A Comparison of Ultra Wideband Signal Functions for Wireless Ad Hoc Networks

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Abstract

This paper describes, simulates, evaluates and compares four time and band limited functions that can be used to create ultra wideband signals. Expressions and methods for generating the functions are presented followed with simulations using uniform sets of parameters. It is shown at the -10bB points the Hermite, prolate spheroidal wave function (PSWF) and wavelets have suitable UWB properties. Using the Daubechies wavelets as examples, we demonstrate that both the scaling and wavelet functions provide suitable UWB bandwidths. Their -10 dB bands are 1.9739 GHz and 3.4644 GHz respectively. The -10 dB bandwidth of the Daubechies wavelet of order 1 is about 0.61 compared to the Hermite function of the same order, but higher than what can be achieved with PSWF.

1 Introduction

As broadband communications becomes widely accepted the era of ultra wideband communication is emerging. Ultra wideband communications has origins in time domain electromagnetic [1]. An ultra wideband (UWB) system is any communication system that fits within the following defining criteria, based on bandwidth, the fractional bandwidth and the range of frequencies in use. A UWB system is defined by the following parameters. Given two frequencies f_h and f_l defining the upper and lower 10 dB cutoff frequencies respectively in the power spectrum of any signal, the UWB centre frequency f_c is defined as:

$$f_c = \frac{f_h - f_l}{2} \qquad (1)$$

and the fractional bandwidth f_p of the signal is defined also by the expression

$$f_p = 2\frac{f_h - f_l}{f_h + f_l} \qquad (2)$$

Then the signal is UWB

$$if \quad f_p > 0.2 \; or$$

$$f_h + f_l > 500 \; MHz$$

The fractional bandwidth f_p should be at least 20% and some authors specify 25%. A detailed history of UWB can be found in [2]. The UWB band is typically in the range of 3.1 GHz to 10.6 GHz. Several important communication systems use parts of this band. The range from 3.1 GHz to 4.7 GHz is used by radar and satellite systems. Therefore fears have been widely expressed that UWB systems will interfere with existing communication systems. Hence the FCC in 2002 imposed spectral masks, the so-called lower limits of signal power which a UWB system is permitted to radiate in specific spectral bands [3] even when no devices near them are using such bands. In general UWB signal powers are very low and relatively imperceptible by devices further away from them by 10 meters.

Ultra wideband techniques have been known for a while mostly in radar technology [4] where its advantages over other used techniques are much more apparent. Seven distinctive advantages must be mentioned. Firstly, short pulse radar provides distinctive high resolution and higher range measurement advantages. The range resolution capability of the signal is determined by its bandwidth. A radar signal at 4 GHz is able to resolve a 7.5 cm range, were

$$\Delta r = \frac{\text{microwave speed}}{\text{bandwidth}} \text{ metres} \qquad (3)$$

The resolution is inversely proportional to the bandwidth short duration pulses are preferred. Secondly, UWB radar provides enhanced detection of slow moving or stationary targets. Thirdly, it provides increased detection probability due to the elimination of lobbing structure from a target's secondary pattern. The fourth advantage is related to the radar cross section. UWB short pulse radar provides immunity from interference from sources such as rain, aerosol, fog and metal strips.

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This is because the radar cross section (RCS) of these particles is comparable to the RCS of the target. The fifth advantage of UWB radar is enhanced target recognition. Sixth, it provides immunity from co-located radar systems of the same nature as a result of decreased pulse-to-pulse probabilities. Lastly because the signal is spread over a very large bandwidth, it is hidden from interceptors, the so-called anti-jam and anti-detection security. These advantages have been known for many decades in radar applications.

Advantages of UWB in other areas of communication are equally numerous. Multipath has always been a major problem in most communication systems. UWB provides immunity to multipath cancellation because it is easier to detect signals arriving at a receiver from the direct path and other paths because they are time-orthogonal to each other. UWB also provides low interference between existing telecommunication systems provided the UWB system is designed properly to minimize spectral leakage and operational security. With one of the best low pulse rates, UWB systems have extremely low duty cycles. This translates to low average prime power requirements. Therefore, energy saving is possible and better battery-operated efficiency is to be expected. In other areas of applications such as geo-localisation, it also provides multipath immunity for leading edge detection. Furthermore, it is efficient for battery operations in applications such as RF identification tags (RFID) and precision time of flight measurements.

2 Methods of Generating UWB Signals

There are two broad methods for generating UWB signals based on impulse or short pulse and modulation methods. The variants of UWB signals based on CDMA and OFDMA techniques target wideband personal area networks (WPAN) and the impulse methods target geo-location and tracking. The short pulse versions are characterized by low duty cycles and high peak-to-average power ratios, while those based on modulation techniques are characterized by constant envelopes resulting in nearly equal peak and average power densities.

Several approaches have been used to generate UWB short pulses. To generate a short impulse a large energy is stored over a long period of time followed with a sudden release. If the release time is of the order of a nanosecond or less, a UWB impulse will be generated. Typically, the energy is stored in capacitors. Historically, UWB signals are generated using the Max-generator, shock excitation of wideband antenna, time-gated oscillators, a combination of conventional heterodyning and gated power-amplifier, pulse modulation and Fourier techniques [5 - 7]. In the frequency domain, four types of functions are popularly used. These are Gaussian mono pulse, Hermite, prolate spheroidal and Wavelets. The Gaussian monocycle pulse $p(t)$ is

$$p(t) = 2A\sqrt{\pi e}\,\frac{t}{\tau_p}\,e^{-2\pi\left(t/\tau_p\right)^2} \quad (4)$$

A is the amplitude of the pulse and τ_p is a parameter that is related to the width of the pulse. The name “monocycle” refers to transmission of a single cycle of a sinusoid which has large relative bandwidth [7]. The received pulse modified by antenna’s differentiating properties is

$$p_r(t) = A'\left(1 - \frac{4\pi t^2}{\tau_p^2}\right)e^{-2\pi\left(t/\tau_p\right)^2} \quad (5)$$

Therefore, given a pulse repetition interval T_f a UWB transmitter would transmit a series of pulses given by

$$s(t) = \sum_{k=-\infty}^{\infty} p(t - kT_f) \quad (6)$$

3 Hermite Functions

Hermite polynomials also have excellent UWB properties [7, 8]. Hermite functions are defined by the expression:

$$h_{e_n}(t) = (-1)^n e^{t^2/2\tau^2}\,\frac{d^n}{dt^n}\left(e^{-t^2/2\tau^2}\right) \quad (7)$$

Where n = 1, 2, 3, . . . and $-\infty < t < \infty$ and is the time-shaping factor that controls the width of the pulse. It therefore also controls the bandwidth of the pulse. Hermite polynomials are in general not orthogonal but can be modified to be orthogonal through the process of multiplying them with an exponential factor:

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$$h_n(t) = c_n e^{-t^2/4\tau^2} h_{e_n}(t) = (-1)^n e^{t^2/4\tau^2} \frac{d^n}{dt^n}\left(e^{-t^2/2\tau^2}\right) \quad (8)$$

The constant c_n determines the energy of the pulse. The coefficients used to generate the first 8 Hermite functions are given in Table 1. The coefficients are related by:

$$h_n(t) = c_n e^{-t^2/4\tau^2} n! \sum_{i=0}^{[n/2]} \left(-\frac{1}{2}\right)^i \frac{(t/\tau)^{n-2i}}{(n-2i)!\,i!} \quad (9)$$

where [n/2] is the integer part of n/2. The pulses have almost equal duration and equal zero crossings for all n, are mutually orthogonal, with nonzero DC components and almost the same bandwidth.

Order	**Const.**	$\frac{t}{\tau}$	$\left(\frac{t}{\tau}\right)^2$	$\left(\frac{t}{\tau}\right)^3$	$\left(\frac{t}{\tau}\right)^4$	$\left(\frac{t}{\tau}\right)^5$	$\left(\frac{t}{\tau}\right)^6$	$\left(\frac{t}{\tau}\right)^7$	$\left(\frac{t}{\tau}\right)^8$
0	1								
1	0	1							
2	-1	0	1						
3	0	-3	0	1					
4	3	0	-6	0	1				
5	0	15	0	-10	0	1			
6	-15	0	45	0	-15	0	1		
7	0	-105	0	105	0	-21	0	1	
8	105	0	-420	0	210	0	-28	0	1

Table 1: Coefficients of Hermite Polynomials

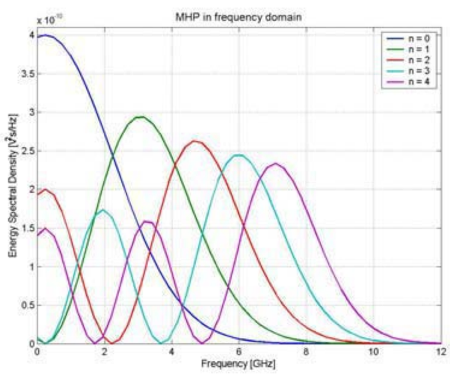

Figure 1: Spectrum of Modified Hermite Polynomials of Order 0 to 4.

Figure 1 shows the spectrum of the first 4 modified Hermite polynomials (MHP) using $\tau = 4 \times 10^{-10}$ sec and a sampling frequency of 50 GHz. A very high sampling rate like this is usually impractical due to the limitation of current analogue to digital converters (ADC). Therefore combination of lower bandwidth pulses and software approaches are in demand.

The zeroth order Hermite function has low pass properties. The rest are bandpass in nature. Two or more of them may therefore be combined to produce a larger bandwidth signal as will be shown in section 3. The choice of τ sets the range of spectrum from zero to 12 GHz in Figure 1. The spectrum for the Hermite function of order n=1 is shown in Figure 2. The -10 dB bandwidth is shown.

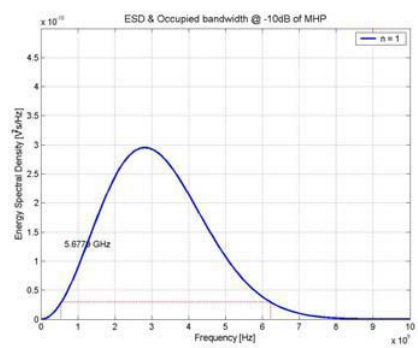

Figure 2: Achievable -10 dB bandwidth of Hermite Pulse of Order n=1

4 Prolate Spheroidal Functions

Prolate spheroidal wave functions have had applications in telecommunications including analogue encryption [8] and biomedical signal processing [11, 12]. Recently Walter and Shen [13] proposed wavelets based on prolate spheroidal wave functions. They replaced the usual wavelet basis consisting of sinc functions with one based on prolate spheroidal wave functions (PSWFs) which have much better time localization than the sinc function. The new wavelets inherited properties of the prolate functions by preserving high energy concentration in both the time and frequency domains.

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Generally, PSWF are highly localised in the time and frequency domains. This means they are time-limited and also band-limited. A signal p(t) is time-limited if for T>0, the signal vanishes for all $|t| > T/2$. It is also band limited with a bandwidth of β if its Fourier spectrum P(f) is identically zero outside the range [-β, β].

The prolate spheroidal functions can be obtained using the integral

$$\int_{-T/2}^{T/2} \psi(x) \frac{\sin \beta(t-x)}{\pi(t-x)} dx = \lambda \psi(t) \quad (10)$$

Prolate spheroidal wave functions are doubly orthogonal, meaning they satisfy the following integral equations:

$$\int_{-T/2}^{T/2} \psi_m(t) \psi_n(t) dt = \lambda_m \delta_{mn}$$

$$\int_{-\infty}^{\infty} \psi_m(t) \psi_n(t) dt = \delta_{mn} \quad (11)$$

They therefore constitute an orthogonal basis of $L^2(-T/2, T/2)$ and also an orthonormal basis of a sub space β of $L^2(-\infty, \infty)$. λ_m is the amount of energy of $\psi(t)$ that lies in the interval $[-T/2, T/2]$. This property guarantees the unique demodulation at the receiver [13]. The concentration of energy λ_n in the duration $[-T/2, T/2]$ can be estimated for the case when m=n. This is given as the ratio:

$$\lambda_n = \frac{\int_{-T/2}^{T/2} |\psi_n(t)|^2 dt}{\int_{-\infty}^{\infty} |\psi_n(t)|^2 dt} \quad (12)$$

Prolate spheroidal wave functions also satisfy the differential equation

$$\frac{d}{dt}(1-t^2)\frac{d\psi_n(t)}{dt} + (\chi_n - ct^2)\psi_n(t) = 0$$

$\psi_n(t)$ are prolate wave functions of order n, $\chi_n(t)$ is the eigenvector of $\psi_n(t)$. The constant c is equal to $c = \beta T/2$ (is the bandwidth and T is the time duration) and the number of degrees of freedom. By re-arranging the differential equation we obtain

$$\left(1-t^2\right)\frac{d^2\psi_n(t)}{dt^2} - 2t\frac{d\psi_n(t)}{dt} + \left(\chi_n - c^2t^2\right)\psi_n(t) = 0 \quad (13)$$

Or

$$\psi_n''(t) = \frac{1}{\left(1-t^2\right)}\left[2t\psi_n'(t) - \left(\chi_n - c^2t^2\right)\psi_n(t)\right] \quad (14)$$

By changing the values of χ_n different orders of the prolate pulses can be obtained. Consequently, this equation is the so-called basic equation for generating multiple pulses. To compute the functions, $\psi_n(t)$ is written with the prolate angular function of the first kind [13]:

$$\psi_n(t) = \psi_n(\beta, T, t) = \frac{\sqrt{2\lambda_n(c)/T}}{\sqrt{\int_{-1}^{1}\left[S_{0n}^1(c,x)\right]^2 dx}} S_{0n}^1(c, 2t/T) \quad (15)$$

Where

$$\sqrt{\int_{-1}^{1}\left[S_{0n}^1(c,x)\right]^2 dx} = \frac{2}{2n+1} \quad (16)$$

and S_{0n}^1 is the prolate angular function of the first kind. Although no exact solution is specified for these functions, in [13] a method for calculating the prolate spheroidal angular function is proposed by firstly calculating the coefficients and then the basic functions. The first kind of the prolate angular function can be written as follows [9]:

$$S_{0n}^1(c,t) = \begin{cases} \sum_{k=even}^{\infty} d_k(c) P_k(c,t); & n \text{ even} \\ \sum_{k=odd}^{\infty} d_k(c) P_k(c,t); & n \text{ odd} \end{cases} \quad (17)$$

$P_k(c,t)$ is an associated Legendre polynomial and $d_k(c)$ satisfy the recurrence relation:

$$a_k d_{k+2}^2(c) + (\beta_k - \chi_n(c)) d_n^k(c) + \gamma_k d_{k-2}^n(c) = 0 \quad (18)$$

and

$$a_k = \frac{(k+1)(k+2)c^2}{(2k+3)(2k+5)}$$

$$\beta_k = \frac{(2k^2+2k-1)c^2}{(2k-1)(2k+3)} + k(k+1)$$

$$\gamma_k = \frac{k(k-1)c^2}{(2k-1)(2k-3)}$$

Following the method in [13], $d_k(c)$ is set equal zero $d_k(c) = 0$ for all $k > 2N+1$, and the following equation is solved to compute $d_k(c)$:

$$(\Theta - \chi_n I) d^n = 0 \qquad (19)$$

where d^n are the eigenvectors and χ_n are the eigenvalues of Θ. Figure 3 shows the spectrums of prolate spheroidal wave functions of orders zero to 6 in the frequency domain.

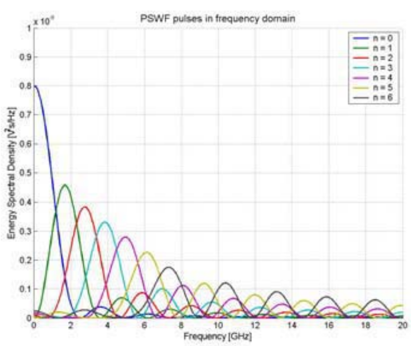

Figure 3: Spectrums of PSWFs of Orders 2 to 6

Each pulse width is 0.4 ns and sampled at 50 GHz. The zeroth order function has low pass characteristics and the rest are band pass in nature. Unlike Hermite functions, PSWF manifest spectral ripples, but are significantly small to be of any major concerns.

A 3 GHz bandwidth can be achieved at the -10 dB point. This is about 0.528 the value obtained with a Hermite function of the same order. It is also less complex to generate the Hermite functions in software.

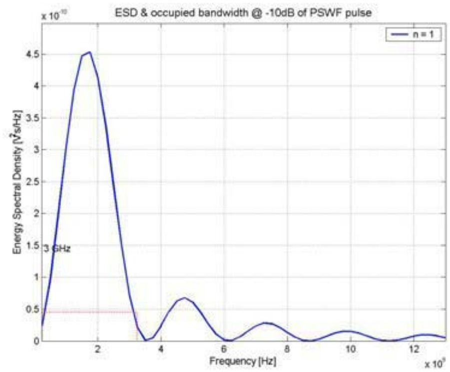

Figure 4: Achievable -10dB bandwidth for PSWF Pulse of Order 1

5 Wavelets as UWB Functions

UWB signals and wavelets share many properties. Major applications of wavelets is in the representation of functions with sharp spikes or edges and are useful for representing functions that are localised in both the time and frequency domains. Wavelets mean "small waves", therefore they oscillate, a desirable property of UWB signals. They must oscillate and occur in very finite durations (localisation in time) to result to ultra wideband. They are also localised in the frequency domain, a property that is particularly useful for UWB to ensure spectral leakage into the spectral regions of existing communication systems is limited. In fact the integral of a wavelet must be zero within its support and it must therefore have a finite energy. Furthermore, the construction of wavelets requires that the mother wavelet generates the basis functions by 'dilation' and 'translation'. Dilation means the duration of the wavelet could be increased or decreased, or the bandwidth can be varied. Translation means the wavelet can be delayed or moved to other regions in the frequency domain, another desirable feature for UWB communications. Translation (shift) in time results to a proportional phase shift in the frequency domain. Similarly, it can be easily shown by using Fourier series techniques that a frequency shift is the result of modulating a time series by a complex signal in the time domain. These Fourier properties facilitate can be used to implement bandwidth expansion.

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Another property shared by wavelets is that of orthogonality. Although orthogonality is used in wavelet analysis to facilitate fast computation, in UWB it is a desirable property for separating multipath signals from two or more sources and to combat interference. In what follows, these properties are explored and investigated. In Figure 5 the achievable spectral bandwidth at the -10bB points for Daubechies wavelets of order=1 are shown. Both the scaling and wavelet functions are compared. The scaling function has low pass characteristics and the wavelet function is band pass in nature. Their -10 dB bands are 1.9739 GHz and 3.4644 GHz respectively. The -10 dB bandwidth of the Daubechies wavelet of order 1 is about 0.61 compared to the Hermite function of the same order, but higher than what can be achieved with PSWF.

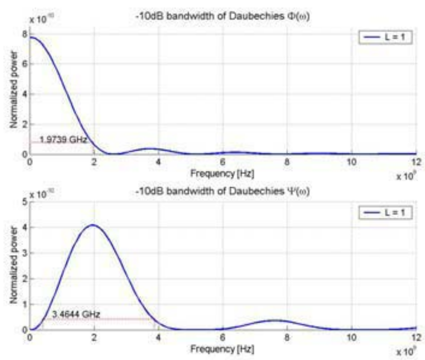

Figure 5: Achievable -10dB bandwidth for Daubechies Wavelets Pulses of Order 1

6 Conclusions

We have investigated and compared UWB signals. Suitable functions must be time and band-limited and possess high resolution properties, be orthogonal and extremely low spectral leakage into the bands used by other systems. Gaussian, Hermite, Prolate spheroidal and wavelet functions provide suitable starting points for generating time and band limited ultra wideband signals.

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