the following argument is made: Let us assume that we are representing the component of a wideband noise signal which is contained within a relatively narrow bandwidth $B$ centered on $\omega_0$, with the two variables P and Q which have been obtained from a lock-in amplifier measurement. One can imagine reconstructing the narrow band signal from P and Q and then re-filtering it with a second lock-in amplifier as illustrated in Figure 7.10.
Figure 7.10 Illustrating narrow band filtering.
(a) followed by 2\textsuperscript{nd} signal detection stage with offset reference frequency which will allow representation of signal by P only (b).
This second stage would have a symmetrical bandwidth of twice the bandwidth of the first stage and its reference frequency would be on the lower band edge of the first lock-in amplifier’s pass band at \( \omega_0 - \frac{B}{2} \). Call the outputs of the second lock-in amplifier \( P_2 \) and \( Q_2 \). The input signal to the second lock-in stage can now be completely represented by just the \( P_2 \) output of the second lock-in amplifier since the \( Q_2 \) output will be the Hilbert transform of the \( P_2 \) output. This is because the signal is now single (upper) sideband, the lower sideband being empty due to the offset in the second lock-in amplifier’s reference frequency.

### 4.2.1 Identical statistics for components of in-band noise when the reference frequency is in the center of the band and when it is at a band edge

If we compare the first and second stages’ \( P \) signals, \( P \) and \( P_2 \), they will not be identical in the time domain and they will have a different spectral content, \( P \) containing frequencies up to \( B/2 \) and \( P_2 \) containing frequencies up to \( B \). However, they will have identical statistics. The reason for this is that they are both derived from the same narrow band signal and the difference in the method of derivation does not affect the statistics of the result. Rice considered a narrow band Gaussian noise signal as made up of a collection of pairs of sine and cosine waves, each pair with a slightly different frequency and each infinitesimally narrow band sinusoidal element with its own random amplitude \([77]\). To make this picture match reality the number of such sinusoids must approach infinity, but it is instructive to take one of these pairs and see how its contribution to \( P \) and to \( P_2 \) compares. From section 3.6.1 above, a frequency element at \( \phi \) rads/sec in the signal below \( \omega_0 \) with sine and cosine component amplitudes \( a \) and \( b \) makes a contribution to \( P \) of:
\[
\frac{1}{2} a \left[ -\sin (\omega_0 - \varphi) t \right] + \frac{1}{2} b \left[ \cos (\omega_0 - \varphi) t \right] \text{ where } (\omega_0 - \varphi) \text{ is positive.}
\]

The same element’s contribution to \( P_2 \) is:

\[
\frac{1}{2} a \left[ \sin \left( \frac{B}{2} - (\omega_0 - \varphi) t \right) \right] + \frac{1}{2} b \left[ \cos \left( \frac{B}{2} - (\omega_0 - \varphi) t \right) \right]
\]

because it is now in the upper sideband. The term \( \left( \frac{B}{2} - (\omega_0 - \varphi) \right) t \) is always positive because \( \frac{B}{2} \) marks the maximum value of the difference between the in-band frequency component and the reference frequency \( (\omega_0 - \varphi) \) which itself is positive because the element we are considering is below the reference frequency. The difference between these contributions to \( P \) and to \( P_2 \) is a variable frequency shift of up to \( \pm \frac{B}{4} \) and an inversion of the sine term. If \( \varphi \) is less than \( \omega_0 - \frac{B}{4} \) then the frequency of the contribution to \( P_2 \) is shifted down when compared to the contribution to \( P \) and if \( \varphi \) is greater than \( \omega_0 - \frac{B}{4} \) then the frequency is shifted up. The amount the frequency is shifted is equal to the difference between \( \varphi \) and \( \omega_0 - \frac{B}{4} \). This shifting behaviour for these lower sideband components is illustrated in Figure 7.11.
How components $\varphi$ with $\varphi < \omega_0$ contribute to $P$ and to $P_2$

Figure 7.11 The shifting behaviour of frequency elements of $P$ and $P_2$ in lower sideband when reference is changed from $\omega_0$ to $\omega_0 - \frac{B}{2}$.

Similarly a frequency element at $\varphi$ rads/sec in the signal with sine and cosine components $c$ and $d$ which is higher in frequency than $\omega_0$ makes a contribution to $P$ of:

$$\frac{1}{2} c \left[ \sin \left( \varphi - \omega_0 \right) t \right] + \frac{1}{2} d \left[ \cos \left( \varphi - \omega_0 \right) t \right],$$

where $(\varphi - \omega_0)$ is positive, and to $P_2$ of:

$$\frac{1}{2} c \left[ \sin \left( \varphi - \omega_0 + \frac{B}{2} \right) t \right] + \frac{1}{2} d \left[ \cos \left( \varphi - \omega_0 + \frac{B}{2} \right) t \right].$$

The only difference being a constant upward frequency shift of $B/2$. 
None of these differences, a fixed frequency shift, a variable frequency shift or an inversion, change the probability distribution of the element. The frequency shift does not change the probability distribution because, as mentioned previously, probability distributions for time varying signals discard all time information in a signal, for example the probability distribution over time of the sinusoid $x \sin (a t)$ is identical to that of $x \sin (b t)$. Neither does an inversion change the probability distribution of the element because the elemental signal is sinusoidal with average zero, spending the same amount of time near each positive value as it does near the equivalent negative value.

Having established that the probability distribution for each elemental contribution to $P$ is exactly the same as the same element’s contribution to $P_2$, we have established that the probability distribution of $P$ is exactly the same as that of $P_2$. This is because the probability distribution of a signal which is made up of a sum of (independent) elements is the convolution of all the probability distributions of the elements together.

### 4.3 Section 4 summary

We have established that, because we can eliminate $Q_2$ in the signal description when the signal is all upper sideband (it is just the Hilbert transform of $P_2$), and because the statistics of $P_2$ are identical to those of $P$, for the purposes of defining the signal’s statistics, we can eliminate $Q$. If we wanted to reconstruct a signal with the same statistics as the original signal then $P$ and $\hat{P}$ will do that. None of this is very surprising since if we have a signal which results from a Gaussian process, then both $P$ and $Q$ are Gaussian with the same mean (zero) and the same variance. The power in the signal is the same whatever the reference frequency, and so $P$ and $P_2$ (and $Q$ and $Q_2$), must have the same PDF.
5 Simulation of measurements

As a final confirmation that we can expect narrow frequency band measurements of noise on the mains to yield results with Gaussian distribution, a simulation was performed which replicates the equipment used in the measurements detailed in the next chapter. This allowed testing of any input noise PDF and easy checking that both the P and Q outputs have the same distribution and are therefore mutually redundant. Before describing the simulation in section 5.4 some peripheral issues are covered including the wide band input representation in section 5.1, the selection of a filter with the correct bandwidth in section 5.2 and the correction applied due to the input not having been present before the time t=0 in section 5.3.

5.1 Sampled representation of white noise for simulation input

As an input to the simulation a noise signal was generated. It is important to be clear about what this signal represents, so a summary is given here. 10 million sample points were used to represent white non-Gaussian noise generated using three summed uniform distributions as in previous examples. These values are just a list of random numbers with no inherent relationship to any points in time but we can arbitrarily choose a sampling period and associated sampling frequency and assign each number to consecutive time intervals. The sampling frequency was chosen to be 10kSPS (Samples Per Second) so the file represents 1,000 seconds of wideband noise data sampled with a Nyquist frequency of 5kHz. If the sampled signal is to represent true white noise there is a problem, aliasing would occur for any component above the Nyquist frequency and each sample would therefore represent a combination of the in-band (<5kHz) components and an infinite number of out of band, aliased components, all added together. This would result in a noise signal of infinite amplitude, so it is clear this
sampled signal cannot represent white noise of unlimited bandwidth. For convenience we will imagine the pre-sampled signal to be white noise filtered by a brick wall 5kHz low-pass filter, our sampled signal therefore fully represents all the white noise components up to 5kHz. Since we are using this signal to look at the performance of narrow band filters and low-pass filters for which signal components above 5kHz are irrelevant, the fact that the signal only represents white noise up to 5kHz is not restrictive.

5.2 Selection of bandwidth of filter to match measurement equipment

Specifying the bandwidth of a “brick wall” filter is simple, but when the fall off of the response is finite then specifying the bandwidth is more ambiguous. There are many ways this can be done for example, stating the -3 dB point and asymptotic roll-off rate. In the next chapter, measurements made with a Stanford Research SR830 lock-in amplifier will be described. Lock-in amplifiers, which select and measure signals within narrow bands by frequency shifting them to around DC and then low-pass filtering, have a universally accepted way of specifying the shape of the filter being implemented. The low-pass filter “time constant”, the roll-off rate, and the Equivalent Noise Band Width (ENBW) are the specification figures used.

To match the simulation work here as closely as possible with the measurement equipment requires a filter closely matched to the equipment’s filter. Most of the measurements presented in the next chapter were made with a 0.300 second time constant, 18dB per octave roll-off and 0.3125Hz ENBW low-pass filter selected. The precise shape of the filter is not specified in the equipment’s manuals but the figures quoted above do allow selection of a filter which should have a similar shape and performance. As the equipment is a measurement device, and there is a requirement to
make accurate measurements of any signal in the pass-band, it is very unlikely that a filter with pass band ripple will have been employed. The logical first choice might be a Butterworth filter of order three to ensure 18dB per octave roll-off. A third order IIR low pass Butterworth filter with a turnover of 1Hz was designed using MATLAB, a trivial task since MATLAB has a function which returns the specified filter’s coefficients. The time constant and ENBW of this Butterworth filter were established using the MATLAB script given in Appendix VII E on page 326 and it was found that the time constant was slightly too high at 0.39 seconds but the ENBW was much too high at 1.05Hz. Since the bandwidth of the filter I am trying to replicate was not specified, it seems sensible to adjust the bandwidth to improve the match but if the bandwidth is increased to get a faster rise-time, the ENBW will increase still further. If a filter type with a better time domain response were chosen to reduce the time constant (for example, Bessel) then the frequency domain response would be expected to be worse (less sharp cut-off), increasing the ENBW to bandwidth ratio. The problem is the equipment manufacture’s use of the term “time constant”. This differs from the standard definition of a time constant for a system, that being the time for the output to reach 63% of its final value after a step input. Measurements showed that the equipment’s displayed “time constant” is related to the equipment’s measurement bandwidth in the manner in which it would be if the filter had been a single pole RC type and the term time constant took its usual meaning. This is a trap for the unwary and it appears to have resulted from the history of lock-in amplifier development. When lock-in amplifiers were of analog construction, a maximum of two single pole low-pass filters were cascaded, and the time constant referred to the rise time for each low-pass filter stage rather than the combined filter [86]. It means that a third order low-pass filter with 3 dB turnover of about 0.3Hz rather than 1 Hz is required. A Butterworth filter low pass filter with these specifications has an ENBW of about 0.32Hz but a rise-time to 63% of final value of about 1.3 seconds.
Since the lock-amplifier is down-shifting then filtering with a low-pass filter, the narrow band over which it detects signals is symmetrical and with a bandwidth twice that of the low-pass filter. The band-pass filter with an equivalent pass-band to the lock-in amplifier’s detection band therefore has a 3dB bandwidth of 0.6Hz.

5.3 The equivalent narrow band filter output

The noise signal described in section 5.1 was passed through the digital IIR bandpass filter by using the MATLAB script given in Appendix VII D on page 325. The filter has a 3dB bandwidth of 0.6Hz and roll-off rate of 18dB/octave and center frequency of 400Hz, typical values for the filtering performed in the measurements in the next chapter. A small (1/100th second) segment of the output is shown in Figure 7.12 showing it looks like a sine wave of about 400Hz as expected.

![Band Pass Filter output](image)

**Figure 7.12** Band-pass filter output from t=2 seconds showing output has sinusoidal nature over short time periods.
The initial three seconds of output from the filter is shown in Figure 7.13 and while at this scale the sinusoidal nature of the signal is obliterated, the slowly varying envelope can be seen clearly.

![Band Pass Filter o/p](image)

**Figure 7.13** Filter output from t=0 at expanded time scale revealing random nature of signal amplitude.

Since there is no signal present before time = 0 and the narrow bandwidth of the filter makes its output slow to change the first section of the output from time = 0 to time = about one bandwidth should be ignored when determining the probability densities of its components, otherwise low values will be over represented.

We could take the P and Q projections of the signal represented in Figure 7.13 and examine their statistics and a MATLAB script for this purpose has been written, but for the very large data sets we are dealing with here it takes a significant time to run. A better solution is to obtain P and Q directly by performing a direct simulation of the lock-in amplifier and this is done in the next section.
5.4 Directly simulating the lock-in

Rather than narrow band filtering the noise signal and then projecting the result on a sine and cosine reference to resolve the Gaussian components of the narrowband noise, the components can be obtained directly from the wideband signal by correlating, with a short integration time, a sine and cosine waveform with the same frequency as that of the center of the narrow band of interest. This is precisely the process the lock-in amplifier performs as was shown in schematic form in Figure 7.5 of section 3.6.1.

This process was implemented in the MATLAB script given in Appendix VII C on page 324 using a reference frequency of 400Hz and Butterworth third order low-pass filters with a rise-time of 0.3Hz and ENBW of 0.32Hz giving a pass band of about 0.6Hz.

5.5 Fit to Gaussian of simulation output

5.5.1 Method

The 10 million point data sample set representing 16.67 minutes worth of white noise below 5kHz (as described in section 5.1) was input to the MATLAB process described in the previous section. The output, after low-pass filtering is very much oversampled. Its spectral content is almost entirely below a few Hz yet it still contains 10,000 Samples Per Second (SPS). To overcome the previously described “stale data” problem (section 2.3.1) the output data was decimated or re-sampled to 1 SPS. The filter’s turnover of 0.3Hz and steep roll-off ensure that samples taken at this rate still properly represent all of the significant spectral components in the signal. This sample rate is also sufficient to represent the underlying PDF of the signal reasonably well, since we still end up with 1000 samples after decimation.
5.5.2 Results

The PDF of the decimated output signals in this experiment matched a Gaussian distribution with Anderson Darlington $A^2$ test statistics of 0.296 and 0.170 for the Q and P outputs respectively. This means the difference between the output and that expected given an underlying Gaussian distribution is not statistically significant at a confidence level of 90% (critical value for $A^2 = 0.6$). The experiment was repeated 12 times with different input data sets, each 10 million points long and each derived in the same manner by combining three uniformly distributed random variables. The P output was recorded in each case and the results of the individual experiments are shown in Table 7.2 which follows in the next section. The $A^2$ statistic for the 12 experiments varied between 0.147 and 0.607 with the average being about 0.32 with a standard deviation of about 0.15. This confirms that the result of the first experiment was not uncharacteristic, and that we can be reasonably sure that the conclusion, that the output is consistent with having come from a process with an underlying Gaussian distribution, is correct. The other obvious check is to repeat the experiment with an input which was taken from a Gaussian distribution. This experiment was also performed 12 times using a 10 million point Gaussian data set with the results also shown in Table 7.2. The $A^2$ statistic for these experiments varied between 0.202 and 0.740 with the average being about 0.44 with a standard deviation of about 0.19. Since these results show the output has no better fit to Gaussian than the previous set of experiments, we can again conclude that the first set of experiments with a non-Gaussian input had an output with as good a fit to Gaussian as can be hoped for.
5.5.3 Results for other, more extreme, distributions

The initial experiment described in the previous section used a distribution for the wideband noise input which was not significantly dissimilar to Gaussian. To test the proposition that the distribution of the input noise signal is largely immaterial and that the resulting output distribution will always be Gaussian, a more extreme distribution for the noise was chosen. This involved a variable with a uniform distribution between two values 0 and 1. The experiment was again repeated another 12 times with the results given in Table 7.2. As in the previous cases, the output signal’s fit to Gaussian was as good as would have been expected had the input been Gaussian rather than uniform. The $A^2$ test statistics varied between 0.189 and 0.505, in all cases indicating acceptance of the Gaussian distribution hypothesis at a confidence level of 90%. The average $A^2$ statistic was 0.35 and the standard deviation was 0.10.

Finally, an even more extreme distribution was tested. The uniform distribution between 0 and 1 was modified so that all input samples less than 0.95 were replaced with a value of zero and all samples above this threshold remained unchanged. This could be described as impulsive noise and has a distribution entirely different from Gaussian. Even in this case, most of the experiments resulted in outputs which had no statistically significant difference from those which would have been expected, had the output been from a process with an underlying Gaussian distribution at a confidence level of 90%. The $A^2$ test statistics varied between 0.194 and 0.987 and the average $A^2$ statistic was 0.44 and the standard deviation was 0.26. The results of the individual experiments are again recorded in Table 7.2.
Table 7.2. Summary of fit of output to Gaussian when the input has different distributions. A fit figure of lower than 0.6 indicates the hypothesis of a fit to Gaussian would be accepted at a confidence level of 90%.

<table>
<thead>
<tr>
<th>Experiment number</th>
<th>Gaussian input</th>
<th>Non-Gaussian input</th>
<th>Uniform input</th>
<th>Impulsive input</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.292</td>
<td>0.222</td>
<td>0.282</td>
<td>0.624</td>
</tr>
<tr>
<td>2</td>
<td>0.65</td>
<td>0.147</td>
<td>0.505</td>
<td>0.858</td>
</tr>
<tr>
<td>3</td>
<td>0.386</td>
<td>0.46</td>
<td>0.249</td>
<td>0.269</td>
</tr>
<tr>
<td>4</td>
<td>0.529</td>
<td>0.539</td>
<td>0.204</td>
<td>0.285</td>
</tr>
<tr>
<td>5</td>
<td>0.74</td>
<td>0.182</td>
<td>0.335</td>
<td>0.987</td>
</tr>
<tr>
<td>6</td>
<td>0.724</td>
<td>0.371</td>
<td>0.389</td>
<td>0.229</td>
</tr>
<tr>
<td>7</td>
<td>0.322</td>
<td>0.429</td>
<td>0.47</td>
<td>0.293</td>
</tr>
<tr>
<td>8</td>
<td>0.324</td>
<td>0.221</td>
<td>0.416</td>
<td>0.194</td>
</tr>
<tr>
<td>9</td>
<td>0.202</td>
<td>0.192</td>
<td>0.443</td>
<td>0.476</td>
</tr>
<tr>
<td>10</td>
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<td>0.301</td>
<td>0.372</td>
<td>0.266</td>
</tr>
<tr>
<td>11</td>
<td>0.213</td>
<td>0.607</td>
<td>0.336</td>
<td>0.322</td>
</tr>
<tr>
<td>12</td>
<td>0.52</td>
<td>0.227</td>
<td>0.189</td>
<td>0.518</td>
</tr>
<tr>
<td>Average</td>
<td>0.44</td>
<td>0.32</td>
<td>0.35</td>
<td>0.44</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.19</td>
<td>0.15</td>
<td>0.10</td>
<td>0.26</td>
</tr>
</tbody>
</table>

6 Conclusion

To characterise a noise signal it is advantageous to split it into narrow spectral bands. The signal in each band is then defined by a reference frequency and a bandwidth (both deterministic) and two random variables which change with time. The two random variables can be Gaussian distributed P and Q (or R and I) or a Rayleigh distributed amplitude and a uniformly distributed phase. The phase is not generally a useful parameter to record because, if the bandwidth is narrow, it behaves the same way for any noise signal. Although a record of the phase would be necessary to perfectly reconstruct the original noise waveform in the time domain, it represents no important characteristics of the in-band noise. By appropriate selection of the reference frequency at the edge of the narrow band, the P and Q components become related by the Hilbert transform so, in this case, only one would be needed to represent the noise. It was shown that if the statistics of the signal are the only thing of importance, then this result can be
extended so that even in the case of the reference frequency being in the center of the band, only one component is necessary to represent the signal. It was shown that, if the bandwidth is small enough, the P and Q components can be expected to have a Gaussian distribution. Finally it was shown by simulation of a process very close to that used in the noise measurements described in the next chapter, that even with an input very far from Gaussian, we can expect our experimental measurements from narrow band filtering to exhibit a Gaussian distribution. They have an average of zero so their PDF can be specified by just a single standard deviation figure for each narrow band measurement. A series of narrow band measurements across the wider band of interest is therefore one way of concisely specifying the noise. This method, as opposed to describing a single PDF for the whole noise signal, retains some time information in the variation of signal power across the spectrum. It does, however fail to represent all time related aspects of the noise signal due to effectively discarding the phase term within each narrow band measurement. If the original noise is impulsive then splitting it up into narrow frequency bands hides this impulsive nature because the statistics of the signal within each narrow band are close to Gaussian. None-the-less, this method of measuring and representing the noise will be employed in the following chapters.
VIII RESULTS

1 Introduction

In this chapter the channel sounding measurement results will be presented along with measurements of the noise across the spectrum. The raw attenuation and phase results can be condensed into a small number of parameters used to specify a transfer function for each of the six channels tested. These transfer functions give a simplified model of the way the input signals are modified as they pass over the channels. The method and reasoning used to obtain the transfer functions is presented but detailed discussion of these results is left until the analysis & discussion chapter. The raw noise measurements will be condensed into a piecewise approximation of the additive noise which the channel produces across the spectrum. As stated previously, for the purposes of modelling the channel we are not interested in the response at frequencies harmonically related to 50Hz where it proves impossible to make any meaningful measurements of the channel characteristics due to the presence of large narrow band noise signals. We are also not particularly interested in the response below about 100Hz because of the proximity of the massive interfering current signal at 50Hz. Above a certain frequency, which is generally between 2kHz and 3.2kHz, the attenuation is too high to make good measurements possible.

2 Treatment of noise signals at mains harmonics

The narrow band noise signals at the mains harmonics were generally avoided by not including points close to them in the list of automatically tested frequencies. However, to illustrate their relative level, the exact mains harmonic frequencies were measured at the co-generation control room and the measured levels of harmonics at the receiver
input are compared with the measured level of received signal in between the harmonics. The results are presented in Figure 8.1.

**Figure 8.1** Amplitude measurements showing relative level of mains harmonics (red) at the lock-in amplifier input compared to the level of the received signal (blue).

It can be seen that the harmonics are very large signals in comparison to the received signals. The lowest frequency harmonic shown is at 150Hz followed by 200Hz, 250Hz and so on. The largest in this case is the 5\textsuperscript{th} harmonic, the measurement system gain was such that this harmonic was 85 amps referred to the 240V supply, about ten times the transmitted signal level. The long term average frequency of the mains in Australia is almost exactly 50Hz because it is controlled by the generating companies to keep synchronous clocks in time. However, the short term variations can be significant and the frequency variation for which Australian metering equipment is designed to cope is 49Hz to 50.5Hz [42]. This means that the range in which the higher harmonics might be found is quite large. The 29\textsuperscript{th} harmonic, for example could be anywhere between 1421Hz and 1464.5Hz. For this reason the gaps between frequencies tested were made larger around the higher harmonics, particularly where the harmonic tended to be at a high level.
3 Treatment of Attenuation and Phase change Measurements

The following set of attenuation and phase change measurements were taken using 10 second dwell times at each frequency point in the list in Appendix V F on page 317. This allowed a relatively long time constant of 300ms to be used for the lock-in amplifier's low pass filters. Lock-in amplifiers typically use a time constant figure to specify the low pass filters used and a long time constant equates to a narrow pass band for the receiver. The roll-off of the low pass filters was set to 18dB per octave. This means that the filter is made up of three 6dB per octave sections in series each having a 3dB turnover frequency of 0.53Hz. The Equivalent Noise Band Width (ENBW) for the whole three section filter is specified by the manufacturer as 0.3125Hz. After each measurement run the receiver was set to repeat the run but this time the transmitter was switched off. This gives a very crude indication of the noise floor but it does apply at the time of the experiment. Later, in section 5.2.3, more thorough noise measurements will be described.

An example of the data collected by transmitting from my office (room 117 in the Physical Sciences building) to the Thomas Cherry substation in the Easter break when the co-generator was shut down and disconnected from the network is shown in Figure 8.2 and Figure 8.3. The first amplitude and phase measurements with the transmitter on were performed at 104.3Hz followed by measurements at each frequency on the list in Appendix V F on page 317 in frequency order up to 3,122Hz. Then a series of check measurements were performed starting again at 104.3Hz. 14 different frequencies were checked in this way, the highest being 1,998Hz and these measurements are marked "check points" in the figures. Next the first run of amplitude measurements was repeated with the transmitter switched off, these are marked "noise" in the figures.
These graphs illustrate a few important points which apply to all the data collected for the channel sounding experiments performed in this research work.

- The received signal becomes too small to measure above some frequency usually between 2kHz and 3.2kHz. In this case about 2,300Hz.
- Check points are generally close to the original measurements showing the channel has not altered much in the intervening 20 minute period.
• The received signal is generally well above the noise floor, but, since these noise measurements constitute just single measurements at each frequency, only an indication of the noise floor can be inferred.

3.1 Inserting delay

As explained in chapter VII, the transmitter equipment added a fixed delay to each edge of the transmit waveform. This is now compensated for by subtracting an appropriate phase from each phase change measurement. The phase subtracted is 11 degrees per kHz to compensate for the 30.5 μs transmitter delay and this is performed by the MATLAB script in Appendix VIII A on page 333. The amplitude and corrected phase data is also formatted by this script ready to be fed into a frequency domain system identification program. This program will use numerical techniques to determine the most likely parameter values for the transfer function given the measured amplitude and phase data.

3.2 FDIDENT, a frequency domain system identification tool

FDIDENT is a third party MATLAB toolbox [87, 88]. It makes a Maximum Likelihood Estimate (MLE) of the transfer function parameters. First we have to propose a linear transfer function model in the Laplace domain. The initial choice is based on previous experience and looking at the complexity of the bode plots obtained from the measurements. We might choose, for example:

\[
H(S) = \frac{b_6 + b_5 s + b_4 s^2 + b_3 s^3 + b_2 s^4}{a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}
\]

or, more conveniently expressed in its factorised form overleaf:
The parameters, which comprise the numerator and denominator coefficients or poles and zeros and the gain, can be thought of as a multi dimensional parameter vector, $p$, 10 dimensional in this case.

The steady state frequency domain measurements we have made include some uncertainties, as all measurement do. Even if the channel remains identical, a repeat measurement will have slightly different values. This means that in using the measurements to obtain the parameters which best represent the data, we will only ever have an estimate of the $p$ which defines the model with the best match to the channel. This estimated value is termed $\hat{p}$ and it is the value which has the maximum likelihood given the data. The FDIDENT tool helps to find $\hat{p}$, and it does this using a core algorithm or technique named ELiS [89-91]. In this algorithm the assumption is made that noise is added to the input as well as the output signal. Along with the estimate for the channel model parameters $\hat{p}$, the algorithm gives a figure for the confidence which can be attributed to that estimate. In many cases the tool will be used with time domain data, the input being a repeated sequence of an excitation waveform. The waveform will contain many frequencies and the fact that it is repeated will reduce the uncertainty in the final estimate for $\hat{p}$. The algorithm first performs a transformation of the time domain input data into the frequency domain. In our case though, the input signal is not repeated and it is already in the frequency domain. We can also see that, in our case, the measurements were quite accurate because the received signal was well above the noise floor. In addition, we know that the input was relatively noise free being at a well controlled frequency and being a powerful signal. For these reasons the ability of the algorithm to treat uncertainty in the measurements in a statistically rigorous fashion is
not a critical requirement. In other words, we expect the value of the estimate \( \hat{p} \) to be very close to the true value for the channel and we need not be too concerned about its stochastic nature. Nevertheless, it is important to note that the ELiS algorithm in FDIDENT uses a parameter estimation technique which belongs to the class of Maximum Likelihood Estimators [92]. This type of estimation is optimal for large samples in that it provides an estimate with the minimum variance and with no bias. When the ELiS algorithm is used on multiple measurements with noise it finds the best estimate, \( \hat{p} \), even though there is uncertainty in the measurements themselves.

Estimating \( \hat{p} \) is an iterative process. It starts with an initial set of parameters \( P \), the finding of which is an ordinary linear least squares problem[93]. It then sets up the cost function to be minimised:

\[
C_{LS} (X, Y, P) = \sum_{k=1}^{F} \left( \frac{(X_{mk} - X_k)(X_{mk} - X_k)}{2\sigma_{sk}^2} \right) + \sum_{k=1}^{F} \left( \frac{(Y_{mk} - Y_k)(Y_{mk} - Y_k)}{2\sigma_{yk}^2} \right)
\]

subject to the constraints \( Y_k = H(j\omega_k, p)X_k, k = 1,2,...,F \). [93]

In the equation above, \( X_k \) is the (complex) noise free input at frequency values \( k \) numbered between 1 and \( F \), and \( Y_k \) is the corresponding output. \( X_{mk} \) and \( Y_{mk} \) are the measured values including noise at each of the frequency values \( k \) and each measurement value has an associated variance, \( \sigma_{sk}^2 \) or \( \sigma_{yk}^2 \). The constraint is that the unknown noise free values for input and output are related by the channel transfer function parameters \( p \), where \( p \) is the unknown set of channel parameters for which we want the estimate \( \hat{p} \). \( \omega_k \) is the \( k \)th frequency at which measurements were taken. The cost function, \( C_{LS} (X,Y,P) \), is a function of two sets of unknowns (the input signal and the output signal) for which we have estimates, and \( P \), a variable defining a particular channel model. ELiS iteratively alters \( P \) (using Gauss-Newton and Levenberg-
Marquardt procedures) in the search for the maximum likelihood values over $X$, $Y$ and $P$ [93]. The limit value of $P$ is the MLE $\hat{p}$. The mathematics behind the ELiS algorithm depends upon noise in the measured signals being Gaussian and it has been shown in the previous chapter that the distribution of the noise components in the in-phase and quadrature resolved narrow band measurements should indeed be Gaussian. The FDIDENT program requires a variance value to be associated with each measurement, however, because the noise was known to be relatively insignificant at most frequencies, minimal effort was made to input accurate variances. A single variance figure for all received signal amplitude measurements of $(0.66\text{mA})^2$ on the 22kV supply was input which equates to -111dB on the amplitude scale in Figure 8.2. This represents a standard deviation of 1% of the largest signal measured (just below 400Hz). The input signal variance used was a constant $(0.21\text{ Amps})^2$, 32dB below the power represented by a transmit signal level of 8.7Amps or a standard deviation of about 2.5% of the transmit level. Although constant measurement variance was chosen because it was simple to implement, it is not such a bad representation of the expected variability. The total variation for each measurement is shown in the lower trace of Figure 8.4

![Amplitude Measurements vs. Variation](image)

**Figure 8.4** Variation figure used with FDIDENT for each measurement (lower trace) compared to measured value (upper trace).
Figure 8.4 shows that, due to the relatively high variation used for the input signals, the total measurement uncertainty represented at 400Hz is raised by roughly 10dB compared with above the peak at 600Hz. As can be seen from Figure 8.2, there is indeed a rise in the noise at the receiver in this vicinity. At frequencies above the noise peak in Figure 8.4 we have variances which are probably too high. The last measurement at 2.2kHz, for example, is below the variance figure associated with it yet the signal is still clearly above the noise floor. Even so, the models resulting from the use of FDIDENT in this way will be seen to represent the measurements on the channel reasonably well. The \( \hat{p} \) values for different measurement sets can be used to compare one channel with another and the cost function minimums can be used to rate the validity of one model against another for the same channel. The aim is to obtain as simple a model as possible without sacrificing accurately in the model's fit to the data.

### 3.3 Transfer function modelling

The amplitude and phase measurements below 2,200Hz (excluding the repeats) from the data presented in Figure 8.2 and Figure 8.3 were prepared by the MATLAB script listed in Appendix VIII A on page 333 and fed into FDIDENT. One of the other basic inputs to the program is the order of the model the program is to identify parameters for.

#### 3.3.1 Initial selection of model order

What is required is the simplest possible model, that is, the lowest order, consistent with a good fit to the data. The other requirement is that the model should have no right half poles because the channel is known to be inherently stable. Through a trial and error process a model with five poles and four zeros was chosen, more complex models did not result in a significantly better fit, the reduction in cost factor being small, and many
other choices of pole and zero numbers resulted in the MLE parameters including right half poles.

The output of the program for a five pole, four zero model is shown in Appendix VIII B on page 334 and a summary is given here. The transfer function is:

\[ H(s) = \frac{-2.8(s + 729.0)(s - 28825)(s + 984 + 4434j)(s + 984 - 4434j)}{(s + 5627)(s + 1340 + 2102j)(s + 1340 - 2102j)(s + 1513 + 3202j)(s + 1513 - 3202j)} \]

where the units of \( s \) are seconds\(^{-1} \) and the scaling of both real and imaginary components of \( s \) is a factor of \( 2\pi \) to give both in radians per second.

There are two real zeros, one on each side of the \( j\omega \) axis and one pair of complex zeros. There is one real pole and two pairs of complex poles, all to the left of the \( j\omega \) axis.

The fact there is no zero on the origin of the \( s \) plane points to an inaccuracy of the estimate of \( \hat{p} \). We know that the channel has no response at DC due to the distribution transformer but there were no measurements made at frequencies below 104Hz. The ELiS algorithm has found that \( \hat{p} \) includes a zero close to the origin at 116Hz. A correction is made by forcing the algorithm to include a zero at the origin. When FDIDENT is run again with this zero specified we get the poles and zeros identified in Appendix VIII C on page 335.

This slightly increases the cost function which indicates a fit to the data not quite as good as the previous model but we know it fits reality better as it has a gain of zero at DC. Using this revised transfer function the magnitude and phase fits of the measurements to the model data are shown in Figure 8.5. These graphs are one of the
outputs of the FIDENT tool and they shall be used compare the measurements with the outputs predicted by the model generated from those measurements.

**Figure 8.5** Measurement to model fit for PS117 Office to substation model.

The pole zero plot for this model, also an output of the FIDENT program, is shown in Figure 8.6.

**Figure 8.6** (a) Pole zero plot for PS117 office to Thomas Cherry substation model. (b) expanded area to reveal pole close to origin.
The line of text just above the $\sigma$ scale at the bottom of the left hand plot indicates that the model is non-minimum phase, is stable and has no un-shown poles or zeros.

Forcing the zero in the last model to be at the origin has not greatly altered the overall structure of this revised model. Apart from the two real zeros and real pole, we still have one pair of complex zeros and two pairs of complex poles while the complex zero is now at a frequency in between the two complex poles. The single real pole has moved closer to the origin and is now partly hidden in the left hand diagram by the zero symbol centered on the origin however it is revealed on the expanded area shown on the right.
3.3.2 Gain

To fully define the transfer function a gain factor must be included as well as the parameters $\hat{p}$. The pole zero representation gives the shape of the transfer function but does not include any information about this gain factor. The magnitude plots that the FDIDENT tool has calculated are scaled in dB and it might be assumed that the power ratio so presented includes the gain factor in which case 0dB would be the level of the input current. This is not the case, 0dB represents 225A on the 22kV lines as for the scale of the graph in Figure 8.2. To make the transfer function relate the measured current on the 22kV line to the input current at the 240V socket a gain factor of -2.8 was included as calculated in Appendix VIII F on page 343. The negative sign is included to make the overall transfer function positive (the RHZ gives a phase inversion).

3.4 Verification of the transfer function model

It is no surprise that the data used to construct the model fits the model quite well considering it had eight adjustable parameters. What is needed is a repeat set of data for the same channel but taken at a different time. If the conditions in the channel, the mains load level and physical distribution of load for example, are different, and the model still fits the new data reasonably well, then we have established a robust model for the channel.

The data used to obtain the channel model above was recorded on 12th April 2007 at 3:30PM during an Easter break shut down period. The receiver location was the Thomas Cherry substation and the co-generator was disconnected from the network.
Older data was taken when experiments were being conducted by measuring between the office and the co-generator control room. The channel from the socket to the Thomas Cherry substation was the same for this older set of data but the MV side of the channel was quite different. The signal had to pass a further 750m to the receiver location and this path included an MV/MV transformer (11kV to 22kV). The signal also had the opportunity to split with some portion of it passing out onto the utility's MV supply which was connected at the Thomas Cherry substation. Data from measurements taken on 23rd October 2006 at midday, which was a normal workday with the university full of students, is compared to the channel model previously described and is shown in Figure 8.7 below:

**Figure 8.7** Verification of model, (a) amplitude and (b) phase plots.

As mentioned earlier, a complete transfer function model includes a gain constant while a pole zero description omits this information. For the verification measurements it was clear that the gain constant was different, the signals received always being much smaller at the co-generator control room than at the Thomas Cherry substation. The amplitude graph of the model has therefore been dropped down to meet the data graph by subtracting about 15 dBs. To indicate that the model is plotted with a different 0dB
reference to that used for the data (which is still the same as was used for Figure 8.2) the logarithmic amplitude axis has been marked "arbitrary reference". Apart from the gain constant the model does match the verification amplitude data up to 1,500Hz. The match with the phase data is not as good although the overall shape is similar.

3.5 Modelling of other channels

In order to determine if the channel model obtained above was universally applicable within the La Trobe distribution network, measurements on five other different channels were made. In each case the measurements were made on a weekend or public holiday to ensure that the co-generator was disconnected from the network. The FDIDENT tool was instructed to insert a zero at the origin as for the modelling described in detail above. In all cases, without this instruction, a zero close to the origin was obtained as in the previous example.

3.5.1 Restricting frequency range of applicability

The channel described above included a 1,500kVA distribution transformer and, for reasons which will be given in the analysis & discussion chapter which follows, it is suspected that this transformer is a dominant factor in the channel model. Most of the other channels had a smaller distribution transformer and these might be expected to have a better high frequency response and this assertion is supported by the fact that the magnitude of the bode plots for the other channels either flattened off or tended to rise slightly at the last few frequency points measured. This resulted in the models with the best fit to the data obtained having more zeros than poles. If a model like this is to be used to represent the channel it has to be stated that it is only applicable within a limited frequency range. In most of the channels tested, the fit for the MLE six zero, five pole models given in the next section was always better than the fit for the four pole, five
zero model as evidenced by the lower cost function. However, as will be expanded upon in the analysis & discussion chapter, the better fit was not the main reason for selecting the six zero, five pole representation. The main reason was that this representation yields a generic form of model with a similar pattern in the MLE parameters for most of the channels tested. The single exception was the first Office to Thomas Cherry channel model discussed previously. In the next chapter it will be shown that this channel can also be well represented by the generic model even though the MLE model is quite different having three right hand zeros as well as a right hand pole which makes it unstable (see the Office channel section of Appendix VIII D starting page 336 for details of this unstable model).
3.5.2 Models of all six channels

Measurements were made of the response of five more channels as detailed in the previous chapter. These were used in the same way as described above to identify the parameters for models for each channel. These models are presented in Appendix VIII D, page 336 in order of increasing transformer size and are summarised with values to two significant figures in Table 8.1. Only a single frequency value has been given for each pole and zero, the real component of the complex poles and zeros can be found in the Appendix. The unstable six zero, five pole model for the first Office to Thomas Cherry channel has been replaced by the four zero, five pole model.

<table>
<thead>
<tr>
<th>Channel name</th>
<th>Position of RH Zero (rads/s)</th>
<th>Damped Freq of 1st complex zeros</th>
<th>Damped Freq of 2nd complex zeros</th>
<th>Position of Real Pole (rads/s)</th>
<th>Damped Freq of 1st complex poles</th>
<th>Damped Freq of 2nd complex poles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bakery</td>
<td>19000</td>
<td>860 Hz</td>
<td>2600 Hz</td>
<td>-530</td>
<td>560 Hz</td>
<td>2100 Hz</td>
</tr>
<tr>
<td>Pavilion</td>
<td>42000</td>
<td>860 Hz</td>
<td>2500 Hz</td>
<td>-580</td>
<td>490 Hz</td>
<td>1900 Hz</td>
</tr>
<tr>
<td>Union bldg</td>
<td>16000</td>
<td>760 Hz</td>
<td>2900 Hz</td>
<td>-488</td>
<td>540 Hz</td>
<td>1400 Hz</td>
</tr>
<tr>
<td>Boiler house</td>
<td>24000</td>
<td>1300 Hz</td>
<td>2800 Hz</td>
<td>-510</td>
<td>510 Hz</td>
<td>1800 Hz</td>
</tr>
<tr>
<td>Office (PS117)</td>
<td>32000</td>
<td>790 Hz</td>
<td>N/A</td>
<td>-430</td>
<td>410 Hz</td>
<td>930 Hz</td>
</tr>
<tr>
<td>Room 127A</td>
<td>19000</td>
<td>2100 Hz</td>
<td>3000 Hz</td>
<td>-189</td>
<td>579 Hz</td>
<td>2000 Hz</td>
</tr>
</tbody>
</table>

Table 8.1. Summary of Maximum Likelihood parameters for models of channels from various locations (Channel name) to the Thomas Cherry substation. All models included a zero at the origin.

4 Cross talk

Due to the distribution transformer being Wye/Delta connected, a single phase current out of the 240V active flowing into the neutral will result in a current in at least two phases of the transformer's 22kV primary. The fact that there is no neutral in the 22kV circuit on site is the reason that a current cannot be confined to one phase only of this
MV supply. There may be other reasons for a signal injected into a single phase of the LV supply to appear in attenuated form on the other phases (e.g. common impedances in the neutral lines) but the transformer's Wye/Delta connection is the main mechanism. Measurements showed that if a signal was injected into the LV Blue phase a signal almost as large as that on the MV Blue phase would appear on the Red phase at the receiver, a small signal also being present on the White phase. Measurements of all possible combinations of transmit and receive phases were made and these are summarised in Figure 8.8. In this diagram the back of the arrow indicates the transmit phase and the tip, the receive phase.

![Diagram of cross talk measurements](image)

**Figure 8.8** Summary of cross talk measurements.

Figure 8.9 and the following paragraph illustrate the reasons for this being the case.
Figure 8.9 Diagram of LV/MV connection showing (a) the reason for dominant cross talk and (b) all of the other possible currents induced in the MV circuit by the signal.

Referring to the top diagram (a), a signal current injected into the LV Blue phase induces a signal current in the delta connected Red - Blue winding which is why "cross talk" is seen on the red phase as a result of a Blue phase signal. Most of this current will be flowing through the Red and Blue phase lines as indicated by the dark blue induced current in the lower diagram (b) and marked current path number 1. A secondary
potential signal path is shown in pink between the White and Red phase lines and marked path 2. This will not result in a large White phase current because, in a balanced system, it is cancelled by another secondary path with the current flowing in the opposite direction, path 3, shown in light Blue between the White and Blue phase lines. Any impedance imbalance in these two paths, 2 and 3, will result in some White phase signal current. The other possible signal current path is the circulatory path, path 4, internal to the transformer. It is well known that in delta wound machine stators and transformers triplen harmonics (3rd, 9th, 15th etc) are trapped and circulate inside the windings [94-96]. This may also be possible for signal currents and this would contribute to attenuation because the circulating portion of the signal would not reach the receiver.

5 The noise measurements
To determine the channel noise characteristics, frequency sweeps of the lock-in amplifier were done with the transmitter switched off. For these experiments the lock-in amplifier’s noise measurement function was selected. The bandwidth was increased by a factor of ten, (time constant of 30ms) and the dwell time at each frequency was extended to 30 seconds. The increased bandwidth enabled more independent measurements to be taken per second; the longer dwell time increased the total number of measurements contributing to the output resulting in a more accurate estimate of the noise. The noise measurement function takes a moving average of the absolute deviation from the mean, for the in-phase “X” channel and for the quadrature “Y” channel. This mean average deviation is converted to an RMS measurement using the assumption that the measurements have a Gaussian distribution. This assumption has been shown to be valid in the last chapter and is confirmed by the results described in section 5.2. The measurement is then adjusted to take into account the equivalent noise bandwidth of the filter. All of these adjustments and conversions are done inside the lock-in amplifier.
which gives a read-out in RMS Volts per √Hz which can then be related to the noise current signal on the mains using the calibrated gain in the receiver equipment's signal path. The bandwidth is small enough to make the assumption that the spectral density of noise across the band is flat, but because it has increased for the noise measurements (compared to that used for signal measurements), tests close to the large odd harmonics of the mains are no longer possible. This results in the spectrum for the noise floor being much more granular in frequency than that for the signal measurements.
5.1 Tests close to odd harmonics

Because the noise signals were so small, and because the receiver bandwidth has been increased as described in the last section, at many of the previously used test frequencies the receiver suffered from trapping some energy from odd harmonic signals. This is a spurious result because the harmonics are narrow band signals and their presence is being felt only due to the skirt of the filter used by the lock-in amplifier. Figure 8.10 shows how serious a problem this is.

![Effect of harmonic at 350Hz](image)

**Figure 8.10** Effect of an odd harmonic on noise measurements.

Measurements at the previously used frequencies between 300Hz and 400Hz are shown. These were taken over two weekends. At each frequency a total of 120 measurements were taken 60 spread evenly over the hours of the first weekend and 60 over the second weekend (a slight frequency offset has been added in the figure so that the two groups can be distinguished). Each individual measurement recorded was calculated from at least 150 deviation from the mean measurements and each of those measurements was taken using a filter time constant of 30ms and a slope of 18dB per octave. This time
constant gives an ENBW for the filter of 3.125Hz [86]. However, the real filter is not "brick wall" and the slope of 18dB per octave on the low pass filter means that the response only drops off at a limited rate as the frequency gets further from the pass band centre frequency. Apart from the automatically recorded noise measurements, some manual measurements around the mains odd harmonic frequency of 350Hz have been added in Figure 8.10. These measurements are not of noise on the mains but are the lock-in amplifier’s output due to an arbitrary amplitude small signal being injected close to the centre frequency in order to demonstrate the filter’s shape. The calculated filter roll-off has been indicated too and those points are marked "extrapolation". It can be seen that only the noise measurements at just above 300Hz and just below 400Hz will be negligibly influenced by the odd harmonic. Unfortunately they will probably include a contribution from the even harmonics which they are close to. It is also clear that the four sets of measurements between 335Hz and 365Hz should not be relied upon to only represent the wideband noise value around those measurement frequencies. Between 300Hz and 400Hz only the measurements at 312Hz, and 381Hz will be used to determine the noise spectral density. The measurements vary between -88dB and -103dB at these two frequencies. Based on this set of measurements, and others performed on week days and week nights, an example of which is shown in as shown in Figure A8.5.1 on page 342, the noise spectral density was determined to be about -95dBV/√Hz between 300Hz and 400 Hz ignoring the narrow band mains harmonics. A similar procedure was used to avoid the influence of the other harmonics as described in section 5.2.3.1.
5.2 Confirmation of a Gaussian distribution for noise components

A series of experiments were performed with the receiver’s reference frequency tuned to different points in the spectrum. The lock-in amplifier outputs, “X” and “Y” were sampled, once every second, for periods of varying length between minutes and hours. The statistics of each set of measurements was compared to a Gaussian distribution and a measure of the fit calculated. Figure 8.11 shows an example of these measurements presented as a histogram of the “X” channel output voltages (1V representing 225Amps at the receiver frequency and in phase with the reference on the 22kV line) together with the best fit Gaussian distribution shown by the continuous line, as determined by use of the EasyFit program [63] described in Chapter VI. The measurements were taken every second over a four hour period with the reference set to 425Hz and a 3Hz equivalent noise bandwidth (ENBW) for the filter formed by the lock-in amplifier.

![Histogram](image.png)

**Figure 8.11** Component of noise between 423.5Hz & 426.5Hz in phase with reference at 425Hz has Gaussian distribution.
The best of the goodness of fit test techniques described in chapter VI, the Anderson Darling test [97], was used to compare the PDF of the measurements to a normal distribution. The Anderson Darling goodness of fit statistic ($A^2$) values for the data sets were compared to a table [72] giving critical values at various significance levels ($\alpha$). The result for the example given above was that $A^2=0.39$ with a critical value of $A^2$ of 0.5 for $\alpha=0.2$. This showed that we can accept that the data came from a Gaussian process. The confidence level of 80% was chosen and, as explained in the chapter VI on statistical test selection, choosing this rather low value for the confidence level in fact means that we are being reluctant to retain the hypothesis of normality. The test of normality is more rigorous than it would have been had a higher confidence level been chosen because the probability of not mistakenly retaining the hypothesis (the power of the test) is increased.

For measurements taken above 400Hz it was generally the case that the hypothesis of normality was accepted i.e. a good fit to a Gaussian distribution was found for data recorded, at one sample per second, over periods of between 5 minutes and 4 hours, the noise being, for the most part, stationary. In contrast, measurements at frequencies below about 400Hz did not show stationary behavior over very long time periods as evidenced by the hypothesis of normality being rejected. However, when the data sets for these lower frequencies were broken up into segments of 12 minutes or less the fit to Gaussian in each segment was generally as good as that demonstrated in Figure 8.11 and the hypothesis of normality was accepted using the Anderson Darling test and the significance level of $\alpha=0.2$. 
5.2.1 Sample measurements shown in the time domain

The measure of fit to Gaussian was found to be good for data taken over those time intervals for which a plot of the data in the time domain indicated reasonably stationary behavior.

5.2.1.1 Frequencies over 400Hz

With the receiver tuned to a frequency of 770Hz individual measurements of the output from one channel of the lock-in amplifier were taken every two seconds for 16 hours using a filter time constant of 30ms and a slope of 18dB/Octave. The data is presented in Figure 8.12 and a segment of it is expanded in Figure 8.13.

![Figure 8.12 Measurements at 770Hz over 16 hours.](image)
Figure VIII.13 The first 2 ½ hours worth of data from Figure VIII.12 re-plotted.

It can be seen that, at this frequency, the noise level is stationary over relatively long periods. For example, the first hour has an RMS level of 4.62 $\mu$V and the second hour, 4.39 $\mu$V which indicates that to an accuracy of about 0.5dB the noise was stationary over the first two hours. When broken up into data sets taken over one hour intervals the fit to Gaussian for each set is good (the hypothesis that the data is coming from a process with an underlying Gaussian distribution is not rejected).

5.2.1.2 Frequencies below 400Hz

In contrast to the higher frequencies, frequencies below 400Hz only show stationary behavior over much shorter time intervals. As an example, data recorded at 175Hz over a 2 ½ hour period is shown in Figure VIII.14.
Only when this data is broken up into shorter segments does the fit to Gaussian become good. A period of about 12 minutes was found to be generally satisfactory in that it lead to acceptance of the hypothesis of Gaussian distribution.

5.2.2 Using the lock-in amplifier's noise function

Having shown that the statistics of the in-band noise are Gaussian over short periods for frequencies below 3kHz we can use the noise measurement function with some confidence. Measurements were made of one of the lock-in amplifier outputs, while simultaneously recording the noise function measurement calculated from the same output signal. When the noise over the measurement period was calculated from the baseband output measurements and put in terms of RMS V/√Hz a match was found with the noise function measurement. This gives further confirmation that the short term measurement distribution is close to Gaussian because the calculations performed in the lock-in amplifier to output a RMS V/√Hz noise reading rely on the measurements on which they are based having such a distribution [86].
5.2.3 Noise measurements taken across the frequency spectrum

For simplicity, the same frequency list as was used for attenuation and phase measurements was used to make automated measurements to determine the noise. The only difference for a noise measurement run being the use of the lock-in amplifier's noise function output, the increase of dwell time at each point to 30 seconds and the reduction of the time constant to 30ms. A great number of these frequency points were too close to the harmonics to produce reliable results but those results were still recorded. Based on the reasoning given in section 5.1, only those frequencies less than 11Hz from an even mains harmonic, and less than 15Hz from an odd mains harmonic, were used to specify the noise spectral density. A complete noise measurement run is recorded in Appendix VIII E on page 342. This data was taken over a 65 ½ hour period mostly during a weekend at the Thomas Cherry substation and comprises 7,600 recordings of the noise output function.
5.2.3.1 Selecting representative noise spectral density figures

An averaging process was used to obtain representative figures for the noise spectral density for sections of the frequency spectrum of interest. These figures do not represent worst case or best case situations but combine many valid measurements into a rough estimate of the noise that can be expected on average. The noise spectral density at the Thomas Cherry substation will usually be less than 10dB from the given figure.

![Noise density measurements](image)

**Figure 8.15** Extract from the noise measurements data in Appendix VIII E showing frequency points to be ignored due to mains harmonics being within measurement equipment’s band of sensitivity (marked with an “X”).

The four frequency points in Figure 8.15 with crosses above, 1,350Hz, 1,561Hz, 1,850Hz and 1,998Hz, are strongly influenced by nearby harmonics and are ignored. The remaining data is averaged to a figure of -115dBV/√Hz for the entire frequency range from 1,300Hz to 2,150Hz.
5.2.4 Final adjustment of noise spectral density measurement

The measurements recorded were of one channel only from the lock-in amplifier, the “X” output. This means they give the component of the in-band noise which is in phase with the lock-in amplifier's reference local oscillator signal. We know from the work in the last chapter that the component of the noise which is unmeasured, that is the component in quadrature phase with the local oscillator, has the same statistics. That is to say, it has the same distribution (Gaussian) and the same RMS value. The actual noise signal on the MV supply is the vector sum of these measured and unmeasured signals. They are uncorrelated, as discussed in the last chapter, and, because they have the same RMS magnitude, the sum is $\sqrt{2}$ times the magnitude of the measured signal. This means we must increase the measured noise by 3dB to arrive at the figure for the RMS noise spectral density on the supply.

The noise measurements discussed so far are those at the input of the lock-in amplifier and are in units of dBBVols per $\sqrt{Hz}$. The signal is actually a current signal on the MV supply so it should be in Amps/$\sqrt{Hz}$. To convert we have previously determined that the calibration constant for the signal path from the MV current to the input to the lock-in amplifier is such that 225 Amps would result in 1V at the input. This gives a second factor of 47dB to be applied to the measurements in order that the result is correct for the current signal on the MV supply. The final graph of the MV noise spectral density after it has had these corrections applied is given in Figure 8.16 overleaf.
6 Conclusion

A large part of this research work into the possibility of using the LV/MV channel for upstream meter communications has centred on determining the characteristics of that channel. At La Trobe University we have a distribution system and collection of distribution transformers of about the size and complexity typical of those in a utility’s zone substation serving a suburb of 1,000 residences and some commercial and industrial loads. This gives us a collection of typical LV/MV channels, six of which have been measured and characterised for current signals. Part of the channel characterisation task was measurement of the noise at the receiver and this too has been performed. Cross talk between phases was also observed but, apart from noting the presence and cause of this effect, no detailed study of this phenomenon was undertaken. All measurements presented were with the receiver measuring current on the same phase as that into which it was injected.

Of the many possible ways of using the measured data to create a model of the channels, the chosen form was to create transfer functions for each channel of the same order, consisting of six zeros and five poles. Although this form does not give a transfer
function representative of the channel at all frequencies, it resulted in a set of generic models to which all six channels could be shown to fit over the frequency range of interest. The noise measurements too could have been made and presented in several forms. Establishing the noise spectral density in narrow bands of the spectrum has been established, in the previous chapter, to allow the assumption of a Gaussian distribution within the bands. This assumption is a very useful one to be able to make and it drove the decision to present the noise in this way. There are disadvantages to this representation of the noise and those are that the partly impulsive nature of the noise is not highlighted, any behaviour of the noise related to the mains frequency is masked and the effect of the large narrow band mains frequency harmonics is ignored.
IX ANALYSIS & DISCUSSION

1 Introduction

In this chapter the new knowledge generated by the research work will be discussed. The measurement results will lead to a set of generic models for the channels in which there are a minimum number of variables and the all-important channel capacity will be calculated. The most significant equipment modification required for progressing from a channel sounding system to a practical communication system is a different transmitter circuit and this will be discussed. Finally, known limitations of this research work will be highlighted.

The chapter will start with a detailed discussion of the results from the measurement experiments presented in the previous chapter.

2 Generic models

The type of model that is straight forward to construct from the results of the measurement experiments is close to a "black box" model in which little information about the expected channel characteristics has been incorporated. The only a priori information that has been included so far is that it is not expected to have a response at DC. In this section the other properties of the models will be considered to see if they can be interpreted in the light of prior knowledge about the channel and to identify commonalities.
2.1 Right half zero (RHZ)

If we consider the simplest stable model which we have for the office to Thomas Cherry channel we see that, like all of the other stable models presented, it has a zero in the right hand half of the s plane. This means that it is describing a non-minimum phase system.

A non-minimum phase system is not the simplest type of system. It does not pass energy from the input to the output at all frequencies at the maximum speed possible, there is some extra delay at some frequencies. This type of system is interesting in that the response to a step or impulse input can result in the output initially moving in the opposite direction to that in which the input moves. A bicycle being peddled at speed is a common example of such a system [98].

If a minimum phase model had been appropriate then the phase response would have been uniquely defined by the amplitude response, the two being linked by the Hilbert transform [99]. In this case it may not have been necessary to measure the phase response at all. Since measuring the phase response requires considerable extra effort in locking the transmit and receive local oscillators much more closely than is required for amplitude measurements, it is worthwhile considering the importance or otherwise of that right half zero (RHZ).

2.1.1 Possible replacement of the RHZ by a left half zero and a delay

Reflecting the RHZ to the left hand side of the s plane has no effect on the amplitude of the system response but the model will now describe a minimum phase system. One effect of this reflection is to alter the phase at low frequencies by a value close to 180 degrees. The model can be changed so that the data still fits reasonably well by
multiplying the gain constant by \(-1\) to absorb this inversion. This was done for the 4 zero 5 pole model of the office to Thomas Cherry channel shown in figures 8.5 and 8.6 in the last chapter. The minimum phase model of that channel is now considered.

![Channel Phase Response and 4 Zero 5 Pole Min phase Model](image1)

**Figure 9.1** Phase response of minimum phase model (a), and non-minimum phase model (b) of the office to Thomas Cherry channel.

The original position of the zero in this case was at +32,000 rads/second which means that for frequencies below 1,500Hz the angle to the zero in the s plane is below 16°. We should therefore expect an almost linearly increasing phase with increasing frequency, along with the subtraction of 180°, to be added to the transfer function phase plot when the zero is reflected from the left half back to its original position. Such a linearly increasing phase is equal to a constant delay and the phase response of this new minimum phase model is shown in Figure 9.1 (a) to be compared to the non-minimum phase model in (b). While it is clear the fit is better with the zero on the right hand side of the s plane, it is also apparent that the minimum phase model could be used in conjunction with a suitably selected delay. This delay would shift the model data down in phase progressively as the frequency increases. A of 32° at 1,500Hz, which is equivalent to 59 \(\mu\)s, would make the phase change for the minimum phase model (with the subtraction of 180°) exactly the same at this frequency as it was for the non-
minimum phase model and would make the "minimum phase plus delay" model fit the data almost as well as the non-minimum phase model does.

The data collected has already been modified by incorporating a delay of 11° per kHz (30.6 µs). This was to compensate for a known and accurately measured delay in the transmitter circuitry. The components of the transmitter causing this delay did not form a part of the channel which we are modelling which is why it was appropriate to make this adjustment. However, there is no known mechanism for the channel itself to be adding a delay of 59 µs to the signal at all frequencies. The channel is too small physically for transmission line behaviour to be a factor. In electric circuits the phase velocity is usually a significant proportion of the speed of light, at half the speed of light a disturbance only takes 1.8 µs to travel one km. The presence of a right hand zero is therefore preferable to replacing it with a left hand zero and an un-explained delay.

2.1.2 Possible replacement of the RHZ by a stable pole

The RHZ could be considered problematic in that it forces measurement of both amplitude and phase response to uniquely define the transfer function. An alternative solution, which might still result in a model with a reasonably good fit to the data, is to replace the right hand zero with an appropriately placed pole in the left half. The effect of the RHZ was to increase the phase approximately linearly over the frequency range of interest and, because it is far from the origin, it only has a small effect on the amplitude response for the frequencies of interest in the office channel case. A real stable pole at the same distance as the zero was from the origin will have the same effect on the phase, adding the same lag with increasing frequency as the RHZ did. As before, when we reflected the zero, the gain constant must be multiplied by -1. The effect of this adjustment is that the match of the phase data to the model is un-altered from the non-
minimum phase model. However, this method of eliminating the RHZ depends upon the zero being at a considerably higher frequency than the highest frequency of interest. Although replacement of the RHZ with an appropriately placed stable pole (combined with subtraction of 180°) has no effect on the phase response, it has a significant effect on the amplitude response close to the frequency of the RHZ. For this reason it cannot be used on all of the channels modelled.

In summary, we must conclude that there is a mechanism in the physical channel which requires a RHZ to model it accurately, and that the transfer function really does exhibit non-minimum phase behaviour.

2.2 *Poles above the highest measurement frequency*

As mentioned in the results chapter, it is clear that the six zero, five pole models cannot form complete descriptions, because having more zeros than poles their response will continue to increase without limit as the frequency increases. No real system behaves in this way; there will always be sufficient poles present to cause the response to roll off at some finite frequency. It is unfortunate that the equipment for this research work had its early development based on trials using the channel which had the lowest frequency at which received signals became immeasurable and this resulted in the system being limited to making measurements below about 3kHz. The assumption must be made that poles are present in all channels which causes roll off at frequencies beyond those measured. The models therefore come with the restriction that they are only valid for a limited frequency band.
2.3 *High frequency complex zeros*

In five of the channel models, a pair of complex conjugate zeros have been identified above 2,500Hz. Measurements for the office channel were not possible above about 2,200Hz however, if measurements had been possible, it is likely we would have discovered a pair of complex zeros in similar position to where they were found for each of the other five channels. Redundant zeros will therefore be added to the office model, making very little difference to the fit of the extended model to the office channel data. The only reason for adding these zeros is that all channels modelled will then fit the same generic model type with six zeros and five poles. The average damped frequency of these zeros over the models is 2,860Hz so a frequency of 3,000Hz was chosen for the un-identified zero pair for the office channel. The higher frequency ensures minimal effect on the model over the frequency range where it exhibits a good match with the measurements. The minimal effect of this zero pair will be apparent in the summary of matches to the generic model presented in section 2.6.

2.4 *Low frequency real pole*

All models had a low frequency real pole, mostly close to 500 rads/second. To achieve a good fit between model and measured data it was found that this pole could be fixed in the same position for all channels. The average position over the six models was 455.5 rads/sec (72Hz) so it will be fixed at 460 rads/second for the generic model. The minimal effect of repositioning this pole will be apparent in the summary of matches to the generic models presented in section 2.6 and the reason for the presence of this pole will be considered in the following section.
2.5 Physical interpretation of the model

2.5.1 High pass filter formed by the zero at the origin and the fixed real pole

The zero at the origin and the real pole we have fixed at 460 rads/second produce a high pass filter with a turnover of 73Hz. However, this does not correspond well with the physical reality of the channel. We have a channel designed for the generator to load current transfer function to have very low attenuation at 50Hz. The explanation for this mismatch between model and reality is that it was impossible to make proper measurements at these low frequencies. The massive interfering current signal at 50Hz produced by the power stations had to be filtered out for any measurements to be made and, for the measurements used for these system identifications, 50Hz and 100Hz notch filters were included within the lock-in amplifier. The 100Hz filter had 30dB attenuation in the notch and would have affected at least the first amplitude and phase measurements at 104Hz. This measurement point should have been excluded for this reason but unfortunately was not. The fact that the model does not reflect reality below about 110Hz is not of great concern because the frequency band between 0 and 110Hz, represents only a small part of the total spectrum available for communications signals and is not much use for communications due to the massive 50Hz interfering signal near its center.

Although the zero at the origin and the real pole do not relate directly to a high pass filter with a turnover at 73Hz within the channel, they are still a necessary part of these models. This is because, although the effect on the amplitude is not significant for most frequencies tested, the effect on phase extends well past 100Hz and does not become negligible until several hundred Hz.
2.5.2 LV section and distribution transformer

The experiments have produced strong evidence that the LV signal path and LV/MV distribution transformer are the dominant determinants of the overall channel response. The surprisingly good match in dynamic behaviour of the channel from the office to Thomas Cherry substation with that from the same office to the co-generation control room, as shown by the model verification measurements described in section 3.4 of the previous chapter, indicates that the MV side of the channel does not contribute significantly to the shape of the transfer function. This is not entirely unexpected as the MV side of the channel is known to have a relatively flat frequency response and a well behaved phase response, often being used for power line communications to the substations by the utilities utilising frequencies up to a MHz [41, 100, 101]. The LV part of the signal path is much more variable because it is onto the LV network that loads are continually connected and disconnected, however it too has a frequency response which allows communications using signals of at least a few hundred kHz [102]. This leaves the distribution transformer as the primary cause of the attenuation of signals above a few kHz. The magnitude plot for the office shows that the attenuation is so high at 2.5kHz that the substantial signal injected at the socket is undetectable at the substation. This limitation of capacity to below a few kHz is the most obvious and important characteristic of the channel and is due to the distribution transformer failing to have a strong transformer effect at these frequencies. Others have studied the behaviour of electrical machines which have been designed for efficiency at 50 or 60Hz, at well above those frequencies. The main reason for these studies has been to determine the response to transients, in particular lightening strikes and switching events. In some of this work [103, 104] the assumption was made that there is no increase in mutual inductive coupling between individual coils on the same winding caused by the presence of the iron. This assumption leads to accurate models of the windings for frequencies
around 1MHz and above. The reason that this surprising assumption is quite correct is that iron laminations designed for power frequencies do not have flux penetration to any appreciable depth at these high frequencies. In our case this phenomenon would prevent the transformer effect at high frequencies. We are operating well above the power frequency but much lower than 1MHz and so we can still expect some transformer effect. The extent to which the flux penetrates the iron, and therefore the reluctance of the path between the primary and secondary, depends upon the iron's resistivity and permeability and the frequency of the current. At 1kHz, typical transformer iron laminations will have a "penetration depth" of only 0.12mm [105]. This penetration will only be a small fraction of the typical 50Hz transformer lamination thickness and so the reluctance of the flux path will be greatly increased. This will lead to much of the signal induced flux which links the secondary not linking the primary and therefore the current transfer ratio of the transformer will be reduced. This physical effect can clearly be seen in all of the models since there is a general reduction in the amplitude of the transfer function as the frequency increases.

2.5.3 **Difficulties in resolving the generic model into a lumped element equivalent**

It might appear to be a useful exercise to perform network synthesis from the transfer function in an attempt to obtain a lumped element model of the channel to help relate the elements of the model to different physical aspects of the channel. This has two problems, the first being the difficulty. There is no known procedure for directly obtaining a lumped element model from a transfer function in the general case, although methods for certain fixed topologies are well established, for example where the resultant network is of a ladder form [106, 107]. At least one method does exist for finding models with appropriate topologies and it continually combines and refines them to obtain a closer match. This approach is called searching by genetic algorithm [108]
and it could conceivably be used to find the simplest lumped element circuit with the same response over the frequency range of interest as our parameterised transfer function has. However, the second problem with using a lumped element representation is that it could not be expected to have a direct correlation with physical reality. We might hope, for example, to be able to identify a particular inductor in the circuit model as representing the leakage inductance of the distribution transformer's secondary winding. However, the inductances in the transformer will all be frequency dependent due to the limited flux penetration effect described in the previous section. A lumped circuit model in which components could be associated with separate physical parts of the channel would necessarily have to have at least some components with frequency dependent values.

2.5.4 Resonance and anti-resonance effects

The models show that there are two major resonances, the first in a position which is fairly consistent across all channels between 400Hz and 600Hz and the second between about 1,000Hz and 2,000Hz. The lower frequency resonance has a damping factor between 0.3 and 0.5 and a similar low frequency peak is also apparent in the spectrum of the noise. The spectrum of the noise measurements give us an indication of how all of the possible channels in the network, with a substation receive point, are behaving. This is because all of the mains connected equipment, which is the primary source of the noise, produces noise at the end of a potential channel. The spectrums of the equipment's noise are not known but let us assume they are flat for every piece of equipment connected. As those noise signals pass through the channels to the single receiver point they are shaped by the different channels. The shape of the received spectrum will therefore be an average of all the channels from all of the equipment locations to the receiver. We might therefore infer that a low frequency resonant peak at a few hundred
Hz is a common characteristic of all channels associated with this network. Unfortunately we are still lacking evidence of the precise physical cause but the sort of mechanisms which lead to resonance effects will now be described.

2.5.4.1 Resonances (poles)

The primary characteristic of large networks of wires is inductance and it is well known that the source impedance of the mains supply at the power frequency has a significant inductive component [109]. There are also distributed capacitances associated with the network wiring due to the proximity of active and neutral connected wires. In addition, there are lumped capacitances in small power factor correction components for fluorescent lighting and larger power factor correction components for industrial loads and there may also be combinations of inductors and capacitors added for harmonic filtering. The power factor correction capacitors are designed to resonate with the load inductance present at the power frequency when their impedance equals that of the local parallel connected inductive load. This tank circuit forms a trap for the power frequency quadrature currents to prevent them from appearing on the supply. At frequencies higher than the power frequency the correction capacitor’s impedance will reduce and it is clear there will be a frequency at which the capacitor’s impedance matches a different inductance in the system, for example, the inductive part of the wiring connecting other parallel loads. This is the type of resonance mechanism which could cause the presence of the lower frequency complex conjugate pole pair. A second pole pair is evident in all models at a significantly higher frequency, between 930Hz and 2,100Hz, and the damping factor for this feature is 0.1 to 0.4. Explanation for this demands either a different inductor or different capacitor, or different inductor and capacitor, to that which caused the first pole pair.
2.5.4.2 Anti-resonances (zeros)

As the frequency increases from that of the first resonance at 500Hz, we can expect other effects to come into play. Some of the high value inductances present in the network as loads will reduce in value due to the decreasing flux penetration depth into the iron laminations which were designed for power frequencies. The leakage inductances of the transformer will increase for the same reason, as explained in section 2.5.2. The resistance values will also begin to increase on the larger conductors due to the skin effect, the skin depth being 2mm in copper at 1kHz. At frequencies where the changing component values form an anti-resonance there is an equivalent circuit formed within the channel, the resonance of which causes a reduction in the current transfer ratio. There are two of these in the generic model proposed, one between 760Hz and 2.1kHz and the second between 2.5kHz and 3.4kHz, both with damping ratios between 0.2 and 0.6.

2.6 Summary of generic models

The final models which are summarised in Table 9.1 opposite form a concise description of the current transfer ratio for the channels. Most MLE values are retained but the high pass filter parameters of a zero at the origin and real pole at 460 rads/second (72 Hz) have been fixed in position. The damping ratio $\zeta$, is the ratio of the real part $\sigma$ of the pole or zero's position to the natural frequency of the pole or zero.
2.6.1 Fit of the generic models to the measured data

An example of the fit of measured data to the generic models with fixed zero and real pole is given below for the Bakery to Thomas Cherry channel. The graphs displaying the fit for all six channels are included in Appendix IX A starting on page 344 and an example of the MATLAB scripts for obtaining these plots is given in Appendix IX B on page 347. All of the channels with the exception of the original office measurements show a similarly good fit to that demonstrated in Figure 9.2.

Table 9.1. Positions of poles and zeros for each of the six channels tested.

<table>
<thead>
<tr>
<th>Name</th>
<th>Zero at origin</th>
<th>Close Real Pole (rads/s)</th>
<th>RHZ (rads/s)</th>
<th>Damped Freq of 1st complex zeros/ζ</th>
<th>Damped Freq of 2nd complex zeros/ζ</th>
<th>Damped Freq of 1st complex poles/ζ</th>
<th>Damped Freq of 2nd complex poles/ζ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bakery</td>
<td>Yes</td>
<td>-460</td>
<td>19000</td>
<td>860 Hz/0.4</td>
<td>2600 Hz/0.2</td>
<td>560 Hz/0.4</td>
<td>2100 Hz/0.2</td>
</tr>
<tr>
<td>Pavilion</td>
<td>Yes</td>
<td>-460</td>
<td>42000</td>
<td>860 Hz/0.4</td>
<td>2500 Hz/0.2</td>
<td>490 Hz/0.5</td>
<td>1900 Hz/0.2</td>
</tr>
<tr>
<td>Union bldg</td>
<td>Yes</td>
<td>-460</td>
<td>16000</td>
<td>760 Hz/0.3</td>
<td>2900 Hz/0.1*</td>
<td>540 Hz/0.4</td>
<td>1400 Hz/0.2</td>
</tr>
<tr>
<td>Boiler house</td>
<td>Yes</td>
<td>-460</td>
<td>24000</td>
<td>1300 Hz/0.6</td>
<td>2800 Hz/0.1</td>
<td>510 Hz/0.5</td>
<td>1800 Hz/0.1</td>
</tr>
<tr>
<td>Office</td>
<td>Yes</td>
<td>-460</td>
<td>32000</td>
<td>790 Hz/0.2</td>
<td>3000 Hz/0.1</td>
<td>410 Hz/0.3</td>
<td>930 Hz/0.4</td>
</tr>
<tr>
<td>Room 127A</td>
<td>Yes</td>
<td>-460</td>
<td>19000</td>
<td>2100 Hz/0.2</td>
<td>3400 Hz/0.3</td>
<td>580 Hz/0.3</td>
<td>2000 Hz/0.1</td>
</tr>
</tbody>
</table>

Figure 9.2 Generic model of Bakery channel compared to measured values of magnitude shape (a) and phase (b).
2.7 Noise

A complete model must include a description of the noise and since the receiving point is the same for all of the channels considered we only need one noise description which is obtained from the measurements described in the results chapter. It was established in chapter VII that noise signal within narrow bands can be expected to behave as if it came from a process with an underlying Gaussian distribution and measurements detailed in the previous chapter confirmed this. It was also established that either the in-phase or quadrature RMS value is sufficient to define the noise in each narrow band. If the noise current in a group of adjacent bands is the same amplitude a single figure can be used to define the noise across the whole group. The assumption is made that the noise within each band is uncorrelated with the noise in any other band so when the noise signals from the bands in a group are added together they sum as if they were orthogonal vectors. The total noise over such a group of bands divided by the bandwidth in the same geometric way gives the noise in a 1Hz bandwidth which is the signal spectral density in Amps per $\sqrt{\text{Hz}}$. We are ignoring the very narrow band noise signals at the mains harmonics in the analysis.

This procedure was applied to the measurements to obtain the noise spectral density graph presented at the end of the last chapter and this, along with the transfer function, forms a description or model of the channel.
3 Use of the model

Having a model of the channel for which a communication system is to be designed is essential, the more accurate the model the better the design can be tailored to optimise performance. The model can then be used to predict the output for any design of signal input, inter symbol interference can be assessed when considering signals in the time domain and equalisation can be designed in the frequency domain. In the following section the models will be used to predict the Shannon capacity limit, that is the maximum theoretical information capacity for error free transmissions.

4 Information capacity of the channel

The main reason for constructing these models is to determine the answer to the primary underlying question, has the LV/MV channel got sufficient capacity for the interval meter AMR application?

A channel’s theoretical maximum capacity, $C$ in bits/s, can be predicted using the signal to noise power ratio, by use of Shannon's capacity equation [110]:

$$C = B \log_2 \left( 1 + \frac{S}{N} \right)$$

where $B$ is the bandwidth of the channel (Hz), $S$ is the signal power spectral density ($A^2$/Hz in our case) and $N$ is the noise power spectral density ($A^2$/Hz). This assumes the signal to noise power ratio is constant across the bandwidth $B$, if not the equation becomes:

$$C = \int_0^B \log_2 \left( 1 + \frac{S(f)}{N(f)} \right) df .$$

$S(f)$ is the received signal power and it depends as much on the spectral shape of the transmitted signal as it does on the channel. In this work we do not presuppose any
particular information modulation scheme for the AMR solution and so the way the signal power might be distributed across the bandwidth available has not been defined. In general a transmitter which is able to tailor the spectrum of its signal to the state of the channel in order to maximise the communication rate will result in better performance than one which does not. For example, the measurements have shown that all of the channels have higher attenuation for signals at frequencies between 2kHz and 2.5kHz than those between 1.5kHz and 2kHz, yet the noise spectral density is about the same over these two 500Hz bands. A transmitter which concentrated all of its signal power in the upper 500Hz band would therefore perform less well than one that concentrated all of its signal power in the lower 500Hz band. A transmitter which divided up the power between the two bands could perform even better. The optimum way of dividing the power has been extensively studied by others and determined to be the scheme using the “water filling” or "water pouring" algorithm [111]. The effect of using this algorithm will be explored shortly, but first the channel capacity is derived for simpler transmit power allocation.

4.1 Transmitter power

The channel capacity is not just a characteristic of the channel, it depends on the signal power input so first we need to settle on a suitable and realistic figure for the transmit power. Let us assume for the moment that we have a single massive transmitter. This differs from the reality in which we might have several thousand meter-located transmitters all communicating with a single receiver simultaneously sharing part, or all, of the same channel. However, taking the simple case of one transmitter and one channel and one receiver allows us to apply the Shannon formula directly. We will make the power of this single transmitter equal to the cumulative power of all the meters. We have seen that it is feasible to make a meter-located transmitter, the SWD type for example,
produce a pulse train at an adjustable frequency with peak pulse values of 10 or 20 Amps, so let us assume we can produce a signal with 5 Amps RMS and have it modulated in such a way as to give a signal with a perfectly flat bandwidth of 1Hz and no frequency components outside that bandwidth. The signal’s spectral density from this single meter-located transmitter would therefore be 5A/√Hz. Let us now consider a single transmitter producing a signal the equivalent of 10,000 meter-located transmitters. Assuming all individual signal components are uncorrelated, the single transmitter could produce either 500A/√Hz over the 1Hz bandwidth or 5A/√Hz over a 10kHz bandwidth, or 10A/√Hz over a 2.5kHz bandwidth. The latter would be most suitable for our channel and it will arrive at the receiver attenuated by different amounts at different frequencies. For the purposes of calculating the information capacity we will initially assume a transmitter signal strength of 10A/√Hz.

4.2 Calculating the information capacity of the poorest channel identified

4.2.1 Quantisation of the spectrum

First we will take a non-optimum but simple course of action and spread the signal power at the transmitter evenly between 100Hz and 2,150Hz which is the part of the spectrum usable in the worst channel identified, the Office channel. In this case the received power will have the spectral shape and amplitude predicted by our model of the transfer function. Because we only have a spectrum for the noise quantised into very few frequency bands for simplicity we will also split the received signal spectrum into a small number of bands and assume that the signal to noise ratio is constant over each band. The average will be taken of the transfer function magnitudes at each frequency point within the same band and this average value will be used to represent the transfer function across that band. The Shannon formula will then be used to find the theoretical maximum capacity of each segment and the capacities will be added up to arrive at the
total channel capacity. This is acceptable if each segment of the channel is assumed to behave independently from all the others [112].

4.2.2 Capacity for constant transmit spectral density between 100Hz and 2,150Hz

In Figure 9.3 the expected received signal spectral density is plotted against the noise spectral density at the receiver for transmission with 10A/√Hz over an approximate 2kHz bandwidth. The spectrum has been quantised into seven bands and the capacity for each band was calculated using Shannon's equation. The capacity for the first band, 100Hz to 200Hz for example is 656 bits per second (bps), the signal to noise ratio being 19.7dB. When the capacity in each of the seven bands is summed the total for this channel is 12,800 bps.

![Indicative received signal/noise ratio for Office channel](image)

**Figure 9.3** Received signal to noise ratio for office channel used to estimate maximum theoretical information capacity of the channel.

4.2.3 Increase in information capacity for water filling transmit power allocation

The total transmit signal in the calculation above was 10A/√Hz over 2,050Hz which is 453 Amps. This could have been spread across the available spectrum in any number of ways but the optimal way is given by the water filling algorithm. In this procedure the bands with a higher gain to noise ratio, will have a higher transmit signal spectral
density allocated. Bands which have a gain to noise ratio below a certain threshold L, will not receive any signal. The level of this threshold depends upon the total signal strength available and the shape of the gain to noise ratio over frequency. The simplest way to apply the algorithm is to plot the inverse of the gain to noise ratio against frequency and to "fill" the lowest areas first with transmit signal power as if pouring it in. When all of the transmit power has been poured in, the level L will determine the power allocation for any segment of the spectrum and parts of the spectrum where the inverse of the gain to noise ratio was above this level will not be allocated any signal. This procedure was applied to the Office channel between 100Hz and 2150Hz using a signal of 453Amps and the increase in information capacity was 2%. The reason for the increase being so low is that, at a transmit signal level of 453Amps (which equates to 10A/√Hz when evenly spread) all bands considered have a quite substantial signal to noise ratio. Water pouring only provides significant improvements in capacity where there are much lower signal to noise ratios. To determine its usefulness in our case a much lower signal level was considered. At 2A/√Hz the even spreading of signal across the bandwidth 100Hz to 2050Hz results in a channel capacity of 4300bps and the signal to noise ratio variation at the receiver is indicated in Figure 9.4 overleaf.
Figure 9.4 Received signal to noise ratio for the office channel used to estimate the maximum theoretical information capacity for a constant signal spectral density of 2A/√Hz.

In this case it is clear that allocating as much signal to the second band (200Hz to 300Hz) with its negative signal to noise ratio, as is allocated to the first band (100Hz to 200Hz) is sub-optimal. The water filling algorithm was again applied and the optimal signal allocation is shown in Figure 9.5. and an increase in capacity to 4,900bps results.

Figure 9.5 Water filling algorithm. The black line shows the inverse of the Gain to Noise ratio and the blue line shows filling level L, 5.56 in this case.

The axes are both linear and the signal spectral density for maximum capacity is proportional to the height of the blue line above the black line. This spread is compared
to the constant spectral density used for Figure 9.4 in the next plot in Figure 9.6:

**Figure 9.6** Comparison of optimal signal spectral density with fixed signal spectral density.

For optimisation of the channel capacity no transmitter power should be used in the 200Hz to 300Hz band and small adjustments should be made to the signal allocated to other bands. This yields a 14% increase in information rate over the rate obtained with a constant $2\sqrt{A/Hz}$ signal spectral density at the transmitter. We can conclude that, in our case, the potential increase in capacity due to water filling is unlikely to result in the substantial capacity increases which would be needed to justify the extra system complexity. This is because the shape of the noise at the receiver is similar to the shape of the received signal with transmit spectral density constant over a fixed band.
4.3 Calculating the information capacity for the other channels

The information capacity for the six other channel models was calculated assuming 10A/√Hz signal spectral density at the transmitter over the range 100Hz to 3kHz. The capacities obtained are summarised below, along with that for the Office channel in Table 9.2.

<table>
<thead>
<tr>
<th>Channel name</th>
<th>Capacity (bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bakery</td>
<td>35200</td>
</tr>
<tr>
<td>Pavilion</td>
<td>32500</td>
</tr>
<tr>
<td>Union bldg</td>
<td>30400</td>
</tr>
<tr>
<td>Boiler house</td>
<td>28300</td>
</tr>
<tr>
<td>Office (PS117)</td>
<td>12800</td>
</tr>
<tr>
<td>Room 127A</td>
<td>24800</td>
</tr>
</tbody>
</table>

Table 9.2 Maximum theoretical channel capacity assuming 10A/√Hz signal spectral density at transmitter.

These capacities are all greater than that which is necessary for Automatic Meter Reading of interval meters as determined in the introduction chapter and, provided a communication system could be designed to approach the theoretical capacity, a population of 10,000 meters could be read in about 80 minutes using these channels and channels like them. This statement contains many critical assumptions which further study could address and some of these are discussed in the next section.
5 A practical transmitter

A communication system would not normally need to have the main parts of the hardware very different from a channel sounding system although it is true the software is likely to be entirely different because information needs to be impressed on the signal at the transmitter and decisions rather than measurements need to made at the receiver. However, in considering the practicality of basing an upstream communication system for meters or smart grid application on the system developed here for channel sounding, one obstacle stands out, the transmitter inefficiency. In order to develop signals detectable at the zone substation many Amps had to be drawn at the transmitter. It was simple to use a resistor but this leads to large power dissipation at the transmitter. The main commercial embodiment of the TWACS® system described in Chapter II does in fact use a resistor in this way but its very low transmit pulse rate of 60 per second, each of which occur when the instantaneous mains voltage is relatively low, coupled with the fact that it is designed only for occasional monthly billing meter reads means that the large power dissipation occurring over short time intervals can be absorbed by the thermal capacity of a small heat sink. For interval meter reading with much more information being sent a resistive load is unsuitable and a load made up of reactances only must be used to produce the large current signals required but with low transmitter power dissipation. The SWD type circuit described in Chapters II and III is perfectly suited, but as the work presented in chapter IV made clear, there are critical complicating issues arising which have not been mentioned in the literature on the subject. Only under certain conditions can the circuit be made to produce the predictable patterns of current pulses required. Now that this behaviour is properly understood it presents no barrier to using the circuit to generate the required signals for a channel sounding system or a communication system. The reason that it was not chosen for the sounding system in this study is that although we now have an understanding of how to
control the circuit to generate the required signals, it would require more development
time to write the necessary algorithms to provide that control. Developing the resistive
circuit with very large water tank heat sinks took much less time.

6 Limitations of this study

The scope of this study was restricted to upstream power line communications using
signals originating at LV endpoints which would pass over the distribution transformer
and reach the zone substation in distribution systems typical of those used in Europe and
Australia. Over the course of the study many questions were raised which had to be put
aside due to resource and time limits and these unanswered questions further narrowed
the scope as will be discussed below. Another limiting factor was that this particular
niche of power line research appears to be unexplored and this will also be outlined here.

6.1 Dearth of available information on existing studies with the same scope

There is great interest in power line communications in general and a mass of
information available from previous studies. However, almost all of it falls outside the
scope of this study and is of very limited relevance. The reason for this is twofold, firstly
most applications with commercial promise, for example Broad band over Power Lines
(BPL), require much higher data rates than the channel in this study can accommodate
and secondly the use of the lower part of the spectrum is restricted by legal regulation.
No commercial power line communication system, other than those which are utility
owned or licensed, will pass the regulatory requirements unless the operating
frequencies are greater than 95kHz and even utility owned systems are generally
restricted to operation above 3kHz in the European Committee for Electrotechnical
Standardization’s CENELEC-A band [113]. Researchers interested in the characteristics
of the channel at the lower end of the CENELEC-A band have noted a lack of data and
the existence of very few publications of relevance [114, 115]. The frequency band of interest in this study is below the lowest frequency of the CENELEC-A band and no previous study has been identified which is concerned with the behaviour of the combined LV/MV channel in this frequency range. However, published literature on three communication systems which used the LV/MV channel was identified [15, 21, 24]. This gave a starting point for the research work but the absence of other studies to compare results against suggests that further research in this area would be useful.

6.2 Lack of verifying data

The modelling of some of the channels within La Trobe University's distribution system is, in essence, a case study. It is my belief that there is no reason for the university's system to behave differently to typical utilities' systems when the co-generation plant is not operating, however this requires confirmation by repeating some of the measurements on other distribution systems. This is not the only verification work still required because the models for each channel were based on only one set of measurements on that channel. In only a single case, the Office channel, was a second set of measurements taken at a much later date to test the validity of the model for that channel. The reason for this was logistical and had more verification work been done, fewer different channels would have been tested. The choice was made to obtain data from as many different channels as possible rather than build confidence in a smaller number of models, but it is recognised that this approach results in many questions about the repeatability and the applicability of the individual models over time remaining unanswered.
6.3 Small number of different channels tested

The development of a reliable and portable set of equipment for performing the channel sounding measurements absorbed most of the time available in this research work. This means that relatively few of the possible channels within the distribution system were characterised. For any channel there are a large number of variables which are likely to impact its behaviour, for example the nature of loads, the size and type of distribution transformer, the length and type of LV cable between the transmitter and distribution transformer and the distance of the distribution transformer to the substation (MV path length). With only six different channels tested insufficient information was available to assess any correlation of the model characteristics with these physical characteristics of the channels. With fourteen separate distribution transformers of various sizes available and hundreds of different potential transmitter locations for each transformer, further study may lead to discovery of correlations between physical channel characteristics and the model parameters. This deeper understanding might allow useful predictions about channel behaviour to be made without the need for measurements in all cases.

6.4 No reason discovered for the markedly different Office channel

One channel stands out as having significantly different behaviour to the other channels, that is the Office channel which has a capacity much lower than the others. Part of the reason for this is that the capacity calculation used the shape of the modelled transfer function rather than the measured data itself to predict received signal strength and in the Office channel's case the match of the modelled transfer function to the measured data above 1,500Hz is much worse than for any other channel. The only apparent physical difference between this office channel and the others is that a large three phase cable was arranged to semi-permanently connect a laboratory bench into the local
switchboard. It is not thought that this difference could account for the more rapid attenuation with frequency that this channel exhibits. There are many unknowns about the exact connections within the various building's electrical supplies and this lack of control and knowledge is another limitation of this research work.

6.5  *Behaviour at frequencies above 3kHz*

As mentioned in section 2.2 above, the system developed was limited to making measurements below 3kHz. It is clear that in most channels there is still some useable spectrum above this frequency. It is not known how far above this frequency the signal to noise ratio in dB terms would have continued to be positive. It is suspected that the limit for measurements would have been quite close to 3kHz because it was so much lower for the office channel and it appears to be a widely held belief that large distribution transformers do not have a good response at frequencies much higher than this. Nevertheless, it would have been much more satisfactory to positively determine the highest useable frequency for each channel.

6.6  *Assumption that individual capacities for LV points add to give total capacity*

In section 4.1 it was assumed that if the signal were being produced by a single large transmitter it would result in the same channel capacity as for the cumulative capacity with many individual meters transmitting proportionally lower signals. However, this is clearly untrue if the signals from one meter appear as extra interference to transmissions from another meter. This would be the case in a Code Division Multiple Access type system, but if each meter were only transmitting on parts of the band unoccupied at the time by other meters a reduction in capacity would not occur. The exclusive use of parts of the spectrum by individual meters at particular times would require a central control system to organise so it may be the case that in practical systems individual meters
unavoidably interfere with each other, resulting in an overall capacity reduction. The extent of this reduction has not been investigated and the validity of the assumption that if meters do not interfere, all of the individual meter's signals can be considered to originate from the same transmitter has not been proved to be valid.

Overcoming some of the limitations described above will be addressed in the last chapter in which final conclusions are drawn.
CONCLUSION

1 Overview
In this chapter the significance and implications of the research work will be summarised, the contribution that the work has made will be highlighted and further work suggested.

2 Main finding
The LV/MV power line channel should not be ignored as a communications solution for half hour interval meter AMR and other smart grid applications. Although the few existing systems already using this channel show performance well below that required for interval metering, with cumulative data rates below 200bps, this research has shown that the channel itself is not the limitation. It has been shown here that it is likely that the characteristics of typical LV/MV channels will theoretically allow us to communicate at overall data rates in excess of 10kbps. As stated in the introduction, a 10kbps information rate would allow 10,000 meters to be read in 80 minutes. The Shannon limits found for the channels studied, when reasonable and achievable transmit signal levels were assumed, were all above this information rate. Subject to proof that the La Trobe University distribution system studied is not a-typical, we can therefore be confident that power line communication for the new AMR application using this channel is at least possible. Considering next the practical possibility, a communication system approaching the Shannon limit is likely to require significant complexity in error correcting code implementation, particularly at the receiver. This is not a great obstacle in upstream communication because a single receiver would be associated with hundreds or thousands of meters effectively amortising the "per meter" receiver cost to a low
value. The owners of the TWACS® system design, described in chapter II, have already shown a willingness to add high performance coding to their equipment [116]. Designing appropriate modulation and multiple access schemes to utilise the entire bandwidth available is another matter, all that has been demonstrated in this work is that using the SWD circuit, described in chapter II and analysed in detail in chapter IV, it is possible to generate large current signals with a degree of flexibility in the frequency content. However, even if the systems which it is possible to develop cannot use the whole channel capacity, it is still likely that an entire zone substation population of meters could be read within a 24 hour period (rather than 80 minutes) and this would still be acceptable in the interval metering application. In summary, the hypothesis that a suitable method for signal generation exists and the channel behaviour with respect to those signals will allow information transmission at the required rate over the LV/MV channel is supported by this research work.

3 Other contributions

1) There is very little literature concerned specifically with the "over the transformer" power line channel. It appears that from a communications viewpoint, the distribution transformer has been thought of as a wholly isolating barrier between two separate networks, the LV (substantially in-building) and the MV (street-to-street) systems, each with reasonably good prospects for use as a communications medium. Although the communication channel between the two networks has historically been of very little use, the new metering infrastructures planned for deployment in jurisdictions all over the world means a mind-set change might be advantageous. This research work demonstrates that the isolating barrier is in fact a useable link.
2) A simple design for reliable and portable equipment to carry out sounding measurements for the LV/MV channel now exists as a result of this research work.

3) Literature on the subject of SWD transmissions in which a resonant circuit is switched repeated and regularly across the sinusoidal mains supply by a triac implies that the current pulses will always alternate in polarity. The theoretical behaviour of this circuit was thoroughly investigated and it was found that the pulses do not always alternate. Undoubtedly previous users of the SWD circuit have noticed this and the other phenomenon described in this research work but several important aspects of the circuit's behaviour are missing from the literature. In chapter IV a closed form solution for the sequential current pulse amplitudes is derived. This solution is based on a gross simplifying assumption and is checked by an iterative procedure without the assumption. The main aspects of the behaviour predicted by the closed form solution are also evident when the more accurate iterative procedure is applied and the same aspects are also evident in experiments with physical circuits. This new knowledge will be indispensible if a transmitter is to be based on the resonant circuit.

4) The concept that bandwidth reducing filtering operations will make the output signal probability distribution tend to Gaussian seems intuitively correct but a proof has not been found. In chapter VII, evidence that this is correct was presented. Similarly, an explanation of the relationship between the real in-phase and real quadrature components of a narrow band Gaussian noise signal (as output by a lock-in amplifier in response to wideband noise) which would allow us to say exactly why the two signals are mutually redundant was not found. Such an explanation was constructed in that chapter.
4 Further work

Many assumptions and estimates are embodied in the main finding of this thesis described in section 2 above and further work is suggested to address those uncertainties. If sufficient confidence in the main result of this thesis were to be built it might justify embarking on a commercial system design for the new interval metering application, the ultimate benefit being reduced ongoing cost to the consumer for electricity metering services.

4.1 More testing required

Six channels were modelled and significant similarities were found between them. To determine if these common characteristics are universal the testing of channels within different utility distribution systems is required. They should also be tested at different times of the day, on week days and weekends and in different seasons to assess the extent to which the noise and transfer function change over time. This work could use the same equipment and methods as was used for the research described in this thesis. Fewer measurement points for each transfer function test would be necessary (because it has been shown that models with only a small number of parameters generally fit well) but those measurements should go to higher frequencies than were used for this study in order to positively identify the highest usable frequency for each channel.

Data from a wide ranging study is expected to show up patterns which will link transfer function characteristics with physical attributes of the channels provided that those physical attributes (wiring path lengths, cable type, correction capacitor locations, transformer size etc) are accurately known for each channel. Once these links are determined the characteristics of any channel could be predicted without testing and so a meter communication system could be designed to suit.
4.2 *Multiple access*

The second major area of further work which would lead towards this effectively free channel being utilised for AMR is study of the applicability of the various modulation and multiple access techniques. It is probable that the channel could be used with a single transmitter and even this simplified realisation does suggest an AMR application: each meter transmitting its data modulated at high frequency on the LV only to a single LV/MV relay transmitter using the low end of the spectrum to pass the data over the distribution transformer to the substation. This topology is similar to the DLC systems described in chapter II, with the concentrators replaced by the relay transmitters but with the advantage of no third party data carrier being required. However, a simpler arrangement is to have each meter transmit directly back to the zone substation and this requires an understanding of the best way of sharing the available spectrum. To obtain the maximum channel capacity the whole bandwidth must be utilised. This can be accomplished in two ways, either each meter's transmissions are made over the whole bandwidth, probably simultaneously, or parts of the available bandwidth are allocated to each meter. In either case it is likely there will be significant meter to meter interference which will reduce the channel's overall capacity. Selection of the optimum method for sharing the channel bandwidth resource, given the limitation in the types of signal the switched resonant circuit can produce, is a large unexplored field of study.
Appendix III A Initial transmitter circuit

The initial transmit circuit is illustrated in Figure A3.1.1

![Initial current pulse transmitter circuit.](image)

A current limiting inductor of 4.7mH adds to the leakage inductance of the transformer which is about 1.3mH, to limit the current and control the current pulse shape. The series resistance of the transformer is about 200mΩ which further reduces the current. The current limiting inductor has a core of powdered iron made up of three stacked toroids each 25mm high, cross sectional width 9mm inside radius 32mm and outside radius 50mm. The material has a relative permeability of 65 giving \[ \mu = 81.8 \times 10^{-6} \text{Hm}^{-1} \]. It has 100 turns so the flux density at 20A is 0.16T, well below the material’s saturation level of 1T. The current is monitored and measured by a 16.3 mΩ four terminal shunt made from about a meter length of nickel chromium shunt material.
Appendix III B Construction of current transformer

Assume a toroid of ferrite material high permeability surrounds the conductor carrying the load current. This will allow construction of a current transformer by including a winding around the toroid core. Let us first assume this winding is open circuit or not present so the only current producing flux in the core is the load current we want to measure.

Almost all the flux surrounding the conductor in the immediate area of the core will be concentrated into the toroidal core due to its low magnetic reluctance. The magneto motive force (mmf) driving magnetic flux around the core is given by the current flowing through the wire. Let us assume that the current peaks at +/-50 Amps which gives a peak m.m.f. of 50 Amp Turns.

Let us assume that a moderately sized core is used with an average magnetic path length of 188mm (average radius 30mm). This would give a Magnetic Field Intensity, or Magnetising force, of 50/0.188 At/m. With a typical relative magnetic permeability of 3000 a ferrite core would have a Magnetic Field density, B, of:

\[ B = \frac{3000\mu_0 \times 50}{0.188} = 1.0 \text{ Teslas.} \]

This flux density would be much greater than a typical saturation level for a Power Ferrite material.

If we now add a secondary and form a measurement current transformer by virtually short circuiting it we get a secondary current determined by the turns ratio and the primary load current. Let us assume a turns ratio of 100 to give a secondary current of half an amp. This secondary current produces an mmf and this acts in opposition to the initial primary m.m.f. (by Lenz’s law). This opposing mmf will also be 50 Amp turns resulting in the flux in the core dropping to a low value which will not saturate the core.
Appendix III C Rogowski coil and amplifier

The amplifier chosen was the Rod Elliot Microphone Preamplifier[29]. This is a discrete two transistor circuit which uses BC549s which are suitable for low noise stages in audio frequency amplifiers and was designed for a low impedance input. It was used in conjunction with a good quality microphone transformer with a turns ratio of 13:1 to further reduce the input impedance (better match to the Rogowski coil) and to provide a balanced input to reduce sensitivity to common mode signals. The amplifier was followed by a simple RC (479 \(\Omega\), 10 \(\mu F\)) integrator to give a signal proportional to the current in the feeder encircled by the Rogowski coil. The output signal was displayed on an oscilloscope, but first the measuring equipment was calibrated. Briefly, the calibration procedure involved passing a known current at a known frequency through a wire which passed 55 times through the middle of the circle formed by the Rogowski coil. The outputs of the amplifier and of the RC integrator were measured on a high quality measuring amplifier (B&K type 2606). The outputs were plotted for an input of 27.5 Amp turns from 25 Hz to 4kHz. The results are shown in Figure A3.3.2. They demonstrate that the amplifier and coil combination is almost linear over this range with a gradient of 0.51 \(\mu V\) per Amp Turn Hz.

![Linearity of Rogowski coil/ Amplifier](image)

**Figure A3.3.2** Calibration of amplifier and coil alone.
The output after the RC integrator was also plotted and is shown in Figure A3.3.3 to 1 kHz. It is approximately constant at 2 µV per Amp Turn.

Figure A3.3.3 Calibration of amplifier, coil and RC filter.

The fact that the RC integrator output is not constant over the frequency range means that there will be some distortion in the shape of the current pulse measurements. Since they are only being used as an indication, it was thought the performance of the simple integrator stage was tolerable.
Appendix III D Phase identification

In order to determine which of the feeders were supplying which sockets in the lab a quick check was done. Two digital oscilloscopes were set up, one measuring the active to neutral voltage at the transmit location’s socket and the other measuring one of the feeder’s currents via the Rogowski coil. If both could be triggered at the same instant, a comparison of the two waveforms would indicate if the phases being monitored were the same. To accomplish this, a commercial UHF band transceiver (“walkie – talki”) was connected up so that it’s microphone would receive a tone from an audio signal generator. At the same instant (to the nearest ms) the transmitter end oscilloscope would be triggered by the same tone signal. The receiver end oscilloscope was monitoring the current from the feeder while at the same time monitoring the audio output from a second UHF transceiver which was set to receive on the frequency of the transmit location’s UHF transceiver. The receive end oscilloscope was set up to trigger from the received tone. If the first oscilloscope’s stored waveform showed the same time delay between its trigger time and the subsequent zero-crossing of the mains as the second oscilloscope showed between the trigger event caused by receiving the tone and the first zero crossing of the current which it was monitoring, then the phases at both ends were the same. There proved to be a small discrepancy of the order of a ms, mainly due to the response time of the UHF transmit link circuitry. It was easy to calibrate this out with the transmit and receive equipment co-located, and then to positively identify each of the feeders when the loads were close to unity power factor.

Appendix III E Design of saturating inductor for commutation failure protection

A ferrite toroid was chosen which had an initial relative permeability 1600 with an effective diameter of 48mm and cross sectional area 300mm². 130 turns gave an initial inductance value of the new component of 67.6mH. This very high inductance obviously limits the current to a low rate of change for a given voltage across it. The core material had a maximum working flux density specification of 0.365 T @75 °C indicating saturation at an H value of around 180At/m which, for 130 turns and flux path length of 0.15m (\(\pi d\)), corresponded to a current of 0.2Amps. As the instantaneous current increased above this level the effective relative permeability of the core material became 1 (the inductor now acted as if it was air cored) and its inductance reduced to 0.04mH. This is negligible compared to the main inductance (leakage inductance of the transformer), so the shape of the current pulse at currents above about an Amp remained almost unchanged. Below the value of about 0.2 Amps, where the inductance was very much greater due to the presence of the new component, the di/dt was greatly reduced.
Appendix III F Initial “home made” receiver

The receiver was based on an Altera CPLD/FPGA UP1 development board coupled to a simple analogue switch based multiplier circuit. The CPLD was configured to produce the switching signals, initially for two pairs of analogue switches. For each pair of analog switches the input to one was the receiver’s input signal (amplified and buffered) and the input to the other was the same signal inverted. The two outputs from each pair were added to give unfiltered P and Q signals. The CPLD generated a variable frequency in the audio range, controllable by an external input. This was the frequency to which the receiver was tuned and the phase of this reference signal defined a reference phase. This reference signal was used to derive the two pairs of switching signals mentioned. The first pair resulted in a multiplication of the circuit’s input with the reference frequency at the reference phase (unfiltered P), while the second pair resulted in multiplication of a quadrature representation of the reference signal by the input signal (unfiltered Q). These two multiplication outputs were low pass filtered to eliminate the high frequency products and to give P and Q outputs from the receiver. The reference signal was a square wave and so the resulting receiver had sensitivity to the odd harmonics of the reference frequency as well as its fundamental. This type of coherent homodyne receiver is standard and is used, for example, in ripple control receivers. Its advantage is the very simple method of multiplication. The control signals to the analog switches are arranged so that the unfiltered P and Q outputs are alternately the direct input (input times one) and the inverted input (input times minus one). This alternating inversion and non inversion of the signal is obviously the same as multiplying it by a square wave which alternates between +1 and -1 and so the circuit is a very simple and cheap way of multiplying two signals together. The restriction of this, the simplest technique, is that one signal must be a square wave and this is where the receiver’s sensitivity to odd harmonics of the square wave’s fundamental comes from.

The Circuit is shown in Figure A3.6.1 opposite.
CPLD listings in VHDL language, Rx_Top_block, Rx_cont, SWD_rec, Rx_clock_div etc, are available on request. The CPLD also contained circuitry configured to control the SWD type transmitter.
Appendix III G Third harmonic cancelation

Third harmonic cancellation for switch based homodyne receivers.
Written 8th March 2006  Author A.Mackie

The Idea.

This is a simple idea for improving the performance of a switch based homodyne receiver. It is novel as far as the author knows. It could probably be extended to cancel not just the one odd harmonic but any number of odd harmonics but, for the application under consideration, cancellation of only the third harmonic should be sufficient.

The application.

A power line receiver installed alongside a domestic electricity meter receives signals from the utility sent along the power lines to enable load control. These signals are keyed audio frequency tones added to the mains voltage. These signals are generally referred to as ripple control signals and the receivers as ripple receivers.

Ripple receivers have in the past used the standard homodyne technique for detecting ripple control signals. The load control application is cost sensitive and so the multiplier component of the homodyne receiver in one practical realization is simply a pair of switches controlled by a pair of locally generated 50% duty cycle square waves. These square waves are at the required receive frequency and are 90° out of phase with each other (to give “phase” and “quadrature” output signals).

This form of circuit is very cheap to make but has the disadvantage that it has sensitivity, not only at the receive frequency, but also at odd harmonics of that frequency. This is a consequence of each switch effectively performing a multiplication of the controlling square wave by the input to the switch, the square wave containing a fundamental frequency and all odd harmonics of it.

To circumvent this disadvantage some form of pre-filtering needs to be performed so that the input to the multiplier part of the receiver does not contain any significant component at odd harmonics of the frequency to which the receiver is tuned. If the receiver is to be tuneable across a range of frequencies then this pre-filter must also be tuneable and must track the frequency to which the receiver is tuned. This pre-filter adds significant expense.

The performance improvement.

If the performance of the multiplier section can be improved so that it has significantly reduced sensitivity to the third harmonic then the specification of the pre-filter can be made significantly less stringent.

This idea is a method of improving the performance of the multiplier at a minimal cost giving rise to a large cost saving in the pre-filter.
Details of the idea.

A standard switch based homodyne receiver would have two switches, one for the “phase” output and one for the “quadrature” output. These switches would switch between the direct input signal and an inverted input signal (so multiplying successively by +1 and -1).

The new idea is to replace the two pole switches by three pole switches. The third pole would be connected to a null point in the circuit ie. earth. In this way the phase and quadrature outputs could be made to switch between the input, earth and the inverted input (so multiplying successively by +1, 0 and -1).

By selecting appropriate ratios of the switch times spent at the +1, 0 and -1 positions the new receiver can be made to have very much reduced sensitivity at any particular odd harmonic. The third harmonic, being the closest odd harmonic to the fundamental, is the most important in this application. The following sequence of switch time ratios gives third harmonic cancellation of about 30dB and this has been experimentally proven by the author:

<table>
<thead>
<tr>
<th>Switch position:</th>
<th>+1 position</th>
<th>0 position</th>
<th>-1 position</th>
<th>0 position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalised time:</td>
<td>10</td>
<td>5</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

This idea could be extended to cancelling more than one harmonic at the same time by either adding more switch positions (e.g. +½, -½) or rapid pulse width modulating between positions, for example 50% duty cycling between positions +1 and 0 would simulate an extra +½ position.

An alternative to adding the extra 0 switch position for third harmonic cancellation might be rapid (ie at a frequency much higher than the receive frequency) 50% pulse width modulating between +1 and -1 switches. This would save the cost of an extra switch but because the cost of the switches is very low it may not justify the higher speed circuitry. This last idea has not been experimentally proven by the author.

Of course the logical extension of this last idea is to smooth pulse width modulate to simulate multiplying by a sine wave which would give no harmonic sensitivities.
Appendix III H Extract from line diagram of university’s distribution network

Figure A3.8.1 Single line diagram of distribution system.
Appendix III I Medium Voltage Network between buildings

In the diagram in Figure A3.9.1 the black lines indicate MV (22kV) cables.

Figure A3.9.1 Physical layout of MV network.
Appendix III J Construction and calibration of current monitoring device

A CALMU meter is a now obsolete commercial three phase meter based on current transformers, it is made by PRI an international meter manufacturer with an Australian branch.

The current transformers from a CALMU meter are used in place in the meter body and connected to the original meter terminals. They have 150 ohm secondary coils with about 3,900 turns around a toroidal core made from very thin Grain Oriented Steel strip. They have 15 turn primaries. The load resistor on the secondary is 39 ohms. They are followed by three single stage inverting op amp circuits. The op-amps are OP21 with a very low input noise. The whole circuit is set for a gain of 11.22V per Amp. The equivalent input noise is $6 \mu \text{A}$ measured with a bandwidth of 20Hz to 20kHz. The output swing of the amplifier is a little less than +/- 15V giving a maximum signal equivalent to an input of about 0.95A and a dynamic range of 104dB which is quite satisfactory.
Appendix III K Measurements of mains harmonics and received signals

These measurements were taken at the co-generation control room.

Data taken from lab log book 2 page 16  20th Dec 2005

<table>
<thead>
<tr>
<th>Harmonic/ frequency</th>
<th>Level at lock-in White phase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time Constant (TC =0.1 Seconds)</td>
</tr>
<tr>
<td>3/150Hz</td>
<td>0.9 mV</td>
</tr>
<tr>
<td>4/200Hz</td>
<td>40 μV</td>
</tr>
<tr>
<td>5/250Hz</td>
<td>4.0 mV</td>
</tr>
<tr>
<td>6/300Hz</td>
<td>20 μV</td>
</tr>
<tr>
<td>7/350Hz</td>
<td>1.6 mV</td>
</tr>
<tr>
<td>8/400Hz</td>
<td>10 μV</td>
</tr>
<tr>
<td>9/450Hz</td>
<td>70 μV</td>
</tr>
<tr>
<td>10/500Hz</td>
<td>2 μV</td>
</tr>
<tr>
<td>11/550Hz</td>
<td>0.3 mV</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Received</th>
<th>Level at lock-in White phase TC = 10 S</th>
</tr>
</thead>
<tbody>
<tr>
<td>180.11 Hz Transmitter on</td>
<td>1.1 μV</td>
</tr>
<tr>
<td>180.11 Hz Transmitter off</td>
<td>0.2 μV</td>
</tr>
</tbody>
</table>

Conditions: 300A load per phase on 11kV lines as measured by control room equipment.
Transmitting on White phase at 2 Amps peak @180.11Hz.
Gain at receiver =11.22V per Amp on the 11kV line.
Attenuation between amplifier housed in CALMU case and input to lock-in amplifier = 30dB. This attenuator was to prevent overloading of lock-in by 50Hz mains signal.
Gain of amplifier and CTs=11.22V/Amp.
Ratio of 11kV current transformer = 450:1.
Appendix IV A  Relationship of theta to phi for maximum capacitor voltage

Considering at which \( \theta \) value will the capacitor reach its maximum voltage of 
\[ V_c = 2\sqrt{2} \times 240 \sec \left( \frac{\phi}{2} \right) \sec . \]
We can best answer this by first considering the continuous version of the equation and looking at the multiplication factor 
\[ \frac{1}{2} \sin \left( \theta + \frac{\phi}{2} + t\phi \right) \pm \frac{1}{2} \sin \left( \theta - \frac{\phi}{2} \right). \]
This expression can only be its maximum of 1 when 
\[ + \frac{1}{2} \sin \left( \theta - \frac{\phi}{2} \right) \] 
so \( \sin \left( \theta - \frac{\phi}{2} \right) = 1 \), or when \( - \frac{1}{2} \sin \left( \theta - \frac{\phi}{2} \right) = \frac{1}{2} \) that is when 
\( \sin \left( \frac{\phi}{2} - \theta \right) = 1 \). For the first situation 
\( \left( \theta - \frac{\phi}{2} \right) = 2m\pi + \frac{\pi}{2} \) where \( m = \pm 0,1,2,3 \ldots \) and for the second situation 
\( \left( \frac{\phi}{2} - \theta \right) = 2m\pi + \frac{\pi}{2} \). The required value is the maximum of these two possibilities.

To determine which of these possibilities is correct we must go back to the two equations derived for the integer \( n \), one valid for even \( n \) and the other for odd \( n \). For even \( n \) the offset term was added and 
\( \left( \theta - \frac{\phi}{2} \right) = 2m\pi + \frac{\pi}{2} \) gives the maximum value of the capacitor voltage. For odd \( n \) the offset term was subtracted and 
\( \left( \frac{\phi}{2} - \theta \right) = 2m\pi + \frac{\pi}{2} \) gives the maximum value. With \( n \) unrelated to 50Hz the substitution of these values into the appropriate odd or even equation gives the same maximum capacitor voltage.
Appendix IV B  Using standard procedure for finding the particular integral

Here we find the particular integral (PI) by proposing a solution of the general functional form:

\[ f(t) = 240\sqrt{2} \sin(2\pi 50t + \delta) \]

The PI for the case of a pure sine wave mains forcing function is:

\[ q = \left[ A \cos(2\pi 50t + \delta) + B \sin(2\pi 50t + \delta) \right] \]

This gives:

\[ \frac{dq}{dt} = 2\pi 50B \cos(2\pi 50t + \delta) - 2\pi 50A \sin(2\pi 50t + \delta) \]

and

\[ \frac{d^2q}{dt^2} = -(2\pi 50)^2 A \cos(2\pi 50t + \delta) - (2\pi 50)^2 B \sin(2\pi 50t + \delta) \]

Substituting these expressions into the differential equation to be solved we obtain:

\[ L \frac{d^2q}{dt^2} = V_{\text{mains}} - R \frac{dq}{dt} - \frac{1}{C} q \]

which gives:

\[ -L(2\pi 50)^2 A \cos(2\pi 50t + \delta) - L(2\pi 50)^2 B \sin(2\pi 50t + \delta) + \]

\[ \frac{B}{C} \sin(2\pi 50t + \delta) = V_{\text{mains}}. \]

Next we substitute \( V_{\text{mains}} = 240\sqrt{2} \sin(2\pi 50t + \delta) \) and collect cosine and sine terms together to give:

\[ \left( -L(2\pi 50)^2 A + R(2\pi 50)B + \frac{A}{C} \right) \left( \cos(2\pi 50t + \delta) \right) + \]

\[ \left( -L(2\pi 50)^2 B - R(2\pi 50)A + \frac{B}{C} - 240\sqrt{2} \right) \left( \sin(2\pi 50t + \delta) \right) = 0. \]

For this to equate to zero both the sine term multiplier and the cosine term multiplier must be zero because of the orthogonality of cosine and sine functions. So

\[ -L(2\pi 50)^2 A + R(2\pi 50)B + \frac{A}{C} = 0 \]

and

\[ -L(2\pi 50)^2 B - R(2\pi 50)A + \frac{B}{C} - 240\sqrt{2} = 0. \]

These are two independent linear equations in two unknowns \( A \) and \( B \). By algebra the solution for \( A \) and \( B \) are:

\[ A = - \frac{240\sqrt{2} \left( \frac{1}{C} - L(2\pi 50)^2 \right) R(2\pi 50)}{\left( \frac{1}{C} - L(2\pi 50)^2 \right)^3 + \left( \frac{1}{C} - L(2\pi 50)^2 \right) \left( R(2\pi 50)^2 \right)} \]

and

\[ B = \frac{240\sqrt{2} \left( \frac{1}{C} - L(2\pi 50)^2 \right)}{\left( \frac{1}{C} - L(2\pi 50)^2 \right)^2 + R(2\pi 50)^2} \]
Appendix IV C  Standard procedure for finding the complementary function

Here we use the standard procedure for finding the complementary function by proposing a solution of the general form \( q = Ae^{\lambda t} \) as the solution to the differential equation with \( f(t) = 0 \) rather than \( f(t) = V_{\text{mains}} \). Substituting into the differential equation:

\[
L \frac{d^2q}{dt^2} = 0 - R \frac{dq}{dt} - \frac{1}{C} q
\]

gives

\[
L\lambda^2 Ae^{\lambda t} + R\lambda Ae^{\lambda t} + \frac{1}{C} Ae^{\lambda t} = 0.
\]

Since \( Ae^{\lambda t} \) cannot be zero if \( A \) is not zero we have:

\[
L\lambda^2 + R\lambda + \frac{1}{C} = 0.
\]

Using the quadratic formula the solution for \( \lambda \) is:

\[
\lambda = \frac{-R \pm \sqrt{R^2 - 4LC}}{2L}.
\]

In our case \( \frac{4L}{C} > R^2 \) and \( \lambda \) will be complex, so let \( \lambda = \alpha \pm j\beta \).

We have found two values for the solution and we cannot assume that the constant \( A \) will be the same in each case so the two solutions will be \( A_1e^{(\alpha + j\beta)t} \) and \( A_2e^{(\alpha - j\beta)t} \). If each of these are solutions to the differential equation then the sum of these will also be a solution because that is a property of this type of differential equation. So our solution is:

\[
q = A_1e^{\alpha t}e^{j\beta t} + A_2e^{\alpha t}e^{-j\beta t}.
\]

Now it is time to remind ourselves that the charge is real so the constants \( A_1 \) and \( A_2 \) cannot take any values. They must both be complex numbers related in such a way as to ensure \( q \) is always real. We can see by looking at the equation for \( t=0 \) that the imaginary parts of \( A_1 \) and \( A_2 \) must be of opposite sign to cancel and leave \( q \) real. We can also see by looking at the equation for \( \beta t = \frac{\pi}{2} \) (and using Euler's formula \( e^{ix} = \cos x + j\sin x \) ) that \( jA_1 - jA_2 \) must be real. For this to be the case the real parts of \( A_1 \) and \( A_2 \) must be equal. So \( A_1 \) must be the complex conjugate of \( A_2 \).

Let \( A_1 = g + jh \) so \( q = (g + jh)e^{\alpha t}e^{j\beta t} + (g - jh)e^{\alpha t}e^{-j\beta t} \)

Using Euler's formula gives:

\[
q = (g + jh)\cos \beta t e^{\alpha t} + (g - jh)\cos \beta t e^{\alpha t}.
\]

All of the imaginary terms cancel leaving:

\[
q = e^{\alpha t}(2g \cos \beta t - 2h \sin \beta t)
\]

where all constants are real. For simplicity set \( G = 2g \) and \( H = -2h \).
Appendix IV D  MATLAB Script to find current pulse function

% Author Andrew Mackie Date 29th Dec 2006
% numerical calc of pulse zero crossings
% for SWD type circuit c=18uF; L=9mH, R=0.1Ohms
format long e;
t=.[0.000000:0.00001:.003];% set q=1 and initial t to find root, set q=0 and comment t= to plot
% behaviour
q=1;
t=2.2*10^-3; %This is aprox position of first zero crossing to start off NR
%method
Theta=00;
r=Theta;
Charge_0=0;
newton=0;
% finding constants G and H
G=((3.566*10^-6)*cos(r))-((0.006208)*sin(r))+Charge_0
H=((5.55555*G/2484.5)-((0.01120/2484.5)*sin(r))-((1.950/2484.5)*cos(r)))
current=((0.06208*2*3.14159*50)*cos(r+2*3.14159*50*t))-(((-3.566*10^-6)*2*3.14159*50)*sin(r+2*3.14159*50*t))-5.555556*(exp(-5.55555*t)).*...
(G*cos(2484.5*t)+H*sin(2484.5*t))+(exp(-5.555556*t)).*(-2484.5*G*sin(2484.5*t)+2484.5*H*cos(2484.5*t));
rate_of_change_of_i=((+0.3520)*cos(r+2*3.14159*50*t))-(612.7)*sin(r+2*3.14159*50*t))-(11.1111.*((exp(-5.555556*t)))).*...
(-2484.5*G*sin(2484.5*t)+2484.5*H*cos(2484.5*t))+(exp(-5.555556*t)).*(6.1727*10^6)*G*cos(2484.5*t)-(6.1727*10^6)*H*sin(2484.5*t)))+
+30.8642*exp(-5.555556*t)).*(G*cos(2484.5*t)+H*sin(2484.5*t));
if q==1
for y = 1:15,
  %y
  t=t-newton
  current=((0.06208*2*3.14159*50)*cos(r+2*3.14159*50*t))-((-3.566*10^-6)*2*3.14159*50)*sin(r+2*3.14159*50*t))-5.555556*(exp(-5.55555*t)).*...
  (G*cos(2484.5*t)+H*sin(2484.5*t))+(exp(-5.555556*t)).*(-2484.5*G*sin(2484.5*t)+2484.5*H*cos(2484.5*t));
  rate_of_change_of_i=((+0.3520)*cos(r+2*3.14159*50*t))-(612.7)*sin(r+2*3.14159*50*t))-(11.1111.*((exp(-5.555556*t)))).*...
  (-2484.5*G*sin(2484.5*t)+2484.5*H*cos(2484.5*t))+(exp(-5.555556*t)).*(6.1727*10^6)*G*cos(2484.5*t)-(6.1727*10^6)*H*sin(2484.5*t)))+
  +30.8642*exp(-5.555556*t)).*(G*cos(2484.5*t)+H*sin(2484.5*t));
  newton=current/rate_of_change_of_i;
  %newton
  %current
  %rate_of_change_of_i
end
end
if q==1
charge=((3.566*10^-6)*cos(r+2*3.14159*50*t))+((0.006208)*sin(r+2*3.14159*50*t))+(exp(-5.555556*t)).*...
  (G*cos(2484.5*t)+H*sin(2484.5*t));
cap_voltag=charge/18*10^-6
end
if q==0
max(current)
end
plot(t,rate_of_change_of_i)
**Appendix IV E  MATLAB Scripts to find maximum capacitor voltage**

% Use function MAXMIN=Vc (C, F,) where C is number of cycles at freq F
% to find the max and minimum values of the capacitor voltage Vc at each
% Starting phase for a fixed tx freq
% note: start Vc is zero.

C=100;
Low_phase=-180;
High_phase=180;
clear result_aray;
resolution_value=1; % number of points to use for each phase degree
result_aray=zeros((resolution_value*(High_phase-Low_phase)),3);
count=0;
F=117;% Frequency in Hz
for P=Low_phase:1/resolution_value:High_phase,
    count = count + 1 ;
    result_aray( count,1)=P;
    maxmin=Vc (C, F,P);
    result_aray(count,2)=maxmin(1);
    result_aray(count,3)=maxmin(2);
end
result_aray;
x=result_aray(:,1);
ymax=result_aray(:,2);
ymin=result_aray(:,3);
Range = result_aray(:,2)-result_aray(:,3);
plot (x,ymax)

**Function Vc called by previous script**

function MAXMIN=Vc (C, F,Phase,Ma,Mi)
na = 1:2*C;  % results are n values for c cycles
f = 50*pi/F;
clear norm_aray  % make array for results check
norm_aray=zeros(C*2,2);
P=Phase;
for i=na,
    n = na(i);
    v(i)=vcn_mod(n,f,P);
    % Final_Cap_Voltage=v(C)/(2*sqrt(2)*240);
    norm_aray(i,1)=i; %fill up results array for checking
    norm_aray(i,2)=v(i)/(240*2*sqrt(2));
end
norm_aray(1:C*2,2);
Ma=max (v);
Mi=min(v);
MAXMIN=[Ma,Mi];
%plot (na, v)
Function vc  n called by previous script

% This is formula for triggered pulses (n)
% Treatment is different according to if n is odd or even
%
% n          non-squared part        squared part
% 1          sum to 0                  sum to 0
% 2          sum to 1                  sum to 0
% 3          sum to 1                  sum to 1
% 4          sum to 2                  sum to 1
% 5          sum to 2                  sum to 2
% Etc.       Etc.
%
% That is, n odd, K = 0 to (n-1)/2
% n even means K = n/2 for non squared part and (n/2)-1 for
% squared part

function v = vcn_mod (n, f,P)

% n is number of gate pulses, f is gate pulse spacing and P is start phase
% angle in degrees
startphase=P*pi/180;%(pi/4);

Odd = n/2-floor(n/2);
if Odd > 0; %n is odd non-squared part is subtracted
    k=(n-1)/2;
    s0 = sin(k*f); % k is the value that the sequence of sines is summed to
    s1 = sin((k+1)*f);
    s0t = sin((k*f)+startphase);
    s1t = sin(((k+1)*f)+startphase);
    v = 2*sqrt(2)*240*(s1*(s1t-s0t)/sin(f));
else        %n is even squared part is subtracted and summed to one less than other
    k=n/2;
    s0 = sin(k*f); % k is the value that the sequence of sines is summed to
    s0t = sin((k*f)+startphase);
    s1 = sin((k+1)*f);
    v = 2*sqrt(2)*240*(s0t*(s1-s0)/sin(f));
end
Appendix V A  

VHDL listing of timing signal generator in CPLD

```vhdl
library ieee;
use ieee.std_logic_1164.all;
use ieee.std_logic_unsigned.all;
use ieee.std_logic_arith.all;

entity fiftyKHZlocked is
  port (Board_clk : in std_logic;
         display_std_logic_vector(1 downto 0); used in de-bugging
         display_std_logic_vector(7 downto 0); used in de-bugging
         external_one_Hz in std_logic ; -- input on pin 33 from GPG
         reset: in std_logic; -- this synchronizes output to GPG excess
         real_fiftyout std_logic-- Reference frequency output
         );
end entity fiftyKHZlocked;

architecture fifty of fiftyKHZlocked is
begin
  -- Component Declarations
  COMPONENT clk
    port (Board_clk : in std_logic;
         display_std_logic_vector(7 downto 0);
         display_std_logic_vector(7 downto 0);
         external_one_Hz in std_logic ; -- input on pin 33 from GPG
         reset: in std_logic;
         error_rate: in std_logic_vector(10 downto 0);
         start_tonpulse : in std_logic;
         re_sync : in std_logic;
         disp : in std_logic_vector(0 to 7)-- feeds through to display component
         );
  end COMPONENT;

  signal error_rate:std_logic_vector(10 downto 0);
  signal error_signal:std_logic_vector(7 downto 0);
  signal display_count:std_logic_vector(8 downto 0);
  signal tick:std_logic_vector(7 downto 0);
  signal tick_down_count:std_logic_vector(16 downto 0);
  signal max_count:std_logic_vector(8 downto 0);
  signal fifty_Khz:std_logic;
  signal display_std_logic_vector(7 downto 0);
  signal display:std_logic_vector(0 to 7);

  begin
    begin
      if error_rate < 0.1 then display_count <= '0';
      elsif Board_clk='1' then display <= '1';
      elsif start_tonpulse = '1' and re_sync = '1' then display <= '0';
      begin
        if display <= '0' then display <= '1' else display <= '0';
      end if;
    end process;

    -- this is the oscillator/tick process
    process (Board_clk) is
      variable fifty_Khz:std_logic;
      begin
        if Board_clk='1' then fifty_Khz <= '1';
        if display <= '1' then display_count <= display_count + '0' else display_count <= display_count - '0';
        display <= display_count;
      end if;
    end process;

    begin
      if error_rate < 0.1 then display <= '0';
      elsif Board_clk='1' then display <= '1';
      elsif start_tonpulse = '1' and re_sync = '1' then display_count <= display_count + '0';
      begin
        if display <= '0' then display <= '1' else display <= '0';
      end if;
    end process;

    -- output divider (temporary) this outputs a test tone for the oscilloscope
    process (fifty_Khz,reset) is
      begin
        if reset = '0' then display_count <= '00000000';
        elsif fifty_Khz='1' then display_count <= display_count + '0';
        end if;
      begin
        if display_count <= '00000000' then display_count <= display_count + '0';
        elsif display_count = '00000001' then display_count <= '00000000';
        end if;
      begin
        if display_count <= '00000000' then display_count <= display_count + '0';
        elsif display_count = '00000001' then display_count <= '00000000';
        end if;
      begin
        if display_count <= '00000000' then display_count <= display_count + '0';
        elsif display_count = '00000001' then display_count <= '00000000';
        end if;
      end if;
    end process;

    process (Board_clk,display,display2,external_one_Hz,reset,
             error Rate,Start_tonpulse,re_sync,disp) is
      begin
        if fifty_Khz='1' then display_count <= display_count + '0';
        elsif display_count = '00000001' then display_count <= '00000000';
        end if;
      end if;
    end process;
```

```
Appendix V B

VHDL listing of CPLD blocks ("clk_error" and "dispdot")

```vhdl
-- Appendices

VHDL listing of CPLD blocks ("clk_error" and "dispdot")

library ieee;
use ieee.std_logic_1164.all;
use ieee.std_logic_485.all;

entity clk_error is
    port ( A : in std_logic_vector(0 to 3);
           clk : in std_logic_vector(0 to 3);
           clk_error : out std_logic_vector(0 to 3)) ;
end;

architecture clk_error_arch of clk_error is
begin
    clk_error <= A xor clk;
end;

architecture dispdot_arch of dispdot is
begin
    dispdot <= (A => B, B => A, C => D, D => C);
end;

architecture dispdot_arch of dispdot is
begin
    dispdot <= (A => B, B => A, C => D, D => C);
end;
```
Appendix V C

VHDL listing of FPGA top block ("test_three")

Page 1 of 3

1  -- test_three_wpgrom  andrew mankie 23rd March 07
2  -- this is a combination of test two with the gps_time component to start at pre determined time
3  -- includes component core with lockin
4  -- similar to test_one except moving to next frequency is optionally automatic
5  -- by use of dip switch
6  
7  -- this steps through frequency list keeping o/p
8  -- locked using CPLL derived 50 MHz input for
9  -- phase measurements at co-go
10  
11  -- derived from test one 8th Feb 2007 A.Makie
12  
13  -- send uart input 24th July, send characters from pc, interfacing invert circuit required
14  -- to get 0 to 5 V input (1600 baud 8 data, no parity no flow cont.
15  
16  -- send "t" to reset (the i is for idle otherwise it stays reset)
17  
18  -- send "a" to start on next frequency.
19  
20  -- added an output to lockin so we can in at transmit end (gate_no_inversions 28th May 2007)
21  
22  
23  
24  
25  library ieee;
26  use ieee.std_logic_1164.all;
27  use ieee.std_logic_unsigned.all;
28  
29  entity test_three is
30  
31  port (  
32  ps1, ps2, ps3 : in std_logic;              -- general inputs and outputs
33  disp : in std_logic_vector(1 to 8);       -- switch 1 down sets to automatic freq sweep
34  display : out std_logic_vector(6 downto 0);  
35  l-dot, r-dot : out std_logic;
36  mains_out : inout std_logic;
37  t-out : out std_logic;                    -- feed to lockin amp
38  pc_train: out std_logic;                  -- feed to PC for record data from lockin
39  lock_in_back : in std_logic;              -- feed from lockin amp
40  uart : in std_logic;                      -- uart input for control by pc
41  g-data_in: in std_logic;                  -- time data input from gps goes to second uart in gps time comp
42  reset, ext : inout std_logic;
43  mains_in : in std_logic;                  
44  gate : out std_logic;                     -- output to transmitter
45  onepps : in std_logic;                    
46  t-watch : out std_logic;
47  t-min : out std_logic;
48  gate_no_inversions : out std_logic;
49  fifty_ms_in : in std_logic;                
50  end;                                     
51  
52  
53  architecture fred of test_three is
54  
55  -- component Declarations
56  
57  COMPONENT disp -- display for use when calibrating
58  port (  
59  A : in std_logic_vector(0 to 9);
60  Eout_std_logic_vector(6 downto 0));
61  END COMPONENT;
62  
63  COMPONENT frequency_list -- lists the set of frequencies to TX on
64  port (  
65  freq_step_count : in std_logic_vector (6 downto 0);
66  max_count : in out std_logic_vector (9 downto 0));
67  END COMPONENT;
68  
69  COMPONENT core_with_lockin -- sends and receives to/from lock in amp
70  port (  
71  Board_clk : in std_logic;
72  start_rec : in std_logic;
73  pc_train_rec : out std_logic; -- feed to PC for record data from lockin
74  lock_in_back : in std_logic;
75  t-out : out std_logic;
76  end;                                     
77  
78  COMPONENT sync_input_mod
79  port (  
80  Board_clk : in std_logic;
81  mains_in : in std_logic;
82  mains_out : out std_logic;
83  quartz_clk_out : out std_logic));
84  END COMPONENT;
85  
86  COMPONENT uart3 rx
87  port (  
88  clC5M, rxd : in std_logic;
89  display : out std_logic_vector (7 downto 0));
90  
91  COMPONENT gps
92  port (  
93  ps1, ps2, ps3 : in std_logic; -- general inputs and outputs
94  disp : in std_logic_vector(1 to 8); -- switch 1 down sets to automatic freq sweep
95  display : out std_logic_vector(6 downto 0);
96  l-dot, r-dot : out std_logic;
97  uart : in std_logic; -- uart input from GPS flex C53 ==pin228
98  mains_in : in std_logic; -- needed just for sync_input_end component
99  
100  COMPONENT sgl
101  port (  
102  signal display : std_logic_vector(0 to 7));
103  signal freq_step_count : std_logic_vector (6 downto 0);
104  signal quant_clk : std_logic; quant_clk : std_logic;
105  signal debounce_count : std_logic_vector (18 downto 0);
106  signal tfifty_mhz : hilo000000; disable : std_logic;
107  signal reset, gate, gate_temp, gate_temp2, gate_temp3, std_logic;
108  signal max_count gử, max_count_temp, max_count_top : std_logic_vector (9 downto 0);
109  signal fifty_mhz_now : std_logic; -- happens after sgl has reset
110  signal fifty_mhz_now_synchronously : std_logic;
111  signal uart : std_logic_vector (7 downto 0); -- UART received
112  signal d_t, t, t_en : std_logic_vector (11 downto 0); -- keeps track of time spent at each freq when in auto mode
113  signal auto_freq_counter : std_logic_vector (11 downto 0); -- keeps track of time spent at each freq when in auto mode
114  signal auto_freq_counter, std_logic_vector (11 downto 0); -- keeps track of time spent at each freq when in auto mode
115  signal d_t, t, t_en : std_logic_vector (11 downto 0); -- keeps track of time spent at each freq when in auto mode
116  signal disable_lp : std_logic_vector (19 downto 0)); -- part of debounce

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VHDL listing of FPGA top block ("test_three") Page 2 of 3

```vhdl
117 signal send: std_logic;
118 signal clkdiv: std_logic_vector (10 downto 0);
119 signal start_rec_time_match: std_logic;
120 signal display1,display2,display3,display4: std_logic_vector(9 downto 0);
121 begin
122 begin
123 -- this process records amplitude freq and phase at the last two seconds of each test
124 process (start_clk) is -- get onepps synced up with internal clock
125 begin
126 if start_clk'event and start_clk = '1' then
127 if (start_rec_time_match = '0' and (auto_step_counter = "00000101" or auto_step_counter = "00001110") then
128 start_rec='0': -- active low, this starts recording of measurements process
129 else if (start_rec = '0') and (auto_step_counter = "00011101" or auto_step_counter = "00011110") then
130 start_rec='1': -- active low, this starts recording of measurements process
131 end if;
132 else
133 start_rec='1';
134 end if;
135 end process;
136
137 gate_no_inversion<=>gate_temp: -- an output to lockin so we can rx at transmit end
138
139 -- mains_out<=>a;
140 display<=>freq_step_counter;
141 button<=>"[p2]
142 r_dot<=>time_match;
143 t_match<=>time_match;
144 gate<=>gate_temp2 or fiftykhz_now_gone or disabled: -- output b while acquiring lock
145 auto_enable<=>not dip[1]: -- auto enable is high when dip switch is down
146
147 process (dip[2]): is -- multiplexers to switch to time match for start
148 begin
149 if dip[2] = '1' then
150 reset <= '0';
151 else
152 reset <= not time_match: -- resets when the time match
153 end if;
154 end process;
155
156 process (dip[3]): is -- multiplexers to switch display to gps/set time or normal
157 begin
158 if dip[3] = '1' then
159 display<=display1;
160 else
161 display<=display2 or display3 or display4;
162 end if;
163 end process;
164
165 process (start_clk): is -- get onepps synced up with internal clock
166 begin
167 if start_clk'event and start_clk = '1' then
168 onepps_sync<=onepps;
169 end if;
170 end process;
171
172 process (start_clk) is -- this is copies from cpid where the signal is external on nz
173 begin
174 if start_clk'event and start_clk = '1' then
175 onepps_sync<=onepps;
176 end if;
177 end process;
178
179 process (start_clk) is -- for external one pulse per second
180 -- This is not a conventional debother but is designed to look for pulses
181 begin
182 if start_clk'event and start_clk = '0' then
183 if (onepps_sync = '0' and debounce_count1 = '0000000000000000') then
184 debounce_count1<=debounce_count1+1;
185 elsif (onepps_sync = '1' and debounce_count2 < '1111111111111111'and disable_ip ="00000000000000000000000000000000") then
186 debounce_count2<=debounce_count2+1;
187 end if;
188 end if;
189 end process;
190
191 process (start_clk) is -- this process produces a start_timing pulse
192 begin
193 if (start_clk'event and start_clk = '1') then
194 if (debounce_count1 = "1111111111111111" and disable_ip ="00000000000000000000000000000000") then
195 start_timing_pulse <= '0';
196 else start_timing_pulse <= '1';
197 end if;
198 end if;
199
200 process (start_clk) is -- test only
201 begin
202 if start_clk'event and start_clk = '1' then
203 if disable_ip = "00000000000000000000000000000000" then
204 disable_ip<=disable_ip+1;
205 and end if;
206 end if;
207 end process;
208
209 process (start_clk) is -- invert phase of gate signal at every mains Hz
210 begin
211 if start_clk'event and start_clk = '1' then
212 if disable_ip = "00000000000000000000000000000000" then
213 end if;
214 end if;
215 end process;
216
217 process (start_clk) is -- invert phase of gate signal at every mains Hz
218 begin
219 end if;
220 end process;
221
222 process (start_clk) is -- invert phase of gate signal at every mains Hz
223 begin
224 end if;
225 end process;
226```
if qart_cik'event and qart_cik = '1' then
begin
  if raiseout = '1' then gate_temp:=gate_temp;
  else gate_temp:=not gate_temp;
  end if;
end if;
end process:
end if;
end process:

-- Turn 50 MHz square wave input into 10MHz
process (qart_click) is begin
if qart_click'event and qart_clk = '1' then
  if (oldา skipped_3 = '1') and (fifty acknowledgment < 1) then -- look for edges
    if (oldа skipped_3 = '1') and (oldа skipped_3 = '1') then
      hundred=0;
    else hundred=0;
    end if;
    end if;
end process:
end if;
end process:
begin
begin
-- debounce button on lpi input
process (qart_click) is begin
if qart_click'event and qart_clk = '1' then
  if oldа button = '1' and button = '0' then;
    if (from_pc = 0')11011' then -- 'a' from UART eq to button press
      debounce_count:=0111111111111111111;
    else debounce_count:=0111111111111111111;
    end if;
  end if;
end if;
end process:
end if;
end if;
end process:
end if;
-- generating external reset for gpi
process (qart_click, reset, from_pc, start_timing_pulse) is begin
if from_pc = '01010101' or reset = '1' then reset_ent := '0';
  auto_step_counter := '00000000';
  start_timing_pulse := '1';
  if (auto-enable = '0') and (debounce_count = '0000000000000000000') or (cmp5 = '1') and (auto-enable = '1') and auto_step_counter = '00000010' and at
  start_timing_pulse = '1' then
    reset_ent := '0';
    -- causes a move to next frequency
  auto_step_counter := '00000010';
  else reset_ent := '1';
  start_timing_pulse := '1' and auto_enable = '1' then
    auto_step_counter := '00000000';
    -- if auto_step_counter = '00011110' then -- after 10 seconds have elapsed
    auto_step_counter := '00011111';
  end if;
  end if;
end if;
end process:
end if;
end process:
end if;
-- increment freq step count when commanded
process (qart_click, reset, from_pc) is begin
if (auto-enable = '0') and (from_pc = '01101101' or reset = '1') or (auto-enable = '1' and reset = '0') then freq_step_count <= '00000000';
  if (auto-enable = '0' and debounce_count = '00000000000000000101') or (auto-enable = '1' and start_timing_pulse = '1' and auto_step_counter = '00110110') then
    freq_step_count <= '00000000';
    max_count_sigs <= '5377.97';
  else
    freq_step_count <= '00000000';
    max_count_sigs <= '5377.97';
    freq_step_count <= freq_step_count+1;
end if;
end if;
end process:
end if;
end if;
-- main output frequency to transmitter gate
process (qart_click) is begin
if (auto-enable = '0' and debounce_count = '00000000000000000101') or (auto-enable = '1' and auto_step_counter = '00101010' and start_timing_pulse = '1') then
  if (fiftySkipped_ack = '1') then
    hundred := hundred + 1;
    if hundred = '1' then
      hundred := hundred + 1;
      if hundred = '1' then
        hundred := hundred + 1;
      end if;
    end if;
  end if;
end if;
end process:
end if;
end if;
end process:
begin
begin
-- external reset for gpi
process (qart_click) is begin
if qart_click'event and qart_clk = '1' then
  if hundred = '1' then
    hundred := hundred + 1;
    if hundred = '1' then
      hundred := hundred + 1;
      if hundred = '1' then
        hundred := hundred + 1;
      end if;
    end if;
  end if;
end if;
end process:
end if;
end process:
begin
begin
-- external reset for gpi
process (qart_click) is begin
if qart_click'event and qart_clk = '1' then
  if hundred = '1' then
    hundred := hundred + 1;
    if hundred = '1' then
      hundred := hundred + 1;
      if hundred = '1' then
        hundred := hundred + 1;
      end if;
    end if;
  end if;
end if;
end process:
end if;
end process:
begin
begin
-- external reset for gpi
process (qart_click) is begin
if qart_click'event and qart_clk = '1' then
  if hundred = '1' then
    hundred := hundred + 1;
    if hundred = '1' then
      hundred := hundred + 1;
      if hundred = '1' then
        hundred := hundred + 1;
      end if;
    end if;
  end if;
end if;
end process:
end if;
Appendix V D  MATLAB Script to create Figure 5.3 transmit waveform

% to plot the transmit waveform

initial_phase_in_degs=0
Initial_Theta=initial_phase_in_degs*pi/180  \% in rads
r=(Initial_Theta);
transmit_FREQ=223
Firing_frequency = 2*transmit_FREQ ;
N = 10000; \% number of points
T = 1 \% length of sim seconds this has to be long enough
t = [0:N-1]/N; \% time interval matrix
t = t*T; \% time in seconds
mains_voltage = (2^.5)*240*sin(2*pi*50*t+r);
tx_sig=sin(2*pi*transmit_FREQ*t);
for z = 1:N,
  if (mains Voltage(z)>0)
    if (tx_sig(z)>0)
      voltage(z)=mains_voltage(z);
    else
      voltage(z)=0;
    end
  else
    if (tx_sig(z)>0)
      voltage(z)=0;
    else
      voltage(z)=mains_voltage(z);
    end
  end
end
y=fft(voltage);
Py=(2^.5)*(((y.*conj(y)).^).5)/N;
Py=Py(1:N/2);
freq = [0:N/2-1]/T; \% find the corresponding frequency in Hz
subplot(2,1,1);plot(t(1:201),voltage(1:201))
axis([0 .02 -550 550])
subplot(2,1,2);plot(freq,Py); \% plot PSD
[Peaks IFreqs] = sort(-Py);  
peaks=abs(Peaks(1:7));
RMS_Voltages=(peaks) \% 
axis([0 300 .02 150])
Appendix V E  MATLAB script modification used to create Figure 5.4

This Appendix shows the substitution of code into that given in Appendix V D to ensure signal is inverted at each mains zero crossing.

REPLACE

for z = 1:N,
    if (mains_voltage(z)>0)
        if (tx_sig(z)>0)
            voltage(z)=mains_voltage(z);
        else
            voltage(z)=0;
        end
    else
        if (tx_sig(z)>0)
            voltage(z)=0;
        else
            voltage(z)=mains_voltage(z);
        end
    end
end

WITH

for z = 1:N,
    if (mains_voltage(z)>0)
        if (tx_sig(z)>0)
            voltage(z)=0;
        else
            voltage(z)=mains_voltage(z);
        end
    else
        if (tx_sig(z)>0)
            voltage(z)=mains_voltage(z);
        else
            voltage(z)=0;
        end
    end
end
### Appendix V F  Main frequency list

<table>
<thead>
<tr>
<th>No.</th>
<th>Frequency (Hz)</th>
<th>No.</th>
<th>Frequency (Hz)</th>
<th>No.</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>104.279</td>
<td>2</td>
<td>109.537</td>
<td>3</td>
<td>119.782</td>
</tr>
<tr>
<td>4</td>
<td>129.739</td>
<td>5</td>
<td>139.915</td>
<td>6</td>
<td>164.311</td>
</tr>
<tr>
<td>7</td>
<td>174.652</td>
<td>8</td>
<td>184.317</td>
<td>9</td>
<td>194.359</td>
</tr>
<tr>
<td>10</td>
<td>204.716</td>
<td>11</td>
<td>215.301</td>
<td>12</td>
<td>225.005</td>
</tr>
<tr>
<td>13</td>
<td>235.611</td>
<td>14</td>
<td>241.304</td>
<td>15</td>
<td>271.471</td>
</tr>
<tr>
<td>16</td>
<td>282.203</td>
<td>17</td>
<td>292.109</td>
<td>18</td>
<td>302.724</td>
</tr>
<tr>
<td>19</td>
<td>312.19</td>
<td>20</td>
<td>322.258</td>
<td>21</td>
<td>332.993</td>
</tr>
<tr>
<td>22</td>
<td>342.125</td>
<td>23</td>
<td>346.878</td>
<td>24</td>
<td>361.954</td>
</tr>
<tr>
<td>25</td>
<td>372.754</td>
<td>26</td>
<td>381.294</td>
<td>27</td>
<td>393.31</td>
</tr>
<tr>
<td>28</td>
<td>406.091</td>
<td>29</td>
<td>416.241</td>
<td>30</td>
<td>419.75</td>
</tr>
<tr>
<td>31</td>
<td>426.922</td>
<td>32</td>
<td>438.156</td>
<td>33</td>
<td>445.986</td>
</tr>
<tr>
<td>34</td>
<td>462.501</td>
<td>35</td>
<td>471.23</td>
<td>36</td>
<td>484.951</td>
</tr>
<tr>
<td>37</td>
<td>494.55</td>
<td>38</td>
<td>504.547</td>
<td>39</td>
<td>509.697</td>
</tr>
<tr>
<td>40</td>
<td>514.95</td>
<td>41</td>
<td>525.787</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Repeats:
Appendix V G Circuit diagram of transmitter

Simplified circuit diagram of the transmitter is shown in Figure A5.7.1 and the IGBT drive circuit is shown in Figure A5.7.2.

**Figure A5.7.1** Switched resistive load transmitter circuit.

**Figure A5.7.2** IGBT gate drive detail.
Appendix V II  Measurements to demonstrate linearity

These measurements were taken at the Thomas Cherry substation.

Data taken from lab log book 9 page 38  10th May 2008

<table>
<thead>
<tr>
<th>Meas. No./ frequency</th>
<th>Level at lock-in Red phase</th>
<th>Time Constant (TC = 1 Second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/312.1Hz</td>
<td>0.703mV</td>
<td></td>
</tr>
<tr>
<td>2/624.2Hz</td>
<td>- undetectable above noise</td>
<td></td>
</tr>
<tr>
<td>5/936.3Hz</td>
<td>0.100 mV</td>
<td></td>
</tr>
<tr>
<td>6/1248.4Hz</td>
<td>- undetectable above noise</td>
<td></td>
</tr>
<tr>
<td>7/362.1Hz</td>
<td>- undetectable above noise</td>
<td></td>
</tr>
<tr>
<td>8/262.1Hz</td>
<td>- undetectable above noise</td>
<td></td>
</tr>
</tbody>
</table>

The water cooled transmitter was used at full load to transmit a signal at 312.1Hz. It will also transmit a third harmonic at 936.3Hz at a level of about 1/3rd of the fundamental. The received signals are shown in the table above. The third harmonic has undergone more attenuation than the fundamental as expected.

Measurements were conducted at second and fourth harmonics of the main transmitted frequency and 50Hz above and 50Hz below the transmitted fundamental. If the channel had exhibited significant non-linearity we would have expected products at the even harmonics and at the 50Hz offset frequencies due to the very large 50Hz signal mixing with our transmitted signal at these frequencies.

Measuring at the even harmonics an experiment was repeated 96 times, half of the time the transmitter was on and the other half it was off. The results were averaged to reduce noise. The result was no detectable difference with the transmitter on at the expected modulation product frequencies of 624.2 and 1248.4 Hz.

The measurements at the offset frequencies were then made 96 times with the transmitter alternating on and off. The averaged level for the transmitter on case was 5% higher than for the transmitter off. This may be statistically insignificant or it may be evidence of very slight non-linearity. The mains signal being used to produce the mixing products is enormous compared to any signal frequency so we can state that any non-linearity is negligible.
Appendix V  I  Measurements to calibrate out system delay

These measurements were taken in the Office by re-locating the receiver next to the transmitter so that the delay of only the measurement system could be determined. The transmitter was swept across 18 frequencies within the band of interest below 3kHz. The phase of the current injected into the mains socket was determined at each frequency, the phase being measured relative to the transmitter (and receiver's) reference oscillator. The current transformer circuit to pick the signal off the mains line was identical to that installed in the receiver locations but with the addition of an attenuator. The graph of the measured results is given in Figure A5.9.1 below:

![Calibration of Transmitter Delay](image)

**Figure A5.9.1** Calibration of transmitter delay

The average slope of this graph is 10.7 degrees per kHz giving the delay as 29.7 us.
Appendix VI A  Three MATLAB scripts used to create and process noise samples

Script 1
% Matlab code
% looking at products of two distributions
% uniform for this example
% A.Mackie Dec 2008
a = rand( 1, 1e7 );
b=rand( 1, 1e7 );
y=a.*b;
pdfplot( y,10000 ); % using third party Matlab function
% written by Alex Bur-Guy, Sept 2003

Script 2
% Matlab code
% creating sum of three uniform distributions
% A.Mackie Dec 2008
a = rand( 1, 2e7 );
b=rand( 1, 2e7 );
c=rand( 1, 2e7 );
y=a+b+c;
w=[y'];
w=w(1:333); % select first 333 sample values
save pdfresuly2.xls w -ascii;

Script 3
% Matlab code
% calculating bin populations fo Chi Square
% A.Mackie Dec 2008
bin=3; % manually change this for each bin number 0-13
lower = bin*3/14; % 3 is the data range
upper = (bin+1)*3/14;
z=0;
clear q;
q=[];
clear w;
w=pdfresuly(1:333);
for i=1:length(w)
    if ((lower)< w(i) )
        if (w(i)<(upper))
            z=z+1; % count number in this bin
            q=[q ; w(i)]; % check all values counted
        end
    end
end
z  % output final count for this bin
Appendices _____________________________________________________________
Appendix VI B Data set used for testing Normality test techniques

Table of 333 value data set used for testing Normality test techniques
1.6735095
2.0613026
0.3945183
1.9008407
1.3784616
1.5745204
1.0264945
1.3837766
1.5427327
1.2263426
1.3979710
0.6925840
1.8219101
1.4720194
1.6209121
2.1596267
1.0870111
2.2444393
1.4264816
1.6492779
1.0494874
2.3467054
0.8427099
1.2903181
2.0678349
1.5976837
1.9556814
0.7931792
0.9493003
1.3575884
0.9151413
1.6300448
2.3562302
1.2786499
1.1391899
1.8036645
1.6912108
0.6691866
1.5334166
1.2980522
0.4761587
1.9158867
1.3148163
1.9954971
1.3000028
0.8306196
0.8758315
0.9987209
1.0611311
0.7855842
0.6197020
1.3800751
1.7588853
0.9968661
1.6024181

322

1.2458733
2.5990806
1.4389941
1.8234590
1.5082368
1.2925338
2.1294569
1.8703495
1.3892352
0.6957962
2.1590254
2.1326912
1.2078055
2.1176489
1.7807177
0.5400058
1.0491417
0.9989921
1.7042289
1.8808453
1.0439999
0.5770877
1.5440674
1.3154549
1.7348709
2.1235972
0.7248009
1.2678989
0.8451472
2.0993325
2.0768800
1.5550940
1.3919098
2.0665671
1.7466948
2.1201869
1.5407328
1.7398730
2.2930003
1.9877176
1.5540951
1.5433778
2.5048440
1.3351788
1.4915783
1.6720284
1.9919834
0.6213911
1.3058975
1.4569332
1.5428809
0.3480775
1.1987670
2.1093793
1.4874833
1.2855290

1.5001070
1.7494223
1.2150589
1.6248669
0.8140528
1.0565080
1.3823805
1.3616385
0.5730566
1.2150218
2.4592870
2.0776986
1.5439475
1.9135821
1.4996438
2.0567321
1.3175075
2.0167848
1.3144384
1.4141408
1.3889045
1.3556291
1.8061095
1.7660077
1.3141177
1.3369784
1.4082412
1.9675865
2.2937158
1.1172246
1.9431928
1.0446737
1.1292703
1.1407879
2.7058020
0.5233639
2.2143723
1.8348036
1.2900582
0.5061438
1.3205209
2.1158213
2.1000672
1.7061126
1.5523549
1.5245346
1.7072817
1.3459863
1.0686409
1.8513280
1.6609777
0.8682165
2.1034081
1.3407673
2.1365869

1.3957832
0.5460673
0.8523130
1.4964531
1.6100546
1.7566043
1.6879118
2.0333046
2.0293487
1.3102234
2.0970393
0.9468211
1.5983344
2.0167172
2.6340237
0.9244932
1.9627275
1.8831347
1.6209729
1.9991480
0.7724013
1.5648039
2.2276346
1.8340980
1.6295672
2.1337003
1.1247686
1.2326923
1.5049226
0.7507506
0.8599896
1.8179953
2.0744768
0.9463537
1.2934774
1.4939547
2.1388551
1.4084894
1.4257367
0.6426465
1.0318179
2.1874936
1.7540196
2.2469309
1.0289317
1.8375670
1.2959526
0.7539085
2.0976740
2.2519895
1.8347478
2.0013408
1.8141441
1.1593556
2.0404141
1.1121136

1.5968354
1.1578041
0.4443523
2.2086225
2.2900710
1.5301866
1.7522706
2.4957187
0.4104374
2.3093473
1.2819070
2.1226591
1.5354546
0.8811619
1.7214789
1.3008433
1.0187148
1.5422503
0.8476738
0.7950110
1.1818429
0.4591198
1.5650151
1.0092423
0.5720313
2.5497078
1.3485963
1.6625531
1.8332675
1.4856963
0.7198553
2.0559835
2.0790234
2.0799095
2.2236396
1.9651388
1.3279536
1.4261042
2.2770107
1.4313771
2.2969093
0.8621419
2.1017509
1.6744182
0.6547020
0.6383868
1.6222248
1.9687898
1.9184626
1.6637177
1.7759140
0.7829412
2.2523454
1.8478621
1.6046338

2.2841964
2.0203222
0.9306443
0.8858115
1.2815870
1.1475112
1.7679479
1.4267618
1.5345557
1.5214221
1.3394097
1.8127717
1.6545711
1.2358731
1.1843320
1.8396223
1.3808799
0.6499558
1.0422726
2.3612139
0.8982374
1.4504437
2.3349282
1.4291466
0.4784861
1.5708654
1.9996456
0.6734101
2.4229885
0.7452645
1.2561856
2.4239301
2.5862752
1.2201121
1.7141690
0.6468060
1.5939938
1.4040592
1.3688920
1.5023365
0.8095982
1.8854406
0.3760023
1.5183940
1.0974946
0.5563285
1.5339433
1.6941589
1.0077547
2.0758679
2.4491230
1.2334446
1.9043473
1.5938908
0.7830789
1.8338775


Appendix VI C  Full quote from EasyFit help file

“The Anderson-Darling test implemented in EasyFit uses the same critical values for all distributions. These values are calculated using the approximation formula, and depend on sample size only. Although this kind of test (compared to the "original" A-D test) is more likely to accept the bad fit, it can be successfully used to compare the goodness of fit of several fitted distributions. “ From EasyFit’s Help file[63].

Appendix VII A  MATLAB script for Butterworth pre-filter

```matlab
% Output envelope expected by bandpass filtering
% A.Mackie Jan 2008
n=100000; % number of samples, w is input
w=pdfresuly;
Wn=[399.4 400.6]/5000; % sample rate is 10KHz
[B,A]=butter(3,Wn) % 400Hz 3rd order filter
f=filter(B,A,w);
plot(f(1:40000))
save init_filt.txt f -ascii;
```

Appendix VII B  MATLAB script for filtering and decimation

```matlab
% MATLAB code Script 4
% Digital low-pass filtering of sample data
% A.Mackie Dec 2008
z=[];
t=2; % 1 + number of zeros
N=2500*t; % length of test output signal is 5000
x =pdfresuly(1:N); % always use the "same" random data set for sample set 1
B = ones(t,1); % transfer function numerator
A = 1; % transfer function denominator
y = filter(B,A,x)/t;
% now decimate (used in some experiments)
for i=1:(N)
    q=rem(i,t);
    if q==0
        z=[z ; y(i)];
    end
end
save pdf_filtered_two.txt y -ascii; % un-decimated data set
save pdf_filtered_two_dec.txt z -ascii; % decimated data set
```
Appendix VII C  MATLAB script for simulating Lock-in Amplifier

% MATLAB code script 5
% correlation as in lockin amplifier, newlockin_ap3
% A. Mackie Dec 2008
n=10000000; % number of samples, w is input
w=pdfresy (after importing)
w=pdfresuly3; % three uniforms or
w = rand(1, 10e6)';
Wn=[1.33]/5000; % sample rate is 10KHz cut off is about 1Hz

[B,A]=butter(3, Wn) % 400Hz 3rd order filter
%[B,A]=cheby2(3,100,44/5000)
s= sin((2*pi/25)*[1:n]'); % use 25 sample representation of sine wave
c= cos((2*pi/25)*[1:n]'); % one cycle in 25/10000 sec is 400 Hz

% save pdfresuly3.txt w -ascii; this is 10 million data set file
%x = pdfresuly(1:n); % always use the "same" random data set
x = DC_in(1:100000);
y = filter(B,A,x); % low-pass filter response to step
figure(1)
plot(y) % step response
axis([0 10000 .00 .7])% 10,000 is 1 second

f=(abs(freqz(B,A,50000)));
figure(2)
plot(f) % freq response
axis([0 100 .001 1]) % 50,000 = 5kHz so 100 is 10Hz

cor_s=s.*w; % correlate random input with sine
% cor_c=c.*w;
figure(3)
P = filter(B,A,cor_s); % low-pass filter to give correlator output

xp=reshape (P,5000,n/5000); % 5000X2000 matrix

decimated_P=xp(5000,:); % the last column

plot (decimated_P)
save pdf_low_pass_P.txt decimated_P -ascii
Appendix VII D  MATLAB script for bandpass filtering

% MATLAB code script 6
% band-pass filtering, newbpfilt.m
% A.Mackie Dec 2008
n=100000;%number of samples, w is input
Wn=[378 422]/5000; %sample rate is 10KHz band is around 400Hz
[B,A] = butter(3,Wn)
[B,A] =cheby2(3,100,Wn)%400Hz 3rd order filter
s= sin((2*pi/25)*[1:n]');% use 25 sample representation of sine wave
c= cos((2*pi/25)*[1:n]');
% save pdfresuly3.txt w -ascii; this is 10 million data set file
%x =pdfresuly(1:n); % always use the "same" random data set
x =w(1:n);
y = filter(B,A,x);  %band-pass filter
plot (y)
%axis([0 10000 -.025 .025])

cor_s=s.*y;  % correlate with sine
cor_c=c.*y;

xsr=reshape (cor_s,25,n/25); % puts the vector which is the multiplication of sine with
% BP output into matrix with columns corresponding to
% each sine cycle (25 samples long)
S_sum = sum(xsr)'; % sums each column and makes a column vecor

xcr=reshape (cor_c,25,n/25); % puts the vector which is the multiplication of sine with
% BP output into matrix with columns corresponding to
% each sine cycle (25 samples long)
C_sum = sum(xcr)'; % sums each column and makes a column vecor
save pdf_band_pass_cor_s.txt S_sum -ascii
save pdf_band_pass_cor_c.txt C_sum -ascii
%fvttool(B,A)
%freqz(B,A,(128*2^10),100000)

% now decimate (used in some experiments)

%for i=1:(N)
% q=rem(i,t);
% if q==0
%  z=[z ; y(i)];
%end
%end

%length (z)

%save pdf_first16000.txt y -ascii;
Appendix VII E    MATLAB script to calculate ENBW

% MATLAB code script 7
% filter's ENBW, myfilter_lp.m
% A. Mackie Dec 2008
%
Power_int=0;

Wn=[1.0]/5000; % sample rate is 10KHz band is around 400Hz
[B,A]=butter(3,Wn);
f=(abs(freqz(B,A,500000))); % scaled up by 100 times
figure(1)
plot(f)
axis([0 100 .001 1])  % 100 is 1Hz

% calculate power in pass band to obtain ENBW

for i=1:500000
    Power_int=Power_int+((f(i))^2);
end

Power_int/100  % this gives ENBW because scale is 100 points to 1Hz
Appendix VII F  Small sample size, no data discarded

Below are some examples of filtering independent data points with rolling average filters. Each column presents results for a different 5000 point sample data set filtered with a different number of zeros in the manner described in chapter V section 2.3. For the different filtering stages the number of zeros is used as a label for convenience. The two zero filter sums three adjacent samples and the three zero filter sums four samples. Each sample in the sums is given the same weighting. To keep the gain unity, the output sums would have to have been divided by three and four respectively, but this makes no difference to the fit of the output distribution. The numbers shown in the table are the $A^2$ test statistic with a smaller number indicating a better fit to Gaussian.

<table>
<thead>
<tr>
<th>filter</th>
<th>sample set 1</th>
<th>sample set 2</th>
<th>sample set 3</th>
<th>sample set 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>no filter</td>
<td>2.4</td>
<td>1.8</td>
<td>3.1</td>
<td>2.3</td>
</tr>
<tr>
<td>one zero</td>
<td>0.98</td>
<td>0.41</td>
<td>0.71</td>
<td>1.0</td>
</tr>
<tr>
<td>two zeros</td>
<td>1.0</td>
<td>0.38</td>
<td>0.37</td>
<td>0.67</td>
</tr>
<tr>
<td>three zeros</td>
<td>1.0</td>
<td>0.81</td>
<td>0.48</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Table A7.6.1 No consistent improvement in fit if data is not discarded with small sample size.

The table above shows there is no consistent improvement in fit to Gaussian by higher order, lower turnover filtering (5000 points) if no data is discarded.

It is surprising that summing three or four (or more) independent samples does not consistently result in a better fit than summing just two samples in a single zero filter, yet summing just two samples always results in a much better fit than the original data.
Appendix VII G  Large sample size, no data discarded

Table A7.7.1 reports results from the same experiment as was performed and reported in appendix A1 but using data sets of 100,000 points. The fit before filtering is much worse than was the case with a 5000 point set because the larger number of points gives a lot more evidence of a non-Gaussian distribution.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Sample set 1</th>
<th>Sample set 2</th>
<th>Sample set 3</th>
<th>Sample set 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>no filter</td>
<td>36</td>
<td>31</td>
<td>34</td>
<td>31</td>
</tr>
<tr>
<td>one zero</td>
<td>6.9</td>
<td>4.3</td>
<td>6.2</td>
<td>9.2</td>
</tr>
<tr>
<td>two zeros</td>
<td>3.0</td>
<td>2.1</td>
<td>2.8</td>
<td>3.3</td>
</tr>
<tr>
<td>three zeros</td>
<td>2.1</td>
<td>1.4</td>
<td>1.6</td>
<td>2.0</td>
</tr>
<tr>
<td>four zeros</td>
<td>1.1</td>
<td>1.1</td>
<td>1.3</td>
<td>1.4</td>
</tr>
<tr>
<td>five zeros</td>
<td>1.1</td>
<td>1.4</td>
<td>0.9</td>
<td>0.7</td>
</tr>
</tbody>
</table>

**Table A7.7.1** Limited improvement in fit if data is not discarded with small sample size.

The table above shows there is limited improvement in fit to Gaussian by higher order, lower turnover filtering and no consistent improvement after four zeros for a large sample size (100,000 points) with no data discarded.
Appendix VII H  

**large sample size, decimation implemented**

Table A7.8.1 reports results from the same experiment as was performed and reported in appendix A2. The original data set of 100,000 statistically independent samples was first “pre-processed” with the two zero rolling average low pass filter. The resulting input data set represents a realistic band limited signal which has been sampled above the Nyquist rate appropriate for the spectral content of the signal. The experiments were repeated using this low-pass signal as the input to the filter and the results are given Table A7.8.1.

<table>
<thead>
<tr>
<th>Filter</th>
<th>No Decimation</th>
<th>With Decimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>no filtering</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>one zero</td>
<td>3.0</td>
<td>1.4</td>
</tr>
<tr>
<td>two zeros</td>
<td>2.8</td>
<td>1.5</td>
</tr>
<tr>
<td>three zeros</td>
<td>2.0</td>
<td>0.84</td>
</tr>
<tr>
<td>four zeros</td>
<td>1.6</td>
<td>0.49</td>
</tr>
<tr>
<td>five zeros</td>
<td>1.5</td>
<td>0.49</td>
</tr>
<tr>
<td>Six zeros</td>
<td>1.9</td>
<td>0.42</td>
</tr>
<tr>
<td>Seven zeros</td>
<td>2.2</td>
<td>0.35</td>
</tr>
<tr>
<td>Eight zeros</td>
<td>2.5</td>
<td>0.28</td>
</tr>
</tbody>
</table>

**Table A7.8.1** Showing effects of filtering on a signal with sample-to-sample correlation.

Note that without decimation there is limited improvement in the goodness of fit to Gaussian. We have started with an over sampled signal and with successive low-pass filtering stages the number of superfluous samples increases, masking the effect of the filtering which would otherwise make the signal appear more Gaussian. The right hand column (where re-sampling by decimation appropriate to the level of filtering has been applied) reveals the effect of the filtering. Although we have lost independence between the input samples for every result in this column and so can no longer explain the result in terms of the central limit theorem, the same effect as was found for independent samples occurs. If we reduce the number of samples which represent our signal as we reduce its bandwidth, the fit to Gaussian improves as more low-pass filtering is applied.
Appendix VII I  The Hilbert transform

In this thesis the Hilbert transform is used to demonstrate the relationship between the operation of a lock-in amplifier and the usual narrow band signal analysis.

Definition and application

The Hilbert transform \([79]\), is defined by the following equation where the “hat” symbol denotes the Hilbert transform operation:

\[
\hat{e}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{e(\tau)}{t - \tau} d\tau
\]

\(\tau\) being a dummy variable of integration.

The Hilbert transform is useful in band-pass signal analysis since it retains all the information in a signal (except DC) and all signals on the mains are, strictly speaking, band-pass because there is no DC component and there is always a high frequency limit. As discussed in section 3.4 the Hilbert transform can be used to construct an “analytic signal” or “pre-envelope” which is very useful in Digital Signal Processing communications systems because it can be used to eliminate negative frequencies (from a real signal). Unfettered down conversion then becomes possible; there is no negative frequency band to cause mixing up of the sidebands in the baseband signal as illustrated in Figure 7.3 of section 3.4 of chapter VII. This means the baseband signal, which can be represented with lower sampling frequencies, is an unambiguous representation of the original signal. In other words, information modulated onto a high frequency carrier can be represented by using an exact equivalent low frequency complex baseband signal.
The effect of the transform in frequency and time domains
In the complex frequency domain, the Hilbert transform shifts all positive frequencies 90° clockwise (multiplies by $-j$) and all negative frequencies anticlockwise by 90°. The time domain equivalent for the Hilbert transform is convolution with $1/\pi t$ [79]. It is worth looking at a plot of this function of time. It is shown in Figure A7.9.1. below:

![Figure A7.9.1 Function $1/\pi t$.](image)

Convolution of this function with a sine wave produces a minus cosine waveform as shown in Figure A7.9.2.
For the present purposes it is sufficient to know that the Hilbert transform converts a sine term to a cosine term as shown in Figure A7.9.2 above, a cosine term to a sine term, a sine term to a cosine term and a cosine term to a sine term keeping all of the amplitudes, frequencies and any phase offsets unchanged. All of these relationships can be seen by using the definition of the transform given in the first sentence of this section, and by plotting the negative and positive (complex) frequency components of each real sinusoid on a complex plane. To summarise, the Hilbert transform process causes the real sinusoid components in a signal to shift 90° backwards in time.

**Figure A7.9.2** Showing convolution of Sine with $1/\pi t$ producing minus Cosine.
Appendix VIII A    MATLAB script to prepare data for FDIDENT

% to create object to use in fdtool
% This one for use with data from the boilerhouse (or any other .xls file)
% import magnitude data and phase data and freqs from excel files stored
% as txt in "work"
clear object;
freqv=Sheet1(:,1);
phasev=Sheet1(:,2);
amplitv=Sheet1(:,3);
phase_list_fident_inv=phasev; % if I invert I get good model with zero in RHS
phasemod=11*(freqv/1000); % next do 11 degs/khz delay adjustment
phasedata3=phasev+phasemod; % plus used because phases were minus
reim=exp(log(amplitv)+j*(phasedata3*pi/180));

% or this I got from help file
%:Y=10.^(tf_mag_dB/20).*exp(j*tf_phase_degrees/360*2*pi);
% thats for mag in dB and phase in degrees

subplot(2,1,2);semilogx(freqv,angle(reim)*180/pi),hold on
subplot(2,1,1);loglog(freqv,abs(reim))
%semilogx(freq_list_fident,(angle(reim)*180/pi)),grid on
%axis([1e2 1e4 -200 180])

length_freq=length(reim);
u([1:length_freq])=1;
ut=transpose(u);

object=fiddata(reim,ut,freqv);
clear u, clear reim, clear phasemod, clear ut, clear phasedata3;
clear length_freq, clear phase_list_fident_inv;
Appendix VIII B Results of frequency domain identification of office channel

Results from FDIDENT on data below 2200Hz (not including repeats) in figure 2 and 3 from chapter 8 for a four pole, five zero model.

Order: 4/5
Domain: s

Zeros (with positions also given in Hz):

- $+2.8825e+004 \text{ radHz}$
- $+4.4344e+003 \text{j radHz}$ ($+705.75 \pm 98.317 \text{ Hz}$)
- $-9.8461e+002 - 4.4344e+003 \text{j radHz}$
- $-7.2905e+002 \text{ radHz}$

Poles (with positions also given in Hz):

- $-5.6276e+003 \text{ radHz}$
- $-1.5126e+003 + 3.2024e+003 \text{j radHz}$ ($+509.68 \pm 211.73 \text{ Hz}$)
- $-1.5126e+003 - 3.2024e+003 \text{j radHz}$
- $-1.3409e+003 + 2.1023e+003 \text{j radHz}$ ($+334.59 \pm 222.39 \text{ Hz}$)
- $-1.3409e+003 - 2.1023e+003 \text{j radHz}$

Figure A8.2.1 Maximum Likelihood 4 zero 5 pole model of office channel.
Appendix VIII C  Results of identification of office channel with origin zero

Same model as appendix 2 but with zero at origin forced

Order: 4/5
Domain: s

Zeros (with positions also given in Hz):
+0.0000e+000 radHz
+3.2082e+004 radHz
-1.2730e+003 +4.9867e+003*j radHz (+793.66 ± 109.58 Hz)
-1.2730e+003 -4.9867e+003*j radHz

Poles (with positions also given in Hz):
-2.7951e+003 +5.8677e+003*j radHz (+933.87 ± 262.47 Hz)
-2.7951e+003 -5.8677e+003*j radHz
-8.2418e+002 +2.5530e+003*j radHz (+406.33 ± 24.521 Hz)
-8.2418e+002 -2.5530e+003*j radHz
-4.3367e+002 radHz

Figure A8.3.1 Model of office channel with zero at origin.
Appendix VIII D  Six zero five pole models of Bakery, Pavilion, Union building, Boiler house, Office and Room 127A

Bakery

Order: 6/5
Domain: s

Zeros (with positions also given in Hz):
+0.0000e+000 radHz
+1.8978e+004 radHz
-2.5499e+003 +1.6193e+004*j radHz  (+2577.1 ± 171.72 Hz)
-2.5499e+003 -1.6193e+004*j radHz
-2.1356e+003 +5.3981e+003*j radHz  (+859.13 ± 66.981 Hz)
-2.1356e+003 -5.3981e+003*j radHz

Poles (with positions also given in Hz):
-2.2829e+003 +1.3120e+004*j radHz  (+2088.1 ± 128.21 Hz)
-2.2829e+003 -1.3120e+004*j radHz
-1.4697e+003 +3.5493e+003*j radHz  (+564.89 ± 38.274 Hz)
-1.4697e+003 -3.5493e+003*j radHz
-5.3268e+002 radHz

Gain constant = -5.5 × 10^{-9} as calculated by the method used in Appendix VIII F.

Figure A8.4.1 Bakery channel model.
Pavilion

Order: 6/5
Domain: s
Zeros (with positions also given in Hz):
+0.0000e+000 radHz
+4.1547e+004 radHz
-3.3953e+003 +1.5947e+004*j radHz  (+2538.1 ± 235.84 Hz)
-3.3953e+003 -1.5947e+004*j radHz
-2.6124e+003 +5.3774e+003*j radHz  (+855.83 ± 101.13 Hz)
-2.6124e+003 -5.3774e+003*j radHz

Poles (with positions also given in Hz):
-1.8940e+003 +1.1665e+004*j radHz  (+1856.6 ± 94.686 Hz)
-1.8940e+003 -1.1665e+004*j radHz
-1.7615e+003 +3.0806e+003*j radHz  (+490.29 ± 49.709 Hz)
-1.7615e+003 -3.0806e+003*j radHz
-5.8279e+002 radHz

Gain constant = \(-4.3 \times 10^{-8}\) as calculated by the method used in Appendix VIII F.

**Figure A8.4.2** Pavilion channel model.
Appendices

Union Building

Order: 6/5
Domain: s

Zeros (with positions also given in Hz):

+0.0000e+000 ± 000 radHz
-2.3437e+002 +1.8430e+004*j radHz  (+2933.2 ± 300.56 Hz)
-2.3437e+002 -1.8430e+004*j radHz
+1.5879e+004 radHz
-1.6485e+003 +4.8011e+003*j radHz  (+764.11 ± 63.012 Hz)
-1.6485e+003 -4.8011e+003*j radHz

Poles (with positions also given in Hz):

-1.9866e+003 +8.9113e+003*j radHz  (+1418.3 ± 54.48 Hz)
-1.9866e+003 -8.9113e+003*j radHz
-1.3760e+003 +3.3956e+003*j radHz  (+540.43 ± 45.105 Hz)
-1.3760e+003 -3.3956e+003*j radHz
-4.8877e+002 radHz

Gain constant = -6.2 × 10⁻⁸ as calculated by the method used in Appendix VIII F.

Figure A8.4.3 Union Building channel model.
Boiler House

Order: 6/5
Domain: s
Zeros (with positions also given in Hz):
  +0.0000e+000 radHz
  +2.3984e+004 radHz
  -1.4459e+003 +1.7640e+004*j radHz  (+2807.6 ± 565.94 Hz)
  -1.4459e+003 -1.7640e+004*j radHz
  -6.5721e+003 +8.0052e+003*j radHz  (+1274.1 ± 393.39 Hz)
  -6.5721e+003 -8.0052e+003*j radHz

Poles (with positions also given in Hz):
  -1.4886e+003 +1.1542e+004*j radHz  (+1836.9 ± 101.62 Hz)
  -1.4886e+003 -1.1542e+004*j radHz
  -1.6934e+003 +3.2177e+003*j radHz  (+512.11 ± 38.6 Hz)
  -1.6934e+003 -3.2177e+003*j radHz
  -5.0950e+002 radHz

Gain constant = -1.8 \times 10^{-8} as calculated by the method used in Appendix VIII F.

Figure A8.4.4 Boiler House channel model.
Office (Room 117)

Order: 6/5, unstable
Domain: s
Zeros (with positions also given in Hz):
+0.0000e+000 radHz
+3.3137e+003 +1.0626e+004*j radHz  (+1691.1 ± 248.01 Hz)
+3.3137e+003 -1.0626e+004*j radHz
-1.4679e+003 +4.3619e+003*j radHz  (+694.22 ± 91.99 Hz)
-1.4679e+003 -4.3619e+003*j radHz
+7.5177e+002 radHz

Poles (with positions also given in Hz):
-9.1241e+002 +2.7964e+003*j radHz  (+445.06 ± 52.095 Hz)
-9.1241e+002 -2.7964e+003*j radHz
+1.7099e+003 radHz  unstable
-1.7567e+003 +6.9790e+002*j radHz  (+111.07 ± 275.62 Hz)
-1.7567e+003 -6.9790e+002*j radHz

A gain constant has not been calculated for this model as it is unstable.

Figure A8.4.5 Office channel model.
Second office (Room 127A)

Order: 6/5
Domain: s
Zeros (with positions also given in Hz):
+0.0000e+000 radHz
+1.8904e+004 radHz
-6.1274e+003 +2.1626e+004*j radHz (+3441.8 ± 456.91 Hz)
-6.1274e+003 -2.1626e+004*j radHz
-3.0578e+003 +1.3538e+004*j radHz (+2154.6 ± 271.43 Hz)
-3.0578e+003 -1.3538e+004*j radHz
Poles (with positions also given in Hz):
-1.1891e+003 +1.2748e+004*j radHz (+2028.9 ± 84.84 Hz)
-1.1891e+003 -1.2748e+004*j radHz
-9.5552e+002 +3.6393e+003*j radHz (+579.22 ± 10.072 Hz)
-9.5552e+002 -3.6393e+003*j radHz
-1.8571e+002 radHz

Gain constant = $-7.1 \times 10^{-9}$ as calculated by the method used in Appendix VIII F.

Figure A8.4.6 Room 127A channel model.
Appendix VIII E  Noise spectral density measurements

A complete set of 7,600 measurements of the current noise in narrow frequency bands on the MV supply at the Thomas Cherry substation is shown in Figure A8.5.1.

Figure A8.5.1 Example of noise measurements at all frequency points.
Appendix VIII F Calculation of gain

The signal fundamental at the transmitter had a level of 8.7A RMS which is 18.8dB Amps. At the frequency where the received signal was a maximum the level was -70 dB relative to 225Amps and this equates to -23dB Amps. The gain of the channel at this frequency is therefore -41.8 dB including the distribution transformer.

The 4 zero 5 pole transfer function as specified by the poles and zero positions without any gain factor included has a maximum value of 0.0029 at 2190 rads/s. To make the complete transfer function equal the measured attenuation of -41.8dB which is 0.0081, we need a gain factor of 0.0081/0.0029 = 2.8. As the low frequency phase of the transfer function is close to -180 degrees, and we have no reason to suppose that the channel includes an inversion, the gain factor is determined to be -2.8 to ensure the complete transfer function reflects the measured results.

When the zero for this first model is moved from close to the origin onto the origin it changes the gain of the transfer function from -2.8 to -2.9. The phase is changed below the frequency of the pole at about 70Hz but that is immaterial as we are not interested in such low frequencies. The higher frequencies still include a 180 degree shift which the negative value of the gain constant will cancel. When the other adjustments in poles and zeros are made to this model to make it fit the generic model type with 6 zeros and 5 poles the gain of the poles and zeros alone changes from 0.0029 to 1,470,000. This is mainly due to the addition of two more zeros. To maintain the gain of the overall transfer function correct so that it matches the measurements a gain factor of \(-5.5 \times 10^{-9}\) is required. The gain constant values were calculated in the same way for the models of the other channels and are given in Appendix VIII D.
Appendix IX A  Generic models for each of the six channels compared with data

Here the Generic models for each of the six channels are compared with the measurement data.

**Bakery:**

![Bakery Channel Amplitude Response compared with generic type 6 Zero 5 Pole Model](image1)

![Bakery Channel Phase Response compared with generic type 6 Zero 5 Pole Model](image2)

**Pavilion:**

![Pavilion Channel Amplitude Response compared with generic type 6 Zero 5 Pole Model](image3)

![Pavilion Channel Phase Response compared with generic type 6 Zero 5 Pole Model](image4)
Appendices

Union:

*Note the damping factor of zero at 2,933 Hz was altered from the Max Likelihood value of 0.01 to 0.08 to avoid the deep notch shown on the original model referenced in the results chapter.

Boiler house:
Office:

Office Channel Amplitude Response compared with generic type 6 Zero 5 Pole Model

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Model</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
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<td>-60</td>
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<tr>
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<tr>
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<tr>
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</table>

Office Channel Phase Response compared with generic type 6 Zero 5 Pole Model

<table>
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<tr>
<th>Frequency (Hz)</th>
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</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>500</td>
<td>-180</td>
<td>-120</td>
</tr>
<tr>
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<td>120</td>
</tr>
<tr>
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<td>120</td>
<td>180</td>
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</tbody>
</table>

New Office (Room 127A):

New Office Channel Amplitude Response compared with generic type 6 Zero 5 Pole Model

<table>
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<th>Measured</th>
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<tr>
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New Office Channel Phase Response compared with generic type 6 Zero 5 Pole Model

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<td>60</td>
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</tr>
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<td>180</td>
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</table>
Appendix IX B MATLAB script to plot model & compare with measured data

% To plot transfer functions and compare with measured data for various locations to %TC
f=[1:20500]; %to 3200 Hz
s=j*f;
% Parameters for room 127A channel are used here
realpole1=1./((s-(-4.60e+002))); %fixed in generic model
complexp1=1./(((s-(-1.1891e+003 +((+1.2748e+004)*j))).*(s-(-1.1891e+003 -((1.2748e+004)*j))));
complexp2=1./(((s-(-9.5552e+002 +((+3.6393e+003)*j))).*(s-(-9.5552e+002 -((3.6393e+003)*j))));
complexz1=(s-(-6.1274e+003 +((+2.1626e+004)*j))).*(s-(-6.1274e+003 -((2.1626e+004)*j)));
closezero=((s+0));
complexz2=(s-(-3.0578e+003 +((+1.3538e+004)*j))).*(s-(-3.0578e+003 -((1.3538e+004)*j)));
rhzero=(s-1.8904e+004);

% use freqv=Sheet1(:,1); and phasev=Sheet1(:,2) and amplitv=Sheet1(:,4); to % separate out from imported spread sheet of measurement data
freqv=Sheet1(:,1);
phasev=Sheet1(:,2);
phasemod=11*(freqv/1000); %next do 11 degs/khz delay adjustment
phasev=phasev+phasemod;
amplitv=Sheet1(:,4);

amplit =
abs(realpole1.*complexz1.*complexp1.*complexp2.*closezero.*complexz2.*rhzero);
ang=((180/pi)*unwrap(angle(realpole1.*complexz1.*complexp1.*complexp2.*closezero.*complexz2.*rhzero)));
logamplit=20*log10(amplitv)-190;%gain factor

% functions for plotting graphs and labelling
graph1(f/(2*pi), logamplit, freqv, amplitv)
graph3 (f/(2*pi), ang+180, freqv, phasev)
REFERENCES


References


References


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