2009 IEEE International Conference on Control and Automation
Christchurch, New Zealand, December 9-11, 2009

Zero Prefix Precoder-Based Blind SIMO Channel Estimation

Song Wang*, Robert Ross* and Jonathan H. Manton†

*Department of Electronic Engineering
La Trobe University
Bundoora, VIC 3086, Australia
Tel: +61-3-9479 3744
E-mail: song.wang@latrobe.edu.au
R.Ross@latrobe.edu.au

†Department of Electrical & Electronic Engineering
University of Melbourne
VIC 3010, Australia.
Tel: +61-3-8344 4974
E-mail: j.manton@ieee.org

***Abstract*— Single-input multiple-output (SIMO) channels can arise from radio propagation between a single transmitter and multiple receivers/sensors. The transmitter precoding strategy introduces algebraic redundancy to transmitted symbols, which can be exploited to facilitate channel estimation. In this paper we propose a zero prefix precoder-based blind SIMO channel estimation method. Compared to the existing blind estimation methods, the proposed method is highly data efficient in that it achieves blind multichannel identification using only a small size of observation data.**

I. INTRODUCTION

Signal precoding builds input diversity in digital communication systems and hence facilitates channel estimation and equalization [1]. A well-known precoding example is a guard interval used in orthogonal frequency division multiplexing (OFDM) transmission. Another widely known precoding scheme is a zero prefix linear precoder, which adds a prefix of a sequence of zeros to each source symbol block.

Single-input multiple-output (SIMO) channels arise from radio propagation between a single transmitter and multiple receivers/sensors, and have extensive applications in modern wireless communications. Blind identification of a SIMO finite impulse response (FIR) channel deals with identifying the impulse response of an unknown SIMO channel using only the channel's output data with no knowledge of the input signal. Blind SIMO channel identification has been extensively researched and there are many publications on this subject; see e.g., [1]-[6].

In this paper we develop a zero prefix precoder-based, short-data effective blind SIMO channel estimation method. By exploiting the transmitter redundancy introduced by the zero prefix and the multichannel signaling property brought by the SIMO channel output structure, we recast the time-domain cross-relation [3] between each pair of receive branches into the frequency domain. As a consequence, the linear convolution is effectively resolved by performing the circular convolution, or more precisely, element-wise multiplication in the frequency domain. The proposed method makes no statistical assumptions on the transmitted symbols. In the presence of additive noise, the SIMO channel can be estimated blindly by solving a nonlinear least squares problem. Simulations show that the proposed algorithm outperforms the existing blind estimation methods [5], [3], [6], which need a larger size of observation data.

The rest of the paper is organized as follows. Section II presents the zero prefix precoder-based signal model. Based on the equivalence between the linear convolution and the circular convolution due to the redundancy introduced by the zero prefix, we derive a new frequency-domain blind SIMO channel estimation algorithm in Section III . Section IV shows the simulation result, where we compare the proposed algorithm with the existing methods. The conclusion is given in Section V.

II. SIGNAL MODEL

Each block of source symbols $\mathbf{s} = [s(0)\ s(1)\ \cdots\ s(r-1)]^T \in \mathscr{C}^r$ is mapped to the encoded vector $\mathbf{x}$ by the zero prefix precoder $P \in \mathscr{C}^{N\times r}$ with $N = r + M$, that is, $\mathbf{x} = P\mathbf{s}$, where

$$P = \begin{bmatrix} \mathbf{0}_{M\times r} \\ \mathbf{I}_r \end{bmatrix}$$

So the encoded vector $\mathbf{x}$ is obtained as

$$\mathbf{x} = [0\ 0\ \cdots\ 0\ s(0)\ s(1)\ \cdots\ s(r-1)]^T \in \mathscr{C}^N$$

The encoded signal is transmitted through L FIR channels with channel coefficients

$$\mathbf{h}_m = [h_m(0)\ \ h_m(1)\ \cdots\ h_m(M)]^T,\ \ m = 1,\cdots,L$$

The maximum order of the L channels is M, where M is assumed to be known a priori. The received signal of the mth channel is given by

$$y_m(n) = x(n) * h_m(n) + w_m(n) \qquad (1)$$

where the symbol $*$ denotes linear convolution, and $w_m(n)$ the additive white Gaussian noise (AWGN) at channel m, which is uncorrelated with the transmitted signal. Since the M prefixed zeros of each transmitted block clear the memory of the channels, the current observation vector $\mathbf{y}_m$ of size N at channel m, $m = 1, \cdots, L$, is only dependent on the currently transmitted block $\mathbf{x}$.

With the signal model (1), our aim is to identify the channel coefficient vector $\mathbf{h}_m$, $m = 1, \cdots, L$. In particular, we focus our attention on a small size of observation data, namely, N is small. This is because short data lengths are practically important in situations where there is high mobility in wireless communication systems.

III. Proposed blind SIMO channel identification method

In the absence of noise, any channel pair (i, j) of (1) satisfies the following relationship:

$$y_i(n) * h_j(n) = y_j(n) * h_i(n), \quad 1 \leq i, j \leq L, \ i \neq j \qquad (2)$$

The above cross-relation was initiated in [3] and has been used in subsequent algorithm development, e.g., [2], [4] and [6].

Due to the redundancy introduced by the zero prefix precoder in the transmitted signal, the linear convolution in (2) is equivalent to the circular convolution of $y_i(n)$ $[y_j(n)]$ with $h_j(n)$ $[h_i(n)]$, so long as the circular convolution is of length $N_s \geq N + M$ [7] . It follows from the property of the circular convolution that taking the N_s-point discrete Fourier transform (DFT) on both sides of (2) yields

$$Y_i(k)H_j(k) = Y_j(k)H_i(k), \quad k = 0, 1, \cdots, N_s - 1 \qquad (3)$$

where $Y_m(k)$ and $H_m(k)$ represent the frequency-domain samples of $y_m(n)$ and $h_m(n)$, respectively. As $H_m(k) = \sum_{n=0}^{N_s-1} h_m(n)e^{-j2\pi kn/N_s} = \sum_{n=0}^{M} h_m(n)e^{-j2\pi kn/N_s}$, we rewrite (3) as

$$[-\mathbf{F}_j \ \ \mathbf{F}_i] \begin{bmatrix} \mathbf{h}_i \\ \mathbf{h}_j \end{bmatrix} = \mathbf{0} \qquad (4)$$

where $\mathbf{F}_m = \text{diag}\{Y_m(0), Y_m(1), \cdots, Y_m(N_s - 1)\}\mathbf{W}$, with $\mathbf{W}$ being the first $M + 1$ columns of the $N_s \times N_s$ DFT matrix. Considering all L channels, from (4) we can construct

$$\mathbf{F}\mathbf{h} = \mathbf{0} \qquad (5)$$

where $\mathbf{h} = \begin{bmatrix} \mathbf{h}_1^T & \mathbf{h}_2^T & \cdots & \mathbf{h}_L^T \end{bmatrix}^T \in \mathscr{C}^{L(M+1)}$, and

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{1,2}^T \ \mathbf{F}_{1,3}^T \ \cdots \ \mathbf{F}_{1,L}^T \ \mathbf{F}_{2,3}^T \ \cdots \ \mathbf{F}_{2,L}^T \ \cdots \ \mathbf{F}_{L-1,L}^T \end{bmatrix}^T$$

with $\mathbf{F}_{i,j} \in \mathscr{C}^{N_s \times L(M+1)}$ defined as

$$\mathbf{F}_{i,j} = [\mathbf{0} \ \cdots \ \mathbf{0} \ -\mathbf{F}_j \ \mathbf{0} \ \cdots \ \mathbf{0} \ \mathbf{F}_i \ \mathbf{0} \ \cdots \ \mathbf{0}]$$

where the ith entry is $-\mathbf{F}_j$ and the jth entry is $\mathbf{F}_i$. The dimension of $\mathbf{F}$ in (5) is $\frac{L(L-1)N_s}{2} \times L(M + 1)$. Hence the channel coefficient vector $\mathbf{h}$ can be determined by solving (5).

Next we show that the channel vector $\mathbf{h}$ is identifiable up to a complex-valued scaling factor if all channels share no common zero and the N_s-point DFT $X(k)$ of the transmitted vector $\mathbf{x}$ are nonzero.

Theorem: *The nontrivial solution* $\hat{\mathbf{h}}$ *to* $\mathbf{F}\hat{\mathbf{h}} = \mathbf{0}$ *uniquely (up to a scaling factor) determines the channel impulse response* $\{\mathbf{h}_m\}_{m=1}^{L}$*, if there is no common zero among all the channels and* $X(k) \neq 0$*,* $k = 0, 1, \cdots, N_s - 1$*.*

Proof: Based on (5), for any channel pair (i, j), we have

$$\mathbf{F}_i\hat{\mathbf{h}}_j = \mathbf{F}_j\hat{\mathbf{h}}_i$$

This can also be expressed as

$$Y_i(k)\hat{H}_j(k) = Y_j(k)\hat{H}_i(k), \quad k = 0, 1, \cdots, N_s - 1$$

which leads to

$$y_i(n) * \hat{h}_j(n) = y_j(n) * \hat{h}_i(n)$$

or equivalently,

$$x(n) * h_i(n) * \hat{h}_j(n) = x(n) * h_j(n) * \hat{h}_i(n)$$

Taking the z-transform yields

$$X(z)H_i(z)\hat{H}_j(z) = X(z)H_j(z)\hat{H}_i(z)$$

As the N_s-point DFT $X(k) \neq 0$, based on the equivalence between the z-transform and DFT, $X(z)|_{z=e^{j2\pi k/N_s}} \neq 0$, which means

$$H_i(z)\hat{H}_j(z) = H_j(z)\hat{H}_i(z) \qquad (6)$$

Since there is no common zero among all channels, it follows from (6) that for any j, {zeros of $H_i(z)$} $\in$ {zeros of $\hat{H}_i(z)$}. Suppose that channel $i = l$ has the maximum order M. Then {zeros of $H_l(z)$} $\in$ {zeros of $\hat{H}_l(z)$}. As $H_l(z)$ is of order M, $\hat{H}_l(z)$ is also of order M. So $\hat{H}_l(z) = \alpha H_l(z)$, where α is a nonzero constant. Substituting it into (6) yields $\hat{H}_j(z) = \alpha H_j(z)$, i.e., $\hat{\mathbf{h}}_m = \alpha\mathbf{h}_m$, $m = 1, \cdots, L$. □

Based on the above development, we make the following remarks:

(i) The blind SIMO channel estimation method developed in the paper is a deterministic method. It assumes no statistical or finite alphabet property of the source symbols.

(ii) The zero prefix precoder enables the receiver to identify the SIMO channels using a block of N observations, where $N \geq M + 1$. As shown by the simulations, blind channel estimation is achieved with an observation length as short as $M + 1$. By contrast, the existing deterministic, non-precoder-based blind SIMO channel estimators in [1], [3], [6] require an observation duration of at least $3M + 1$ to estimate the channels.

(iii) The main computational load of the proposed method comes from the singular value decomposition (SVD). Specifically, the proposed algorithm requires one SVD on $\mathbf{F}$ in (5) with computational complexity amounting to $\mathscr{O}(\frac{L^3(L-1)(M+1)^2N_s}{2})$ according to [8]. As the observation data size N is small, the proposed method is computationally inexpensive.

(iv) In practice, the channels must be estimated from noisy observations. The blind identification problem is thus solved in the least squares sense, i.e., find $\mathbf{h}$ which minimises $\|\mathbf{F}\mathbf{h}\|$; see (5). The proposed algorithm is described as follows.

Algorithm summary:

(1) For each channel $m = 1, \cdots, L$, take the N-point DFT on the received data block $\mathbf{y}_m$ to obtain $Y_m(k)$, $k = 0, 1, \cdots, N-1$.

(2) Use $Y_m(k)$, $k = 0, 1, \cdots, N-1$, to form the matrix $\mathbf{F}_m$ in (4). Based on $\mathbf{F}_m$, construct the matrix $\mathbf{F}$ in (5).

(3) The channel estimate $\hat{\mathbf{h}}$ is obtained by

$$\hat{\mathbf{h}} = \arg\min_{\|\mathbf{h}\|=1} \|\mathbf{F}\mathbf{h}\|$$

IV. Simulation

Simulations were performed to compare the proposed method with the subspace algorithm [5] and the FFT/IFFT-based algorithm recently presented in [6] based on short-duration observations. A SIMO system with four FIR channels was used in the simulation. The channel coefficients were the same as in [5], which are listed below.

$$\begin{aligned}
\mathbf{h}_1 &= [(-0.049, 0.359), (0.482, -0.569), (-0.556, 0.587), \\
&\quad (1.0, 0), (-0.171, 0.061)]^T \\
\mathbf{h}_2 &= [(0.443, -0.0364), (1.0, 0), (0.921, -0.194), \\
&\quad (0.189, -0.208), (-0.087, -0.054)]^T \\
\mathbf{h}_3 &= [(-0.211, -0.322), (-0.199, 0.918), (1.0, 0), \\
&\quad (-0.284, -0.524), (0.136, -0.19)]^T \\
\mathbf{h}_4 &= [(0.417, 0.03), (1.0, 0), (0.873, 0.145), \\
&\quad (0.285, 0.309), (-0.049, 0.161)]^T
\end{aligned}$$

The system was driven by a BPSK (binary phase shift keying) sequence. The proposed method encodes the source symbols through a zero prefix precoder. AWGN was applied to channel outputs. The mean-square-error (MSE) is used as the performance measure for channel estimation, defined as

$$\text{MSE (dB)} = 10\log_{10}\left(\frac{1}{t}\sum_{i=1}^{t}\|\hat{\mathbf{h}}_i - \mathbf{h}\|^2\right)$$

where t is the number of Monte Carlo runs, $\mathbf{h}$ the true unit-norm channel vector, and $\hat{\mathbf{h}}_i$ the estimated channel vector (with unit norm) from the ith run.

In the simulation, the proposed method used a block of five output samples to estimate the channels. It is shown in Fig. 1 that with such a small sample size, the proposed method outperforms the subspace algorithm [5] and the FFT/IFFT-based algorithm [6], both of which used ten times more, i.e., 50, output samples. The MSE of the cross-relation algorithm [3] is not depicted in Fig. 1, because its performance is the same as that of the algorithm in [6] when the channel order is known.

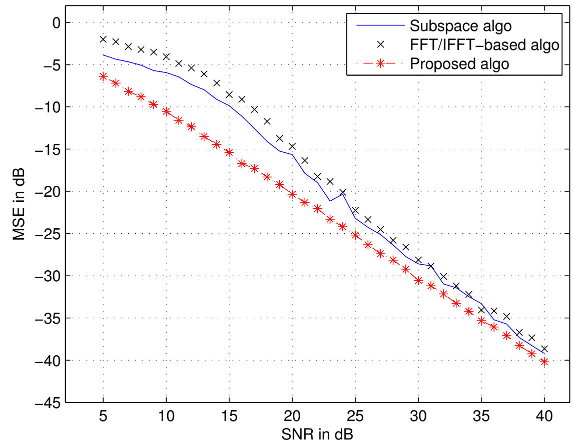

Fig. 1. Performance of the subspace algorithm, the FFT/IFFT-based algorithm and the proposed method.

V. Conclusion

A zero prefix precoder-based blind multichannel identification method has been proposed. The strength of the new method lies in its effectiveness to blindly estimate the SIMO channels using short-duration observations. The data sample size required by the proposed method is smaller than that in the existing blind identification methods. The proposed method is well suited for short-data applications in wireless communications when there is fast channel variability.

References

[1] G. B. Giannakis, Y. Hua, P. Stoica and L. Tong, editors. *Signal Processing Advances in Wireless and Mobile Communications. Vol. 1: Trends in Channel Estimation and Equalization*. Prentice-Hall PTR, 2001.

[2] L. Tong and S. Perreau, "Multichannel blind identification: from subspace to maximum likelihood methods," *Proc. of IEEE*, vol. 86, no. 10, pp. 1951-1968, 1998.

[3] G. Xu, H. Liu, L. Tong and T. Kailath, "A least-squares approach to blind channel identification," *IEEE Trans. Signal Processing*, vol. 43, no. 12, pp. 2982-2993, 1995.

[4] Y. Huang and J.B. Benesty, "A class of frequency-domain adaptive approaches to blind multichannel identification," *IEEE Trans. Signal Processing*, vol. 50, no. 1, pp. 11-24, 2003.

[5] E. Moulines, P. Duhamel, J.F. Cardoso and S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters," *IEEE Trans. Signal Processing*, vol. 43, no. 2, pp. 516-525, 1995.

[6] S. Wang and J.H. Manton, "Blind SIMO channel identification using FFT/IFFT," *Signal Processing*, vol. 88(12), pp. 3007-3010, 2008.

[7] A.V. Oppenheim and R.W. Schafer, *Discrete-time Signal Processing*, 2nd ed. Prentice-Hall, 1999.

[8] G.H. Golub and C.F. Van Loan, *Matrix Computations*, 3rd ed. Johns Hopkins Univ. Press, 1996.