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Snobs and Quality Gaps

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Abstract

The paper characterizes the optimal provision of quality by a monopolist facing a population of consumers with private valuation for quality. Unlike previous models by Mussa and Rosen (1978) and others, this paper assumes there is a mass of consumers who prefer the highest quality goods. I liken these consumers to snobs who demand the highest valued goods. I show that the quality supplied jumps discontinuously as the highest valued consumers are encountered and the variety of products is reduced as the population of snobs increases. I also show that only snobs may be supplied once their population grows to a critical size.

KEYWORDS: screening, product line, quality gap

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1. INTRODUCTION

Most papers in the screening literature use models where the distribution of types is either discrete (e.g. Adams and Yellen, 1976) or absolutely continuous with respect to the Lebesgue measure (e.g. Mussa and Rosen, 1978). Two notable exceptions are the papers by Lewis and Sappington (1993) and Cremer, Khalil, and Rochet (1998). In these papers with some probability a consumer remains uninformed about her own type. Mathematically, such a problem is equivalent to a problem in which all the consumers are informed about their types, but the types distribution has a mass point at the average type in the population.

Lewis and Sappington (1993) and Cremer, Khalil, and Rochet (1998) showed that quality offered by the monopolist jumps discontinuously at the mass point. The variety of the product qualities served to types below the population average decreases and the distortions in this region increase in the probability that a consumer stays uninformed. Moreover, there is a pooling region to the right of the mass point and consumers whose type coincide with the mass point are served efficiently.[1]

In this paper I consider a model in which the distribution of types is absolutely continuous on $(0, 1)$ but possesses an atom at one. I call the consumers of the highest possible type *snobs*.

One can justify the assumption of a positive mass at the right end of the type space in several ways. For example, think of a monopolist as a tourist agency that offers different types of accommodations. Let the quality of accommodation be determined by its distance from a beach. Suppose that after the agency posts its offers, potential consumers hear the weather forecast. If the weather forecast is favorable, which happens with some positive probability, everyone has the same highest possible marginal rate of substitution between the quality of accommodation and money. If, on the other hand, the weather forecast is not so favorable, the marginal rates of substitution are different among the potential consumers and are distributed on $(0, 1)$ according to some density function. Since the agency has to commit to the set of offers *before* the weather forecast is known, from the agency's point of view, the distribution of types has an atom at one.

Another way to justify a positive mass at the right boundary of the type space is to assume that the population contains individuals who are determined to get the highest quality product (e.g. to drive a Cadillac) and are therefore, willing to pay for a marginal increase in quality at least as much as any other member of the population. This kind of behavior justifies the name *snobs* I employ for the consumers of the highest possible type in this paper.

The first result, probably not surprisingly, is that top types are still served efficiently while the lowest type earns no information rents. An important difference from the usual "no distortion at the top" result, however, is that the quality does not approach the efficient level as the type converges to its maximal possible value, i.e. the product line has a *quality gap*. In other words,

[1] Note, however, that a posteriori, (i.e. after the consumers learn there type) the measure of consumers who are served efficiently is zero in these models.

the downward distortion for the types infinitely close to the highest type is finite. To get some intuition for that result, recall that the main trade-off the monopolist faces is between minimizing the information rents left to the higher types and serving the lower types more efficiently. The downward distortion of quality for the lower types serves to decrease the information rents paid to the higher ones. Since a positive fraction of consumers are of the highest possible type, as opposed to the zero mass of the consumers of any other type, the need to minimize the information rents left to them is particularly pressing. As a result, the downward distortion of quality for the types infinitesimally close to it becomes finite.

The second result is that the variety of product qualities decreases and distortions increase in the mass of the snobs. Moreover, if the mass of snobs is above some critical level, then snobs are the only ones served at equilibrium. For a wide class of distributions this level depends only on the density of the types distribution at the vicinity of point one and is unrelated to the finer details of the distribution. The second result is the direct consequence of the first. Indeed, both stronger distortions and smaller product variety are consequences of the discrete downward jump in allocation at the right end of the types distribution.

It is interesting to compare the results of this paper with those of Lewis and Sappington (1993) and Cremer, Khalil, and Rochet (1998)[2]. Both of those models have a quality gap and the variety of products decreases as the mass of the atom in the distribution increases. The crucial difference, however, is that in those models the distortions in the pooling region (in the right neighborhood of the mass point) are smaller than they would have been had all the consumers been informed. In my model, on the contrary, the distortions are exacerbated for all types by the presence of snobs.

Another contribution of this paper is methodological. The techniques used to solve the problem are much simpler than those of Lewis and Sappington (1993) and can be easily adapted for their problem. Moreover, it is straightforward to generalize the techniques to probability measures that possess any finite number of atoms.[3]

The paper is organized in the following way. In Section 2 I introduce the model and discuss its general properties. In Section 3 I solve an example. Section 4 concludes.

2. THE MODEL

Consider a continuum of consumers each of whom is interested in buying at most one unit of an indivisible good. Different units of the good may, however, differ in quality, x. The marginal rate of substitution between quality and

[2] Cremer, Khalil, and Rochet (1998) in fact consider a harder problem. They allow the principal to offer two tariffs: one intended for informed and the other for uninformed consumers.

[3] A general probability measure on the real line can be decomposed into an absolutely continuous part, a discrete part, and a singular part. Using the ideas of this paper, one can approach a problem where the type distribution is presented by a probability measure with a discrete and an absolutely continuous components.

money, α, does not depend on quality but differs across consumers, i.e. utility takes the form

$$u(\alpha, x, t) = \alpha x - t, \tag{1}$$

where t is the amount paid to the monopolist. I assume that α is private information of the consumer. However, it is common knowledge that α is distributed on $[0, 1]$ according to probability measure

$$\mu = (1 - \gamma)\lambda + \gamma\delta_1, \tag{2}$$

where $\gamma \in (0, 1)$ is the mass of the snobs, measure λ is absolutely continuous with respect to the Lebesgue measure with the Radon-Nykodim derivative $f(\cdot)$, while δ_1 is the Dirac's measure concentrated at one. We assume that f is twice differentiable, strictly positive, and weakly increasing on $[0, 1)$ and

$$\lim_{\alpha \to 1} f(\alpha) \equiv f(1) < \infty. \tag{3}$$

Let $F(\cdot)$ be the cumulative distribution function, corresponding to density $f(\cdot)$. Define the virtual type

$$v(\alpha) = \alpha - \frac{1 - F(\alpha)}{f(\alpha)}, \tag{4}$$

and assume that it is strictly increasing in α. This assumptions allows us to concentrate on the so-called relaxed problem, i.e. the problem in which we drop the constraint that allocation $x(\cdot)$ is increasing (see, Mussa and Rosen, 1978).

The utility of the outside option is the same across consumers and is normalized to be zero. The cost of production is convex in quality and linear across consumers and is given by a twice differentiable, strictly increasing, strictly convex function $c(\cdot)$. For technical reasons, I will strengthen the convexity assumption assuming that

$$\exists \eta > 0 \text{ such that } c''(x) > \eta \text{ for } \forall x \in R_+. \tag{5}$$

I also assume that

$$c(0) = c'(0) = 0. \tag{6}$$

The above consideration can be summarized by the following model. The monopolist selects a measurable function $t : R_+ \to R_+$ to solve

$$\max_{t(\cdot)} \int_0^1 (t(x(\alpha)) - c(x(\alpha)))d\mu(\alpha), \tag{7}$$

subject to

$$x(\alpha) \in \arg\max(u(\alpha, x) - t(x)) \tag{8}$$

$$\max(u(\alpha, x) - t(x)) \geq 0. \tag{9}$$

The first of these constraints is known as the incentive compatibility constraint and guarantees that each consumer selects the quality optimally, while the second is the individual rationality or participation constraint that states that each consumer should get utility, which is at least as large as that of the outside option.

Introducing the consumer surplus by

$$s(\alpha) = \max_{x \geq 0}(\alpha x - t(x)) \tag{10}$$

one can write the monopolist's objective as:

$$\int_0^1 (\alpha x - c(x) - s) d\mu(\alpha). \tag{11}$$

Using (2) this expression can be transformed to:

$$(1-\gamma)\int_0^1 (\alpha x - c(x) - s) f(\alpha) d\alpha + \gamma(x(1) - c(x(1)) - s(1)). \tag{12}$$

Note that equation (10) and the envelope theorem[4] imply:

$$s'(\alpha) = x(\alpha) \tag{13}$$

for almost all $\alpha \in (0,1)$. To proceed further we need the following result:

Theorem 1 *Function $s(\cdot)$ defined by equation (10) is absolutely continuous.*

Proof. Take any $\varepsilon > 0$ and let $\alpha_0 < \alpha_1 < ... < \alpha_{2n+1}$ be such that

$$\sum_{i=0}^{n} (\alpha_{2i+1} - \alpha_{2i}) < \delta. \tag{14}$$

for some $\delta > 0$. Let P denote the optimal product line.[5] Using equation (10) one obtains:

$$|s(\alpha_{2i+1}) - s(\alpha_{2i})| \leq (\max_{x \in P} x)(\alpha_{2i+1} - \alpha_{2i}). \tag{15}$$

Our assumptions on the cost function and the compactness of the type space imply that $\exists x^* > 0$ such that $\forall x > x^*$ and $\forall \alpha \in [0,1]$

$$\alpha x - c(x) < 0. \tag{16}$$

Since qualities above x^* generate negative surplus they will be never offered, i.e. x^* provides a uniform (independent of the types distribution) bound on the product line. Therefore,

$$\sum_{i=0}^{n} |s(\alpha_{2i+1}) - s(\alpha_{2i})| \leq x^* \sum_{i=0}^{n} (\alpha_{2i+1} - \alpha_{2i}) < x^* \delta. \tag{17}$$

[4] See Milgrom and Segal (2002) for the most general formulation of the envelope theorem.

[5] The set of the product qualities offered by the monopolist is known in the literature as the product line.

Choosing $\delta = \varepsilon / x^*$ we establish that

$$\sum_{i=0}^{n} |s(\alpha_{2i+1}) - s(\alpha_{2i})| < \varepsilon, \quad (18)$$

provided

$$\sum_{i=0}^{n} (\alpha_{2i+1} - \alpha_{2i}) < \delta, \quad (19)$$

i.e. $s(\cdot)$ is absolutely continuous.

Q. E. D.

Equation (13) and Theorem 1 allow us to write (see, Kolmogorov and Fomin, 1970):

$$s(1) = s(0) + \int_0^1 x(\alpha) d\alpha. \quad (20)$$

Transforming the first integral in (12) using integration by parts (see, Mussa and Rosen, 1978) and taking into account (20), the monopolist's objective takes the form:

$$(1-\gamma) \int_0^1 [(v(\alpha)x - c(x))f(\alpha) - \frac{\gamma x}{1-\gamma}] d\alpha + \gamma(x(1) - c(x(1)) - s(0). \quad (21)$$

The optimally conditions now imply,

$$s(0) = 0. \quad (22)$$

i.e. the lowest type gets her reservation utility,

$$c'(x(1)) = 1, \quad (23)$$

the "no distortion at the top" property and

$$c'(x) = \max(v_\gamma(\alpha), 0) \quad (24)$$

for $\alpha \in [0, 1)$. Here

$$v_\gamma(\alpha) = v(\alpha) - \frac{\gamma}{(1-\gamma)f(\alpha)}. \quad (25)$$

Note, that under our assumption on the distribution of types $v_\gamma(\alpha)$ increases in α for all γ. Therefore, since the cost is convex in x, the allocation defined by (24) is increasing and therefore, implementable.

Also note that

$$v_\gamma(\alpha) < v(\alpha) \quad (26)$$

for all α. This has three consequences. First, if type α is served under both conditions $\gamma = 0$ and $\gamma > 0$, equation (26) implies that the downward distortion is stronger under the second regime. Second, the exclusion region increases in γ in the set theoretic sense, i.e. if $\gamma_1 > \gamma_2$ then $\Omega_0(\gamma_2) \subset \Omega_0(\gamma_1)$, where $\Omega_0(\gamma)$

is the exclusion region for a given value of γ. Third, the product line defined by:

$$[0, v_\gamma(1)) \cup \{x(1)\} \tag{27}$$

is decreasing in γ (i.e. the variety of products offered by the monopolist decreases in γ) and is not connected for $\gamma > 0$. Note that if

$$v_\gamma(1) \leq 0 \tag{28}$$

for all values of α, the snobs are the only ones served in the equilibrium. This happens if

$$\frac{\gamma}{1-\gamma} \geq f(1). \tag{29}$$

From this equation one can observe that γ depends only on the value of $f(1)$ (it is increasing in $f(1)$) and not on the finer details of the distribution. Also, since $f(\cdot)$ is assumed to be non-decreasing and integrate to one, $f(1) \geq 1$. Therefore, in order to exclude all consumers apart from the snobs γ should be at least 1/2.

3. A NUMERICAL EXAMPLE

Let us consider a specific case of the model introduced in the previous section. Assume that

$$f(\alpha) = 1, \tag{30}$$

the cost of production is quadratic

$$c(x) = \frac{1}{2}x^2, \tag{31}$$

and $\gamma = 1/3$. Then

$$v_\gamma(\alpha) = 2\alpha - \frac{3}{2}. \tag{32}$$

The exclusion region is $[0, 3/4]$ which is a superset of $[0, 1/2]$, the exclusion region for $\gamma = 0$. The product line is

$$[0, \frac{1}{2}) \cup \{1\}, \tag{33}$$

where qualities in range $[0, 1/2)$ are purchased by the consumers whose types belong to $[3/4, 1)$, while the snobs purchase the good of quality one. A good with quality $x \in [0, 1/2)$ can be purchased at a price

$$t(x) = \frac{x^2 + 3x}{4}, \tag{34}$$

while the good of quality one can be purchased at

$$t(1) = \frac{15}{16}. \tag{35}$$

Note that the snobs are served efficiently and are indifferent between selecting their contract or purchasing a good of quality 1/2 at $t(1/2) = 7/16$. However, since the good of quality exactly 1/2 is not offered by the monopolist, they strictly prefer their contract to any other deal offered by the market.

4. CONCLUSIONS

In this paper I take a step to building a screening model, where the type distribution is given by an arbitrary probability measure of the real line. For this purpose, I revisit the Mussa and Rosen (1978) model. However, unlike Mussa and Rosen, I assume that there is a positive mass of the consumers of the highest possible type (snobs) and develop a technique that allows us to solve the problem in this case.

The technique used in the paper can be easily generalized to cases when the mass point is interior (as it is, for example, in Lewis and Sappington (1993) and Cremer, Khalil, and Rochet (1998) papers) or when it is not unique. The main challenge to building a general unidimensional screening model with a continuum of types is the problem of dealing with singular distributions of types. To the best of my knowledge, there are no papers in the literature that address this question.

The main results of the paper are the following: the snobs are served efficiently, the optimal quality is discontinuous at the right hand of the types distribution, i.e. the optimal product line is not connected, and the product line decreases in the mass of snobs. Moreover, if the mass of snobs exceeds some critical level then they are the only consumers who are served at the equilibrium.

To understand these results, recall that the downward distortion of quality for the lower types serves to decrease the information rents paid to the higher ones. Since a positive fraction of consumers are of the highest possible type, as opposed to the zero mass of the consumers of any other type, the need to minimize the information rents left to them is particularly pressing. As a result, the downward distortion for the types infinitesimally close to it becomes finite. This finite downward jump, in turn, results in a decreased variety of the products served by the monopolist. If the effect is sufficiently strong, the quality can jump all the way to zero, i.e. nobody but snobs is served at the equilibrium.

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