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Auctions with Opportunistic Experts

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Auctions with Opportunistic Experts*

Suren Basov and Svetlana Danilkina

Abstract

In this paper we revisit the first price and the second price sealed-bid auctions, but, unlike the standard model, we assume that bidding is conducted by an expert on behalf of the client, and that the client does not completely trust the expert's qualifications. In particular, if the client does not win the auction, but could have won it by submitting a bid below her valuation or won but feels she could have paid less for the object, the client asks the expert to justify the strategy. The objective of this paper is to incorporate the concern for the justifiability into the expert's objective function. We show that under some assumptions about the justification process the requirement of justifiability increases the optimal bid in the first price sealed-bid auction, while bidding the client's true value remains the optimal strategy in the second price auction sealed-bid auction. Hence, the first price auction may raise more revenue than the second price auction and thus it will be preferred by the seller. Both auctions allocate the good to the client with the highest valuation. However, the second price sealed-bid auction is more efficient, since the experts do not incur costs from the failure to justify their strategies.

KEYWORDS: auctions, experts, efficiency

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1 Introduction

Suppose you are called upon to advise a client who has to participate in a sealed-bid first price auction. There are two bidders and their values come from a uniform distribution on $[0, 1]$. Assume that the realized value of your client is 0.8. What bid would you recommend her to submit? If you believe that your client's opponent will play the unique equilibrium strategy, your advice should be 0.4.[1] Now suppose that your client's opponent submitted a bid of 0.45 and your client lost the auction. In this case you may be forced to justify your advice. Your client might ask why you shaded your bid by so much. After all, had you submitted a bid of 0.5, for example, your client would have won and enjoyed a considerable surplus of 0.3. She may have little patience for formal arguments, in which case you will be fired.

The objective of this paper is to incorporate the concern for justifiability into the expert's objective function in the auction-theoretic context. The concept of the justifiability of a strategy in a game was first introduced by Spiegler (2002). He assumes that a strategy in an *extensive form* game is justifiable if a player is armed with "smashing" counter-arguments against ex-post critiques. Having a smashing counter-argument means that faced with a critique of her action the player will accept the logic of the argument and show that the critic's ex-post recommendation would have exposed the player to the same critique in the opposite direction. The concept is, however, specialized for extensive form games and cannot be used in our context.

In this paper we take a different approach to justifiability. We will assume that the client doubts the qualifications of the expert if she did not win the auction but could have won it by submitting a bid below her valuation, or won but paid too much. It is a rather common reaction to doubt the argument one does not understand. For example, some people doubt that smoking is harmful or exercise is beneficial because they cannot follow the argument that leads to this conclusion.

If the client doubts the expert's qualifications, she will ask the expert to justify her choice. Since the client is not sure whether the expert has given the best advice, she will judge the expert's action against some reference point, which is provided by actions of other experts and exogenous factors.[2]

[1] The unique Nash equilibrium in this game is to submit a bid $b(v) = v/2$. This strategy is, in fact, optimal against any *linear* strategy of the opponent.

[2] See Rosenkranz and Schmitz (2007) for a description of the role of reference points in

It is important at this point to draw a distinction between the lack of confidence in the qualifications of the expert and regret. The former is possible only if the client is boundedly rational and cannot figure out for herself the optimal course of action.[3] Regret, on the other hand, can be felt by a fully rational client whose preferences are not fully captured by the expected utility (see, for example, Mas-Colell, Whinston, and Green, 1995). In this paper we assume that the client experiences no regret, but is not fully confident in the abilities of the expert. In modeling terms regret usually will be stronger if the expert just missed the other expert's bid. Therefore, under regret, all else being equal, the experts will have incentives to submit significantly different bids. Concern for justifiability, on the contrary, will push the bids of the experts closer to each other. The logic is that the words of one expert carry more weight if the other expert behaved similarly.

In this paper we concentrate on the effect of the justifiability concern on the bids. We consider two auction formats: the first price sealed-bid auction and the second price sealed-bid auction without a reserve price. We show that concern for justifiability affects bidding in the first price sealed-bid auction, while in the second price sealed-bid auction bidding the client's true value remains the optimal strategy. Hence, the first price and second price sealed-bid auctions will, in general, raise different revenues. We determine the sufficient conditions for the case when the first price auction raises more revenue than the second price one. We find these conditions to be intuitively appealing and this case to be more relevant. Both auctions allocate the good to the client with the highest valuation. However, the second price auction is more efficient, since in this case the experts do not incur any costs from the failure to justify their strategy.

2 The Model

Assume there are N buyers who want to buy one unit of a good. The seller possesses only one unit and sells it through a sealed-bid auction. We will consider two auction formats: the first price and the second price sealed-bid auctions. The buyers' valuations are independent and come from distribution $F(\cdot)$ whose support is the unit interval. The distribution is assumed to be

auction theory.

[3] A fully rational principal will not need an expert in the first place.

absolutely continuous with respect to the Lebesgue measure with the Radon-Nikodym derivative $f(\cdot)$. Assume that $f(v) > 0$ for any $v \in [0, 1]$.

Each buyer hires an expert to bid in the auction. The expert observes the buyer's value and is promised some fraction $\beta > 0$ of her surplus. If the buyer is not satisfied with the result of the auction, she will doubt the expert's qualifications and ask the expert to justify her choice. If the expert fails to justify her decision she will incur cost $c \geq 0$. This cost can be interpreted as associated with the loss of reputation or the psychological cost of her failure to persuade the buyer of the viability of the strategy. Another interpretation is that the expert is fired and c represents the difference in values of being employed and unemployed.

The principal is not satisfied with the result of the auction if she did not win the auction, but could have won it by submitting a bid below her valuation or won but feels she has paid too much compared to what she expected to pay. Our first result is independent of the specific justification process, and states that justifiability concerns are irrelevant in the second price sealed-bid auction. More precisely, let $b(v)$ denote the bid of the expert when her client's valuation is v. The following proposition holds.

Proposition 1 *In a second price sealed-bid auction with a concern for justifiability each expert has a weakly dominant strategy to bid the true value* $b(v) = v$.

Proof. Given this bidding strategy, if a buyer loses she could not have done any better (because the winning bid was above the buyer's valuation) and if she wins she could not have paid less because the price she has to pay is independent of her expert's bid. Therefore, the buyer has no reason to be unhappy with the advice of the expert. The proposition now follows from the standard result for the second price auction (Vickery, 1961).

Q.E.D.

Let us now turn to the first price sealed-bid auction. Assume that the client asked the expert to justify her bid. The justification will proceed with reference to an "appropriate bid." To form an estimate of the appropriate bid the buyer uses the bids of the other experts, a reserve price (if publicly known), and a vector of exogenous random factors (for example, prices in similar auctions or market prices of similar goods). We will assume that the buyer's estimate of the appropriate bid is given by:

$$b_a = g(b^*_{-i}, r, z), \tag{1}$$

where b^*_{-i} is the vector of bids of the other experts, r is the reserve price[4], and z is the vector of random factors that can include prices of related items, bids observed in similar auctions in the past, etc. For example, if the buyer believes that the appropriate bid is the weighted average of the other experts' bids and the reserve price, then an estimate of the appropriate bid is:

$$b_a = \xi\bar{b} + (1-\xi)r, \tag{2}$$

where $\bar{b}$ is the average bid of the other experts. Formula (2) is particularly attractive when the number of experts is large.

We assume that the probability that the expert fails to justify her bid increases with the distance between the appropriate bid b_a and the actual bid b, where $b_a < b$ corresponds to the situation when buyer won the auction but paid too much in her opinion, and $b_a > b$ when the buyer lost the auction but could have won. The probability is given by:

$$L(b_a - b), \tag{3}$$

where $L(\cdot)$ is an appropriate convex function, differentiable everywhere with a possible exception of zero, which satisfies

$$L(0) = L'(0) = 0.^5 \tag{4}$$

We now look for a symmetric equilibrium, in which everybody bids according to a differentiable function $b(v)$.

For simplicity of exposition we will assume from now on that $N = 2$. The results can be easily generalized for a situation with $N > 2$ but the notation becomes cumbersome. We will also assume that the auction allows for a differentiable, symmetric, strictly increasing Bayes-Nash equilibrium. This assumption is not too restrictive and is satisfied in the example we explicitly solve below.

If the client's true valuation is v, the expert bids as if it is $\widetilde{v}$, and in equilibrium everybody bids according to a strictly increasing differentiable

[4] See Rosenkranz and Schmitz (2007) for a discussion of the role of the reserve price as a reference point.

[5] $L'(\cdot)$ denotes the left derivative in the case when $L(\cdot)$ is not differentiable at zero. Since $L(\cdot)$ is assumed to be convex, the one-sided derivatives always exist.

function $b(v)$, then the expert's expected utility is:

$$\Pi(v,\widetilde{v}) = \beta(v - b(\widetilde{v}))F(\widetilde{v}) - cE_z \int_0^{b^{-1}(v)} L[g(b(x),r,z) - b(\widetilde{v})]f(x)dx. \tag{5}$$

The first term in the expression (5) is standard and represents the expected monetary payment to the expert (the fraction β of the buyer's gain $v - b(\widetilde{v})$ multiplied by the probability of winning the auction $F(\widetilde{v})$). The second term is the expected cost of the failure to justify the strategy. It is equal to cost, c, multiplied by the probability that the expert fails to persuade the client. Notice that the expert doesn't have to justify the strategy to the client when the other buyer bids above the client's true valuation v; therefore, the integration is over the area where the bid of the other buyer is between zero and v, i.e. $0 \leq b(x) \leq v$ or $0 \leq x \leq b^{-1}(v)$.

The Revelation Principle (see, for example, Mas-Colell, Whinston, and Green, 1995) implies that in equilibrium it should be optimal for the expert to bid according to the principal's true valuation, i.e.

$$\frac{\partial \Pi(v,\widetilde{v})}{\partial \widetilde{v}}|_{\widetilde{v}=v} = 0. \tag{6}$$

Carrying out differentiation one gets the following functional-differential equation for $b(\cdot)$:

$$\begin{gathered} \beta(v - b(v))f(v) + \\ cE_z \left(\int_0^{b^{-1}(v)} L'[g(b(x),r,z) - b(v)]f(x)dx \right) b'(v) = \beta b'(v)F(v). \end{gathered} \tag{7}$$

This equates the marginal cost of changing a bid to the corresponding marginal benefit. We now consider this in more detail. Assume that while bidder two bids according to $b(\cdot)$, bidder one decides to lower her bid by db. As a result, when the client wins the auction, she has to pay less, and the expert's expected payment is increased by the marginal benefit:

$$MB = \beta F(v)db. \tag{8}$$

The marginal cost from lowering the bid by db is due to the two effects: the decrease in the probability of winning and the increase in probability that the expert will fail to justify her advice. It is given by:

$$MC = \beta(v - b(v))f(v)dv + \\ cE_z\left(\int_v^{b^{-1}(v)} L'[g(b(x), r, z) - b(v)]f(x)dx\right)db \tag{9}$$

where dv represents the increase in the measure of types who will overbid her and is linked to db by:

$$dv = \frac{db}{b'(v)}. \tag{10}$$

Equating the marginal benefit to the marginal cost results in equation (7).

A sufficient condition for the concern for justifiability to increase the equilibrium bid is established in the following proposition.

Proposition 2. *If*

$$E_z(L'[g(b(v), r, z) - b(v)]) \geq 0, \tag{11}$$

for all $v \in [0, 1]$ *then the equilibrium bid is increasing in* c.

Proof. Equation (5) implies:

$$\frac{\partial^2 \Pi}{\partial \widetilde{v} \partial c} = E_z\left(\int_0^{b^{-1}(v)} L'[g(b(x), r, z) - b(\widetilde{v})]f(x)dx\right)b'(\widetilde{v}) \geq 0, \tag{12}$$

where the last inequality follows from (11) and the fact that the equilibrium bid increases in the valuation. Therefore, $\Pi(v, \widetilde{v}, c)$ satisfies increasing differences in incremental returns in variables $\widetilde{v}$ and c, which implies that

$$\arg\max_{\widetilde{v}} \Pi(v, \widetilde{v}, c) \tag{13}$$

increases in c (Topkis, 1998). Therefore, the equilibrium bid increases in c.

Q.E.D.

Given our assumptions about function $L(\cdot)$ condition (11) means that the expert is called upon to justify her action only if she loses the auction. This condition is sufficient for the equilibrium bid to increase in the cost of justification. However, as we will demonstrate by an example, it is not necessary. What is important is that the expert should find it easier to justify her bid in the case when she has won the auction. This seems plausible, since winning the auction is good news that will make the client more receptive to the expert's argument.

It is also worth noting that, in the case where $c = 0$, the solution for (7) is given by

$$b(v) = v - \int_0^v \frac{F(t)}{F(v)} dt < v. \tag{14}$$

This is the equilibrium strategy in the first price sealed-bid auction without justifiability considerations.

Example. Consider a first price sealed-bid auction among two bidders without a reserve price (for example, assume that the seller is the government and it tries to achieve an efficient allocation rather than maximize revenue). Let us further assume that the buyer considers the bid of the other expert to be the appropriate bid:

$$E_z(g(b, r, z)) = b, \tag{15}$$

and the probability of failure by the expert to justify her advice is given by

$$L(y) = \gamma \max(0, y) - \zeta \min(0, y) \tag{16}$$

for some $\zeta, \gamma > 0$. Though literally taken equation (16) suggests that the *probability* of justifying the bid is different for the cases when the expert loses the auction or wins but pays more than expected, one can reinterpret the expert's objective assuming that the probability is the same but the penalty, d, in the case of overpaying is linked to the penalty, c, for losing by

$$d = \frac{\zeta c}{\gamma}.$$

Then equation (7) is reduced to:

$$\beta(v - b(v))f(v) + c(\gamma F(b^{-1}(v)) - (\gamma + \zeta)F(v))b'(v) = \beta b'(v)F(v). \tag{17}$$

This equation should be solved subject to the initial condition $b(0) = 0$. If we further assume that $F(v) = v$, i.e. the values are distributed uniformly on the unit interval, then $b(v)$ can be found in a form:

$$b(v) = \delta v. \tag{18}$$

Substituting this into (17) one obtains:

$$\delta = \frac{\beta + c\gamma}{2\beta + c(\gamma + \zeta)}. \tag{19}$$

If the expert finds it easier to justify her choice provided she won the auction ($\gamma > \zeta$) then δ increases in c from a value of $1/2$ (for $c = 0$) towards the value $\gamma/(\gamma + \zeta)$ as $c \to \infty$, i.e. concern for justifiability increases the bid. The opposite is the case if $\gamma < \zeta$. We believe that the case $\gamma > \zeta$ is more natural, since winning the auction is good news.

3 Conclusions

In this paper we incorporated the concern for justifiability into auction theory. We showed that this concern has no effect on the equilibrium bid in the second price sealed-bid auction but affects the bidding in the first price sealed-bid auction. Though it can affect the equilibrium bid in either direction, depending on whether the expert finds it harder to justify losing the auction at a bid below the valuation or winning but paying more than the client finds appropriate, we believe that typically the concern for justifiability will increase the equilibrium bids. The reason for this belief is that winning the auction will leave the client with a positive utility, which will make her more receptive to the expert's arguments. Therefore, the first price auction will tend to generate higher expected revenues for the seller than the second price auction, i.e. the introduction of the justifiability concern violates the Revenue Equivalence Theorem. The efficiency of *allocation* under both auction formats is preserved. However, since the expected costs for the experts arising from the failure to justify their strategy are zero in the second price auction and positive in the first price auction, the second price auction is more efficient.[6]

Note that if the expert is asked to justify her actions only if she loses the auction, the concern for justifiability affects the equilibrium bids in the same

[6] We thank Ian King for pointing this out.

direction as risk-aversion. However, while the justifiability concern affects only the marginal cost curve, the introduction of risk aversion affects both the marginal cost and the marginal benefit curves in such a way that the ratio of the marginal cost to the marginal benefit is increased.

Some extensions of the basic model are possible. It would be interesting to extend the results to the common value auction framework. One can also consider the case when only one of the principals has an expert, while the second might be an expert herself. One can also endogenize the choice to have an expert. To do this in a meaningful way will, however, require a theory of boundedly rational behavior, since for the client to be willing to hire an expert she must be unable to compute equilibrium herself. These are topics for the future research.

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