Phase contrast radiography: Image modeling and optimization

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We consider image formation for the phase-contrast radiography technique where the radiation source is extended and spatially incoherent. A model is developed for this imaging process which allows us to define an objective filtering criterion that can be applied to the recovery of quantitative phase images from data obtained at different propagation distances. We test our image model with experimental x-ray data. We then apply our filter to experimental neutron phase radiography data and demonstrate improved image quality. © 2004 American Institute of Physics.

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I. INTRODUCTION

The refractive index from x rays contains both a real and imaginary part, meaning that x rays can be both absorbed and refracted. Until recently, although acknowledged,1 it was generally assumed that the refractive effect was negligible for x-ray imaging. The advent of third-generation synchrotrons produced copious quantities of x rays with a relatively high degree of spatial coherence and it was quickly found, at least in this context, that refraction need not be negligible; indeed it was found to be a dominant source of intensity contrast in projection images.2 Subsequent work has shown that refraction-induced x-ray contrast can be turned into a very valuable analytical tool, allowing phase to be measured,3 and allowing tomographic phase images to be acquired.4

Extending the synchrotron-based results, Wilkins et al.5 showed that refraction could also be observed using laboratory sources and augmented the contrast due to absorption. This observation has generated a considerable amount of interest in the development of laboratory (as opposed to synchrotron) source x-ray phase contrast imaging. Allman et al.6 further showed that refraction contrast is also readily observable using thermal neutrons, leading to the possibility of extending the synchrotron-based results similar to those discussed by Pogany et al.8 can also be used here. In cases where the composition is unknown, the dimensionality of the problem demands that two images be acquired and the transport of intensity equation method12 may be applied. However this method can be rather unstable to noise in the low spatial frequencies and the images often display slowly varying distortion across them. The aim of the present article is to develop an imaging model that explicitly incorporates partial coherence and use it to develop methods for dealing with poorly reconstructed low spatial frequencies.

This article begins by developing an image formation model suitable for short wavelengths in the region for which the transport of intensity equation12 is valid. We then use it to predict the sensitivity of the imaging process as a function of spatial frequency. This model is then used to propose a rational method for combining images acquired over a range of propagation distances. In Sec. III, the imaging model is experimentally tested using an x-ray radiography set-up and we find that the results are in excellent agreement. In Sec. IV, we use neutron radiography data to combine neutron phase images (or phase maps) acquired at different distances into a single image according to the method proposed in Sec. II. We find that the resulting images are substantially improved.

II. X-RAY PHASE IMAGE FORMATION

A. A simple imaging model

We consider the case of projection geometry where a finite polychromatic source is used. We will assume that an appropriate polychromatic effective phase can be used13 and will concentrate here on the effect of the spatial extent of the source. We suppose that the source has a spatial distribution $\sigma(r)$ and is located a distance $z_1$ from the object. We denote the coherent intensity image of the transmission function ob-
served at the measurement plane placed a distance \( z_2 \) from the sample as \( I_{\text{coh}}(r,z_2) \). We assume that the sample influences only the phase of the beam. Moreover, we will regard the imaging system as incoherent insofar as each point on the source will produce an image of the sample that is statistically independent of that produced by a neighboring point. Thus, the measured intensity distribution can be treated as a simple convolution of the coherent image with the source intensity distribution after appropriate scaling. A simple transformation of the Fresnel diffraction integral allows us to write the resulting partially coherent image intensity as

\[
I(r) = \frac{1}{M^2} \int I_{\text{coh}} \left( \frac{1}{M}, \frac{1}{M}, \frac{1}{M}, \frac{1}{M} \right) \alpha \left( \frac{1}{M^2} \left| r - r' \right| \right) dr',
\]

where

\[
M = \frac{z_1 + z_2}{z_1}.
\]

Consider a uniform transmission object with a one-dimensional harmonic phase variation:

\[
S(r,0) = S_0 \exp[2 \pi i \Phi_m \cos(k_x \cdot r)],
\]

where \( \Phi_m \) is the maximum phase shift produced by the object. We will assume, without loss of generality, that \( k_x \cdot r = k_x x \). Assume a small propagation distance such that

\[
I_{\text{coh}}(r_2) = I_{\text{coh}}(r,0) + z_2 \frac{\partial I_{\text{coh}}(r,0)}{\partial z},
\]

where “small” means that

\[
z_2 \ll \left| I_{\text{coh}}(r) \left( \frac{\partial I_{\text{coh}}(r)}{\partial z} \right)^{-1} \right|.
\]

We describe this propagation using the transport of intensity equation

\[
\frac{\partial I_{\text{coh}}(r)}{\partial z} = \frac{1}{k_0} \nabla \cdot (I_{\text{coh}}(r) \nabla \Phi(r)),
\]

where \( k_0 = 2 \pi/\lambda \) and \( \Phi(r) \) is the phase distribution of the wave leaving the object, and is here given by the argument of the exponential term in Eq. (3).

Given the assumed form of the object, the transport of intensity equation takes the form

\[
\frac{\partial I_{\text{coh}}(r)}{\partial z} = -2 \pi S_0^2 \frac{\Phi_m}{M^2} \frac{k_x^2}{k_0} \cos \left( \frac{k_x}{M} x \right).
\]

Thus Eq. (5) becomes

\[
z_2 \ll \frac{M}{2 \pi \Phi_m k_x^2}
\]

If we now include the effect of the source intensity, written in the form that corrects for the projection, then

\[
I(r) = S_0^{\frac{2}{M^2}} \left( 1 - 2 \pi \Phi_m \frac{k_x^2}{k_0} \z_2 \int \cos \left( \frac{1}{M} k_x x \right) \right. \\
\times \left. \alpha \left( \frac{1}{M^2} \left| r - r' \right| \right) dr' \right).
\]

In order to be explicit, let us model the source as a Gaussian

\[
\alpha(r) = \frac{1}{\sigma_x \sqrt{2 \pi}} \exp \left[ - \frac{\left| r \right|^2}{2 \sigma_x^2} \right],
\]

so that Eq. (9) may be written

\[
I(r) = S_0^{\frac{2}{M^2}} \left( 1 - 2 \pi \Phi_m \frac{k_x^2}{k_0} \z_2 \sigma_x \right) \exp \left[ - \left( \frac{M^2 + 1}{M} \right)^2 \sigma_x^2 k_x^2 \right] \cos \left( \frac{k_x}{M} x \right).
\]

We introduce the following dimensionless variables:

\[
N_F = \frac{k_0 \sigma_x^2}{2 \pi z_1}
\]

which is the Fresnel number of the source at the sample plane.
which can be regarded as the number of resolution elements contained in the source distribution. That is, when $j > 1$ the modulation in the image has a period that is smaller than the projected source size. In terms of these dimensionless variables, Eq. (11) may be rewritten:

$$I(Y, X) = S_{o}^{2} \frac{1}{M^{2}} \left[ 1 - \Phi_{m} \frac{M-1}{M} \frac{\xi^2}{N_{F}} \right] \times \exp \left[ -\frac{1}{2} \left( \frac{M-1}{M} \right)^{2} \xi^2 \right] \cos \left( \frac{\xi X}{M} \right)$$

where we also use the dimensionless distance $X = x / \sigma_{r}$. As expected, it can be seen that for a point source the visibility of the phase structure increases with both the separation distance, $z_2$, and the spatial frequency of the phase modulation, $k_r$. Equation (14) may be conveniently written as

$$I(Y, X) = S_{o}^{2} \frac{1}{M^{2}} \left[ 1 - \Phi \frac{M-1}{M} \frac{\xi^2}{N_{F}} \right] \times \exp \left[ -\frac{1}{2} \left( \frac{M-1}{M} \right)^{2} \xi^2 \right] \cos \left( \frac{\xi X}{M} \right)$$

where we introduce a general phase visibility function:

$$V(\xi, M) = \frac{M-1}{M} \xi^2 \exp \left[ -\frac{1}{2} \left( \frac{M-1}{M} \right)^{2} \xi^2 \right]$$

This visibility function describes the contrast in the image as a function of spatial frequency for this model system and is plotted in Fig. 1.

The visibility is easily shown to be maximized when

$$\xi = \sqrt{2} \frac{M}{M-1}$$

Thus, as $M \to \infty$, then $\xi \to \sqrt{2}$. That is, when the magnification is much greater than unity, the sensitivity to spatial frequency becomes largely independent of the magnification; there is little point in increasing it further. However, as $M \to 1$, the peak sensitivity to spatial frequency rapidly moves to higher spatial frequencies, and can be significant for structure that is significantly smaller than the scale dictated by the source size.

We note for completeness that the validity condition given by Eq. (8) becomes

$$\frac{M-1}{M} \Phi_{m} \xi^2 \ll N_{F}$$

which, for large $M$ and $\Phi_{m} \sim 1$ reduces to $\xi^2 \ll N_{F}$.

**B. Optimal image filtering**

Subject to the validity condition [Eq. (18)], there is an optimal distance at which data concerning a particular spatial frequency should be acquired, and this distance is highly dependent on that spatial frequency. The visibility curve given by Eq. (16) describes how the spatial frequencies are recorded for different imaging conditions. Interestingly, as $M \to 1$ it is possible to resolve phase structures that are much finer than the source size.

A strategy to optimize the combination of multiple images is to weigh their contribution to the spatial frequency spectrum of the combined image according to the magnitude of the transfer function. In particular, suppose that we have acquired $N$ images of a given sample by placing the detector at a number of different distances. The high spatial frequencies will be recorded well for the low magnification data sets, and the low spatial frequencies will be recorded best for the
large magnifications. In this article we propose to combine images by weighting them according to the sensitivity to spatial frequency as follows:

\[
\hat{I}(\xi) = \sum_{i=1}^{N} \frac{\bar{V}(\xi,M_i)}{V_T(\xi)} \hat{I}_i(\xi),
\]

where

\[
V_T(\xi) = \sum_{j=1}^{N} \bar{V}(\xi,M_j).
\]

\(\hat{I}_i\) is the Fourier transform of image \(i\); and \(\bar{V}\) is the visibility function [Eq. (16)] normalized to have a maximum value of unity.

C. Implications for quantitative phase recovery

Equation (19) proposes a method for optimally combining images acquired over different propagation distances. This method is experimentally explored in Sec. IV. In this subsection we consider the effect this averaging will have on a quantitative phase image.

The generation of phase contrast arises from a spatial frequency dependence of fringe visibility of the form \(V \propto \xi^2\), arising from the structure of Eq. (6). This component is clearly apparent in the small \(\xi\) limit of Eq. (16). The exponential term in Eq. (16) describes the effect of source blurring to reduce the visibility of higher spatial frequencies. In the absence of source blurring and under the approximation of Eq. (4), all data sets contain the same information, but with a visibility that increases with \(z_2\). In this limit, all the data sets [as described by the left-hand side of Eq. (6) estimated using Eq. (4)] can be added so as to ameliorate the effects of noise.

In a noise free environment, the best phase recovery will be obtained from very small \(z_2\) because source blurring will be negligible. However, in practice, such a recovery will have a very poor reproduction of low spatial frequency information because the contrast will be very low. Increasing \(z_2\) will better reveal slowly varying structures but will result in source blurring of the higher spatial frequencies. Equation (19) produces a frequency-weighted average such that the contribution of a given data set to a particular spatial frequency is weighted by the relative visibility with which that frequency is recorded. Note, however, that apart from the effects of noise and source blurring, the averaging process does not intrinsically change the frequency spectrum of the image: it is simply a weighted average of the data sets. We therefore conclude that a better quantitative phase reconstruction should result.

In the next section the basis for the weighting is experimentally tested and a sample experimental application is described in Sec. IV.
III. AN EXPERIMENTAL TEST OF THE MODEL

A sample was created that would produce a phase modulation similar to that described by Eq. (3). Laser ablation was used to etch a grid of lines on a Kapton film (composition C\textsubscript{22}H\textsubscript{16}N\textsubscript{2}O\textsubscript{4} and density 1.45 g cm\textsuperscript{-3}). A mask projection micromachining system using a Lambda Physik LPX210i krypton fluoride excimer laser operating at 248 nm was used to etch grids with periods of 43 μm, 20 μm, and 10 μm. The depths of the grids were measured using optical microscopy and were found to be 40±5 μm, 30±5 μm, and 20±3 μm, respectively. Micrographs of three samples are shown in Fig. 2.

The experiments were performed with a conventional Feinfocus micro focus x-ray tube source (model FXE-160.50 fitted with x-ray tube FXT-160.20) containing a Cu target. The tube voltage was 20 kV and current 400 μA. The spectrum of the source was measured and corresponded to average x-ray energy of 11.7 keV. This is the energy used to calculate the phase shift. At this energy, the transmission age x-ray energy of 11.7 keV. This is the energy used to etch grids with periods of 43 μm, 20 μm, and 10 μm. The depths of the grids were measured using optical microscopy and were found to be 40±5 μm, 30±5 μm, and 20±3 μm, respectively. Micrographs of three samples are shown in Fig. 2.

The source size was measured by observing the shadow of an edge and was found to have a horizontal FWHM of 14±5 μm and a vertical FWHM of 19±5 μm. Thus the theory could be tested for two different source sizes by placing the sample either horizontally or vertically. The sample was placed at a distance of 10.0±0.5 cm from the source and the detector was placed at 171±2 cm from the sample. The image was recorded using a direct-detection CCD camera (liquid nitrogen cooled Photometrics CH260 fitted with a TK 512 CCD chip with 512×512, 27 μm×27 μm pixels). A sample image is shown in Fig. 3.

The intensity images were corrected for the dark current image and for nonuniformities in the imaging system. The visibility is then obtained from the images and compared to the prediction of Eq. (16). The results are shown in Fig. 4 for the two sample orientations. This figure also shows the predictions of Eq. (16) using a horizontal FWHM of 10 μm and a vertical FWHM of 18 μm, consistent with the independent source size measurement. There is a good agreement between theory and experiment which gives us confidence that our model can be used to develop an approach to optimal filtering.

IV. AN OPTIMALLY FILTERED IMAGE

In order to explore phase imaging, neutron data was collected at the National Institute of Standards Technology (NIST), NG0 Neutron Depth Profiling Facility, NIST Center for Neutron Research (NCNR), Gaithersburg, MD, using a thermal distribution of neutron wavelengths about a peak of 4.32 Å. The experimental set up is shown in Fig. 5. The source was a beam of neutrons delivered by a neutron guide. This beam was incident on a 200 μm pinhole, then another 25.4 mm aperture about 1.5 m further downstream, defining the spatial coherence, and maximum beam divergence respectively. The sample was placed 1.8 m from the first pinhole. The sample consisted of a tapered lead slug approximately 15 mm in length and 8 mm in maximum diameter, with an approx. 1.5 mm hole drilled through the center (Fig. 6). Intensity images were recorded using an NE426 neutron scintillator screen coupled with an optical CCD camera with 512×512 pixels of 50 μm size. Three such planes of detection were collected at differing longitudinal distances from the sample allowing two different phase images to be reconstructed. The phase images presented here were reconstructed using the data from planes located approximately at distances of 11.7 cm and 64.0 cm, and 64.0 cm and 471.2 cm from the sample, respectively.

The first reconstruction can be assumed to be made at the mid plane between the two data sets and so is at a distance of approximately 38 cm from the sample, corresponding to M ≈ 1.21. The second reconstruction will correspond to a distance of 268 cm, with M ≈ 2.49. The two phase images are recovered using the TIE (Ref. 6) and in Fig. 7 we see that each of these images contain significantly different information. This is a manifestation of the frequency dependence on longitudinal distance from detector described earlier. As expected the closer image [Fig. 7(a)] contains more information concerning higher spatial frequencies. In contrast the far image [Fig. 7(b)] contains more information regarding the lower spatial frequencies. The image combination strategy described by Eqs. (19) and (20) was used to combine the two images and the result is shown in Fig. 7(c).

This image combination led to a marked increase in image quality across the spatial frequency spectrum. This can be seen in the obvious improvement in the image quality apparent in Fig. 7. However the quantitative improvement is apparent via a comparison between the standard deviation of the phase reconstructions with a calculation based on our knowledge of the composition and size of the sample and the standard deviation of the combined phase image from the predicted image was found to be improved by a factor of 8 over the image in Fig. 7(a), and by a factor of 2 over the image in Fig. 7(b).

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