Blind Multichannel Identification via Frequency Selection

Song Wang * and Jonathan H. Manton†

*Department of Electronic Engineering
La Trobe University
Bundoora, VIC 3086, Australia
Tel: +61-3-9479 3744, Fax: +61-3-9471-0524
E-mail: song.wang@latrobe.edu.au

†Department of Information Engineering
Research School of Information Sciences & Engineering
Australian National University
Canberra ACT 0200, Australia
Tel: +61-2-6125-1531, Fax: +61-2-6125 8651
E-mail: j.manton@ieee.org

***Abstract*— In a recent study on blind multichannel identification, we presented a fast Fourier transform based algorithm, called the BI-FFT algorithm. The BI-FFT algorithm is capable of dealing with channel outputs of a very short length, which is of practical importance in communication applications where wireless channels involve high mobility. However, when the output sample size is relatively large, the BI-FFT algorithm becomes cumbersome and computationally costly. To address this issue, we develop a frequency selection approach in this paper. The proposed method strategically chooses those frequency bins that significantly contribute to the error dynamics in the channel estimation process. As a result, the row dimension of a key matrix in the identification equation is considerably reduced, effectively decreasing the computational cost. With a comparable performance, the new method is much more efficient in computation than the original BI-FFT algorithm in case of long data sequences.**

I. Introduction

Blind identification of multiple FIR channels driven by a common input is a problem of fundamental interest in signal processing and has found abundant practical applications in data communication systems, see [1] and the references therein. The problem of blind multichannel identification is to estimate the channel impulse response using only output observations as the input signal is either unknown or inaccessible. Despite the early works of higher-order statistics (HOS) based blind identification, e.g., [2] - [5], since the pioneering work of [6], [7], the study of second-order statistics (SOS) based blind channel identification has seen rapid development; see [1] and [8] and the references therein. Using SOS for blind identification considerably improves the convergence rate compared to HOS based algorithms. There is a rich literature on SOS based blind identification techniques, which include the following representative algorithms: the subspace [9], cross relation [10], oblique projection [11], and shifted correlation [12] [13] algorithms.

In a recent study [15], we presented a fast Fourier transform (FFT) based blind channel identification method, called the BI-FFT algorithm. By exploiting the cross relation (CR) between each channel output pair, which had been explored in the time domain in [10], we extended the CR property to the frequency domain via the discrete Fourier transform (DFT) in [15]. The BI-FFT algorithm successfully handles very short data sequences, for which the existing SOS based methods, e.g., [9] - [13], are known to suffer performance degradation due to inaccurate statistics. However, when the output sample size is large, the computational load of the BI-FFT algorithm increases in a squared term, making it computationally expensive. Therefore, in order to utilise the BI-FFT algorithm in large data sample situations, it is necessary to reduce its computational cost.

In this paper, we develop a frequency selection approach to improve the computational efficiency of the BI-FFT algorithm [15] when channel output sequences are comparatively long. The proposed approach strategically chooses those frequency bins which potentially make significant contributions to the error dynamics in the channel estimation process. By this means, the row dimension of a key matrix in the identification equation is effectively cut down, resulting in a sizable reduction in computation. The new method can be viewed as a computationally efficient version of the BI-FFT algorithm [15] in case of a large size of observation data. Simulations show that the performance of the frequency selection approach is comparable to that of the original BI-FFT algorithm.

The remainder of the paper is organised as follows. Section II sets forth the multichannel model and reviews the BI-FFT algorithm. Section III describes the frequency selection approach. In Section IV, a simulation example with a relatively large sample size is presented to compare the performance of the proposed approach and the BI-FFT algorithm. Conclusion

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is given in Section V.

II. Overview of the BI-FFT Algorithm

In this section, we briefly review the BI-FFT algorithm [15], which is based on the following discrete-time FIR L-channel model:

$$x_m(n) = s(n) * h_m(n) + w_m(n), \quad m = 1, 2, \cdots, L \qquad (1)$$

where $x_m(n) \in \mathcal{C}$ is the mth channel output at time n with $\mathcal{C}$ denoting the set of complex numbers, $s(n) \in \mathcal{C}$ is the common input, $h_m(n) \in \mathcal{C}$ the impulse response of the channel m, and $w_m(n) \in \mathcal{C}$ the additive noise at the channel m, which is uncorrelated with the source signal. The symbol $*$ in (1) denotes convolution. The maximum order M of all channels is assumed to be known. The BI-FFT algorithm extends the CR property between channel outputs of (1) to the frequency domain. Specifically, when each channel output sequence has a finite duration of length N_s, for any $1 \leq i, j \leq L$, $i \neq j$, the time-domain CR

$$x_i(n) * h_j(n) = x_j(n) * h_i(n)$$

can be transformed to the frequency domain via the DFT as

$$X_i(k)H_j(k) = X_j(k)H_i(k), \quad k = 0, 1, \cdots, N-1 \qquad (2)$$

where $X_m(k)$ and $H_m(k)$ represent the N-point frequency-domain samples of $x_m(n)$ and $h_m(n)$, respectively, with $m = i, j$, and the DFT size $N \geq N_s + M$. By the definition of the DFT,

$$H_m(k) = \sum_{n=0}^{N-1} h_m(n) e^{-j2\pi kn/N} = \sum_{n=0}^{M} h_m(n) e^{-j2\pi kn/N}$$

equation (2) can be rewritten as

$$[-\boldsymbol{F}_j \quad \boldsymbol{F}_i] \begin{bmatrix} \mathbf{h}_i \\ \mathbf{h}_j \end{bmatrix} = \mathbf{0} \qquad (3)$$

where $\mathbf{h}_m = [h_m(0) \quad h_m(1) \quad \cdots \quad h_m(M)]^T$, and

$$\boldsymbol{F}_m = \text{diag}\{X_m(0), X_m(1), \cdots, X_m(N-1)\} \cdot \boldsymbol{V} \qquad (4)$$

with $m = i, j$, $V = e^{-j2\pi/N}$, and

$$\boldsymbol{V} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & V & V^2 & \cdots & V^M \\ \vdots & & \vdots & & \vdots \\ 1 & V^{N-1} & V^{2(N-1)} & \cdots & V^{M(N-1)} \end{bmatrix} \in \mathcal{C}^{N \times (M+1)}$$

Considering all L channels, in the absence of noise, from equation (3), we can construct the following augmented equation:

$$\boldsymbol{F}\mathbf{h} = \mathbf{0} \qquad (5)$$

where $\mathbf{h} = \left[\mathbf{h}_1^T \quad \mathbf{h}_2^T \quad \cdots \quad \mathbf{h}_L^T\right]^T \in \mathcal{C}^{L(M+1)}$, and

$$\boldsymbol{F} = \left[\boldsymbol{F}_{1,2}^T \; \boldsymbol{F}_{1,3}^T \; \cdots \; \boldsymbol{F}_{1,L}^T \; \boldsymbol{F}_{2,3}^T \; \cdots \; \boldsymbol{F}_{2,L}^T \; \cdots \; \boldsymbol{F}_{L-1,L}^T\right]^T$$

with $\boldsymbol{F}_{i,j} \in \mathcal{C}^{N \times L(M+1)}$ defined as

$$\boldsymbol{F}_{i,j} = [\mathbf{0} \; \cdots \; \mathbf{0} \; -\boldsymbol{F}_j \; \mathbf{0} \; \cdots \; \mathbf{0} \; \boldsymbol{F}_i \; \mathbf{0} \; \cdots \; \mathbf{0}]$$

where the ith entry is $-\boldsymbol{F}_j$ and the jth entry is $\boldsymbol{F}_i$, $1 \leq i, j \leq L$, $i \neq j$. The dimension of $\boldsymbol{F}$ in (5) is $\frac{NL(L-1)}{2} \times L(M+1)$.

The identification equation (5) is the core of the BI-FFT algorithm [15]. In the absence of noise, the channel coefficient vector $\mathbf{h}$ can be uniquely (up to a constant scalar) identified by solving (5). In the presence of noise, the channel estimate $\hat{\mathbf{h}}$ is given by $\hat{\mathbf{h}} = \text{argmin}_{\|\mathbf{h}\|=1} \|\boldsymbol{F}\mathbf{h}\|_2$.

III. The Frequency Selection Approach

The main computational load in the implementation of the BI-FFT algorithm [15] comes from the singular value decomposition (SVD) of the matrix $\boldsymbol{F}$ in (5). Because the row dimension of $\boldsymbol{F}$ increases with the output data length, resulting in the growth in SVD operations with a factor of N^2, as pointed out in [15], the BI-FFT algorithm is more suitable for a small sample size. In case of a large number of output data, the BI-FFT algorithm becomes cumbersome and computationally expensive. In this section, we develop a method to strategically reduce the computational cost of the BI-FFT algorithm [15] when output data sequences are relatively long, while maintaining the estimation performance comparable to that of the original BI-FFT algorithm.

In general, equation (5) represents an overdetermined system. Due to the fact that in the absence of noise, the channel impulse response can be identified uniquely up to a constant scalar, the matrix $\boldsymbol{F}$ in (5) is of rank $L(M+1) - 1$, i.e., $\boldsymbol{F}$ is rank deficient. That is, in the absence of noise, $\boldsymbol{F}$ must have linearly dependent rows, which can be regarded as "unimportant" as they make no contribution to determining the channel coefficient vector $\mathbf{h}$. Based on this line of thinking, when noise is present, if those important or useful rows of $\boldsymbol{F}$ can be picked out, forming a new matrix with fewer rows than $\boldsymbol{F}$, then we effectively lower the computational cost of SVD.

In the presence of noise, the CR between each channel output pair fails and an error vector $\boldsymbol{E}_{i,j} \in \mathcal{C}^N$ for channel pair (i, j), with $1 \leq i, j \leq L$ and $i \neq j$, can be defined as

$$\boldsymbol{E}_{i,j} = \boldsymbol{F}_{i,j}\mathbf{h}$$

It follows that

$$\|\boldsymbol{E}_{i,j}\|_2^2 = \sum_{k=0}^{N-1} |E_{i,j}^{(k)}|^2$$

where $E_{i,j}^{(k)}$ denotes the kth element of $\boldsymbol{E}_{i,j}$. In fact, $E_{i,j}^{(k)}$ represents the error signal for channel pair (i, j) at frequency k, $k = 0, 1, \cdots, N-1$. The 2-norm of the overall error vector for the multichannel model (1) is

$$\begin{aligned} \|\boldsymbol{F}\mathbf{h}\|_2^2 \;\; = \;\; & \|\boldsymbol{E}_{1,2}\|_2^2 + \|\boldsymbol{E}_{1,3}\|_2^2 + \cdots + \|\boldsymbol{E}_{1,L}\|_2^2 + \\ & \|\boldsymbol{E}_{2,3}\|_2^2 + \cdots + \|\boldsymbol{E}_{2,L}\|_2^2 + \cdots + \|\boldsymbol{E}_{L-1,L}\|_2^2 \end{aligned}$$

For each channel pair (i, j), with $1 \leq i, j \leq L$ and $i \neq j$, we attempt to find those frequencies at which $E_{i,j}^{(k)}$ significantly contribute to $\|\boldsymbol{E}_{i,j}\|_2$. Then we can approximate $\|\boldsymbol{E}_{i,j}\|_2^2$ by adding up $|E_{i,j}^{(k)}|^2$ over those selected frequencies.

Let $\boldsymbol{F}_{i,j}^{(k)}$ denote the kth row of $\boldsymbol{F}_{i,j}$. The error signal $E_{i,j}^{(k)}$ at frequency k is described by

$$E_{i,j}^{(k)} = \boldsymbol{F}_{i,j}^{(k)}\mathbf{h}$$

According to the vector norm properties [14],

$$|\boldsymbol{F}_{i,j}^{(k)}\mathbf{h}| \leq \|\boldsymbol{F}_{i,j}^{(k)}\|_2\|\mathbf{h}\|_2 \quad (6)$$

and

$$|\boldsymbol{F}_{i,j}^{(k)}\mathbf{h}| \leq \|\boldsymbol{F}_{i,j}^{(k)}\|_\infty\|\mathbf{h}\|_1 \quad (7)$$

Hence, the magnitude of $E_{i,j}^{(k)}$ is upper bounded by the right-hand side of (6) or (7). In other words, for different frequency bins, the "strength" of $E_{i,j}^{(k)}$ is reflected by $\|\boldsymbol{F}_{i,j}^{(k)}\|_2$ or $\|\boldsymbol{F}_{i,j}^{(k)}\|_\infty$. Because $\|\boldsymbol{F}_{i,j}^{(k)}\|_\infty \leq \|\boldsymbol{F}_{i,j}^{(k)}\|_2$, $\|\boldsymbol{F}_{i,j}^{(k)}\|_\infty$ is a better indicator for gauging $|E_{i,j}^{(k)}|$ than $\|\boldsymbol{F}_{i,j}^{(k)}\|_2$. Therefore, we may select p frequency bins at which $\|\boldsymbol{F}_{i,j}^{(k)}\|_\infty$ constitute the p largest values over all frequencies. The range of p is $\frac{2L(M+1)-2}{L(L-1)} < p < N$, with the lower bound of p obtained by ensuring $\frac{pL(L-1)}{2} > L(M+1)-1$. Thanks to the special structure of $\boldsymbol{F}_{i,j}^{(k)}$,

$$\|\boldsymbol{F}_{i,j}^{(k)}\|_\infty = \max_{1\leq i,j\leq L,\ i\neq j}\{|X_i(k)|, |X_j(k)|\} \quad (8)$$

We now summarize the frequency selection approach.

(i) For each channel pair (i, j), with $1 \leq i, j \leq L$ and $i \neq j$, according to (8), compute $\|\boldsymbol{F}_{i,j}^{(k)}\|_\infty$ for $k = 0, 1, \cdots, N-1$.

(ii) For each channel pair (i, j), sort $\|\boldsymbol{F}_{i,j}^{(k)}\|_\infty$ in either ascending or descending order.

(iii) Choose p frequency bins which produce p largest values of $\|\boldsymbol{F}_{i,j}^{(k)}\|_\infty$ over all frequencies for each channel pair (i, j). Pick out those rows of $\boldsymbol{F}$ corresponding to the p frequency bins with respect to each $\boldsymbol{F}_{i,j}$ and construct a new matrix $\tilde{\boldsymbol{F}} \in \mathcal{C}^{\frac{pL(L-1)}{2}\times L(M+1)}$. The channel coefficient vector $\mathbf{h}$ is estimated by $\min_{\|\mathbf{h}\|=1}\|\tilde{\boldsymbol{F}}\mathbf{h}\|_2$.

Remarks:

1) In the proposed approach, frequency bins are selected for each individual channel pair. This is because each channel pair plays an equal role in forming $\boldsymbol{F}$ in the identification equation (5). This balancing should be maintained in the course of frequency selection.
2) The number of frequency bins to be chosen is a trade-off between estimation accuracy and computational efficiency. The allowable range of frequency bins is $\frac{2L(M+1)-2}{L(L-1)} < p < N$. The fewer frequency bins selected, the less computation in the identification process, but the more estimation inaccuracy. If the number of frequency bins is set to be N, then the proposed method coincides with the BI-FFT algorithm. From our extensive simulations with channel outputs of a long duration, choosing half of the frequency bins renders a performance comparable to that of the original BI-FFT algorithm [15]. When we select half of the frequency bins, the computational cost of the proposed method is about a quarter of that of the original algorithm.

IV. SIMULATION

As the frequency selection method is intended to reduce the computational cost of the BI-FFT algorithm [15] in the face of long data sequences, we conducted computer simulations to test the proposed method and compare its performance with that of the BI-FFT algorithm [15]. We used the same 4-channel example in [15] with channel order $M = 4$. The impulse response of each channel is given in Table I.

TABLE I
CHANNEL IMPULSE RESPONSE

	$h_1(n)$	$h_2(n)$	$h_3(n)$	$h_4(n)$
$n=0$	-0.049+0.359i	0.443-0.0364i	-0.211-0.322i	0.417+0.030i
$n=1$	0.482-0.569i	1.0	-0.199+0.918i	1.0
$n=2$	-0.556+0.587i	0.921-0.194i	1.0	0.873+0.145i
$n=3$	1.0	0.189-0.208i	-0.284-0.524i	0.285+0.309i
$n=4$	-0.171+0.061i	-0.087-0.054i	0.136-0.19i	-0.049+0.161i

The number of output samples from each channel was 500, and the FFT size chosen to be 512. White Gaussian noise was added to the channel outputs. We computed the mean-square-error (MSE) to be the performance measure:

$$\text{MSE (dB)} = 10\log_{10}\left(\frac{1}{T}\sum_{i=1}^{T}\|\hat{\mathbf{h}}_i - \mathbf{h}\|^2\right)$$

where T is the number of Monte Carlo runs ($T = 100$ was used), $\mathbf{h}$ is the true channel coefficient vector (with unit-norm), and $\hat{\mathbf{h}}_i$ is the estimated coefficient vector (with unit norm) from the i-th run.

For the frequency selection approach, we selected half of the frequency bins, i.e., 256 frequencies for each channel pair. Fig. 1 compares the MSE (in dB) of the proposed frequency selection method and the original BI-FFT algorithm [15] over a wide range of SNR. It can be seen that the new method performs comparably with the BI-FFT algorithm. In particular,

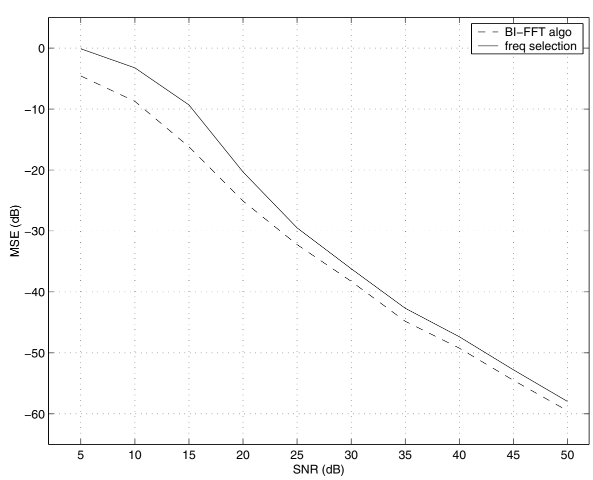

Fig. 1. Performance comparison of the frequency selection method and the BI-FFT algorithm. Output sample size is 500.

in the high SNR region (say SNR more than 30dB), the simulation result shows that the performance of the proposed method is very close to that of the BI-FFT algorithm. For the multichannel example in the simulation, since we only chose half frequency bins, in other words, the row dimension of $\boldsymbol{F}$ in (5) was halved, the computational load of the frequency selection method was just a quarter of that of the original algorithm.

V. Conclusion

We have developed a frequency selection method to address the issue of computational cost associated with long data sequences in the implementation of the BI-FFT algorithm [15]. By selecting the frequency bins that make significant contributions to the error dynamics, we effectively downsize the row dimension of a key matrix which plays a main role in the computation of channel impulse response estimate. The proposed approach greatly improves the computational efficiency of the original BI-FFT algorithm [15] when the output sample size is relatively large. The performance of the frequency selection method is close to that of the original algorithm, as illustrated by the simulation result.

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