A Successive Mean Splitting Algorithm for Contrast Enhancement and Dynamic Range Reduction

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***Abstract*— This paper presents a successive mean splitting algorithm for both contrast enhancement and dynamic range reduction of digital images. The proposed algorithm is an extension to the recently published successive mean quantization transform (SMQT) for contrast enhancement. We explore the conditions under which the proposed algorithm is "optimal". We also discuss the relationship between the proposed algorithm and the moment preserving quantization (MPQ) algorithms, especially the absolute moment block truncation coding (AMBTC). We observe that AMBTC can be regarded as a special case of the proposed algorithm. Therefore, the proposed algorithm also has the moment preserving property.**

I. Introduction

In digital image processing, contrast enhancement is often desirable because it makes an image sharper and richer in detail to the human eye [1]. This procedure has been widely used in applications such as remote sensing and medical imaging. The desired result can be achieved using a number of approaches, with intensity mapping among the most commonly employed [2]. Generally speaking, the intensity mapping approach attempts to map the original intensity levels of pixels into new ones such that the intensity difference between pixels is amplified. Therefore, it tends to spread out the intensity distribution and stretch the dynamic range of the image. One algorithm in this category is the well-known histogram equalization (HE). Recently, Nilsson et al. [7] proposed an intensity mapping method using the successive mean quantization transform (SMQT). It has been demonstrated that the SMQT-based method out-performs the HE method.

Another often desirable digital image processing procedure is dynamic range reduction, which reduces the number of bits needed to represent the intensity value of a pixel. This is often required when the receiving or display device has a lower dynamic range than the original image. In terms of the manipulation of the intensity distribution, dynamic range reduction can be regarded as somewhat opposite to the intensity mapping approach of the contrast enhancement. Like in the case of contrast enhancement, various techniques [4]–[6] have been proposed for dynamic range reduction.

In this paper, we perform a further study on the SMQT algorithm. We will present a generalized successive mean splitting (SMS) algorithm, which is slightly different from the SMQT, for both contrast enhancement and dynamic range reduction. We will explore under what conditions the proposed algorithm is "optimal". We will also explore the relationship between the proposed algorithm and the moment preserving quantization (MPQ) algorithms [9], especially the absolute moment block truncation coding (AMBTC) [10]. We will observe that AMBTC can be regarded as a special case of the proposed algorithm. Therefore, the proposed algorithm will also have the moment preserving property.

The rest of this paper is organized as follows. In section II, we briefly introduce the SMQT algorithm. The SMS algorithm, a generalized version of SMQT, will be described in section III. In section IV, we present some discussions on the SMS algorithm, including when the algorithm is "optimal" and how it is related to the MPQ algorithms. Experimental results will be given in section V, with enhancement results in section V-A and dynamic range reduction results in section V-B.

II. A Brief Introduction of SMQT

The SMQT [7] is an image-based succesive mean splitting algorithm, which was proposed by M. Nilsson et al. and was initially employed in the fields of automatic enhancement of gray-scale images. Its center piece is a scheme to split a group of data into two according to how each datum is compared with the mean of the group. A "bitplane" of the group will be generated to indicate which of the two sub-groups each datum is going into. This process will happen succesively until a designated level L is reached. For an 8-bit image, L is set to 8. It is easy to see that at a certain level l ($l = 1, 2, \cdots, L$), there will be a total of 2^{l-1}groups. More specifically, let x be a pixel and $y(x)$ be the intensity of pixel x. Let $D_l^n(x)$ denote the nth group of pixels at level l. The process of SMQT can be summarized in the following steps.

1) Identify the data $D_l^n(x)$ and calculate the mean $m_l^n(x)$.
2) Using the mean as the threshold, quantize each pixel in the current set $D_l^n(x)$,

$$b(x) = \begin{cases} 1, & \text{if} \quad y(x) \geq m_l^n(x) \\ 0, & \text{if} \quad y(x) < m_l^n(x) \end{cases},$$

where $b(x)$ represents the quantized value of $y(x)$. The bitplane $B_l^n(x)$, which is the concatenation of $b(x)$, is consequently generated.

3) Split the data $D_l^n(x)$ into two subsets, namely a "higher" subset with data greater than $m_l^n(x)$ and a "lower" subset with data smaller than $m_l^n(x)$.
4) Repeat steps 1-3 until all the groups at level l are processed. Then go to level $l+1$ and repeat steps 1-3 unless $l = L$.

To reconstruct the data, we simply put bitplanes at different levels into different bit position by weighting them with the weight 2^{L-l}, where l is the index of level where the bitplane is generated. Therefore, the result of the SMQT enhancement is given by

$$M(x) = \sum_{l=1}^{L} \sum_{n=1}^{2^{l-1}} B_l^n(x) \cdot 2^{L-l}. \qquad (1)$$

III. The SMS algorithm

The SMS algorithm presented in this paper is a generalized version of SMQT for both contrast enhancement and dynamic range reduction. It is the same as SMQT when used for contrast enhancement. When used for dynamic range reduction, the SMS algorithm still uses the same mean splitting strategy as SMQT but is slightly different in the reconstruction stage. Instead of using the bitplanes, we make use of the means of the subsets at each level. More specifically, the reconstructed image with SMS is given by

$$M(x) = \sum_{l=1}^{L} \sum_{n=1}^{2^{l}} m_{l+1}^n(x) \cdot \frac{2^{l-1}}{2^L - 1}. \qquad (2)$$

We will see in the next section that this modification will lead to the nice property of moment preserving.

IV. Some discussions

A. When is the mean an optimal threshold?

It would be interesting to know under what conditions the mean is an optimal threshold for the splitting process. Let us assume that the data samples we see come from two data sources ω_1 and ω_2 that emit data according to two Gaussian probability distribution functions, that is,

$$p(x|\omega_1) \sim N(\mu_1, \sigma_1),$$

$$p(x|\omega_2) \sim N(\mu_2, \sigma_2).$$

We then have to decide whether an observed data sample x is from ω_1 or ω_2. We may define a classifier with discriminant function $g(x)$ for this. Let $g_1(x)$ and $g_2(x)$ denote the values of the discriminant function corresponding to ω_1 and ω_2, respectively. The classifier will assign sample x to ω_1 if

$$g_1(x) > g_2(x). \qquad (3)$$

Otherwise, x will be assigned to ω_2. Using the minimum error probability as the criterion for the optimal classifier, we may employ a Bayes classifier [**?**]:

$$g_i(x) = p(\omega_i|x), \quad i = 1, 2. \qquad (4)$$

The reason for choosing this classifier is that the maximum discriminant function value in (4) will correspond to the minimum error probability.

According to Bayes rule[6],

$$p(\omega_i|x) = \frac{p(x|\omega_i)P(\omega_i)}{p(x)}. \qquad (5)$$

Thus equation (4) becomes

$$g_i(x) = \frac{p(x|\omega_i)P(\omega_i)}{p(x)}. \qquad (6)$$

Note that $p(x)$ in equation (6) can be regarded as a scaling factor. Therefore it will not affect the relative amplitude of $g_i(x)$. We may simplify the definition of the discriminant function as

$$g_i(x) = p(x|\omega_i)P(\omega_i). \qquad (7)$$

We may further simplify the definition by taking $\log$ of the right-hand side of (7):

$$g_i(x) = \log p(x|\omega_i) + \log P(\omega_i). \qquad (8)$$

The equation for the decision boundary seperating the two classes is given by

$$g_1(x) = g_2(x). \qquad (9)$$

From (8) and (9), we have

$$\log p(x|\omega_1) + \log P(\omega_1) = \log p(x|\omega_2) + \log P(\omega_2). \qquad (10)$$

If we do not have any prior knowledge about which of ω_1 and ω_2 is more likely to occur, we may set the priors as

$$P(\omega_1) = P(\omega_2) = \frac{1}{2}. \qquad (11)$$

From (11) and the Gaussian pdfs, we can get the solution to equation (10) as

$$x = \frac{\sigma_2\mu_1 + \sigma_1\mu_2}{\sigma_2 + \sigma_1}. \qquad (12)$$

From (11), we can also obtain the mean of the data we observe as

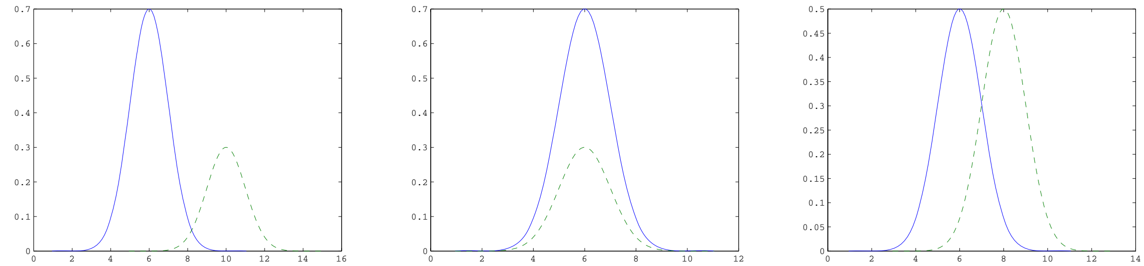
$$\mu = \mu_1 P(\omega_1) + \mu_2 P(\omega_2) = \frac{1}{2}(\mu_1 + \mu_2). \qquad (13)$$

We can now discuss when the mean in (13) is the optimal threshold in (12).

Case 1: $\mu_1 = \mu_2 = \mu$, "overlapping" distributions as shown in Fig.1(b). In this case, we have $x = \mu$. Therefore, the mean is the optimal threshold.

Case 2: $\mu_1 \neq \mu_2, \sigma_1 = \sigma_2$, "symmetric" probability distributions as shown in Fig. 1(c). In this case, equation(12) yields $x = \frac{\mu_1+\mu_2}{2} = \mu$. Again, the mean is the optimal threshold.

Case 3: $\mu_1 \neq \mu_2$, $\sigma_1 \neq \sigma_2$, arbitrary normal distributions. In this case, $x = \frac{\sigma_1}{\sigma_2+\sigma_1}\mu_2 + \frac{\sigma_2}{\sigma_2+\sigma_1}\mu_1$, which is not exactly



(a) Arbitrary normal distributions. (b) Overlapping normal distributions. (c) Symmetric normal distributions

Fig. 1. Possible relationship between two probability distribution functions. In the plots, the continuous line represents $p(x|\omega_1)$ and dashed line represents $p(x|\omega_2)$.

the mean of the data. If $\sigma_2 > \sigma_1$, the decision boundary x would be closer to μ_1, on the other hand, if $\sigma_1 > \sigma_2$, x tends to μ_2.

From the above discussion, we can see that the mean will be the optimal threshold if either the means or the variances of the two Gaussian sources are equal. Otherwise, the optimal threshold will be a weighted average of the two means. If the two variances σ_1 and σ_2 are not very far from each other, the mean will still be a reasonable choice.

B. Relationship with MPQ

The SMS algorithm is closely related to the moment preserving quantization (MPQ) [9], although the former is image-based while the latter is block-based. Let us first look at the simple case of a two level quantizer. We use random variables X and Y to represent the input and output of this quantizer, respectively. Obviously, we need three parameters to fully define the operation of the quantizer, namely (1) a threshold x_1, (2) the output value y_1 when the input is less than or equal to x_1 and (3) the output value y_2 when the input is greater than x_1. One can determine these parameters by solving three equations according to certain design criteria. In MPQ, the criterion is to preserve the first three moments of the input, that is,

$$E[Y] = E[X]$$

$$E[Y^2] = E[X^2]$$

$$E[Y^3] = E[X^3]$$

To further simplify the calculation, we can set x_1 as the mean of the data and solve y_1 and y_2 to preserve the first two moments of the input. This is known as the block truncation coding (BTC). A variant to BTC is to preserve the absolute moments rather than the standard moments. This leads to the absolute moment block truncation coding (AMBTC) [10].

It can be shown that the basic idea of the AMBTC algorithm is actually the same as the SMS algorithm when $L = 1$. The former is a block-based algorithm, while the latter is an image-based. Therefore, the SMS algorithm also has the nice moment preserving property. In addition, we note that although the MPQ can be generalized to N-levels with the criterion to preserve $2N-1$ moments, the solution becomes very complicated for larger Ns. On the other hand, the SMS algorithm can be easily used for N-level ($N = 2^n$) quantization.

V. EXPERIMENTAL RESULTS

Here we show some experimetal results for the SMS algorithm. In section V-A, we compare the performance of SMS enhancement algorithm with that of the histogram equalization (HE). We also look at underlying reasons for the good performance of the SMS from the entropy-keeping perspective. In section V-B, we verify the moment preserving property of the SMS algorithm in dynamic range reduction.

A. SMS enhancement performance

To evaluate the performance of the SMS algorithm in enhancement, we show the enhancement results for images "Jim" and "bird" in Figs. 2 and 3. We also compare the results with those obtained with the well-known histogram equalization technique [3]. From the enhanced images in Figs. 2 and 3, we can see that the SMS results show less over-saturation and artifacts when compared to the HE results.

TABLE I

ENTROPY VALUES FOR DIFFERENT VERSIONS OF IMAGES.

Image	bird	camera-man	Jim	goldhill
Original	6.7744	7.0097	6.1332	7.4716
H E	5.7668	5.8632	5.1200	5.8856
SMS enhancement	6.7486	6.9135	6.1139	7.3702

To understand why the SMS algorithm performs better than the HE technique, we look at the probability distributions and the entropy of the images before and after the enhancements. In Figs. 2 and 3, we plot the probability distributions of the different versions of the images. As expected, the probability distribution of the image enhanced with the HE technique has been flattened and stretched across the whole dynamic range

TABLE II

MOMENT PRESERVING TEST RESULTS IN DYNAMIC RANGE REDUCTION OF THE IMAGE "CAMERA-MAN". THE MEAN AND STANDARD DEVIATION OF THE ORIGINAL IMAGE ARE 118.72 AND 62.34, RESPECTIVELY.

	Mean		Standard deviation	
Bits per pixel	SMS	BT	SMS	BT
1	118.80	98.96	55.51	58.77
2	118.90	120.59	60.95	71.86
3	118.73	117.76	61.84	65.66
4	118.78	119.55	62.28	61.48
5	118.69	119.22	62.27	62.37
6	118.75	119.22	62.36	62.37

and made close to the uniform distribution. On the other hand, while the SMS algorithm also stretches the distribution, the shape of the original distribution remains largely unchanged. This means that the entropy of the enhanced image should be close to that of the original image. To verify this, we list the entropy values of the original and enhanced images in Table I. From the table, we can see that the SMS algorithm will cause a very small drop in entropy value, while the HE technique will cause a substantial drop. This observation helps explain the reason for the good performance of the SMS algorithm. The SMS algorithm is much better at keeping the information content in the original image than the HE technique.

B. Dynamic range reduction results

To test how well the proposed SMS algorithm can preserve the first two moments, we apply both the SMS algorithm and simple bit truncation to the dynamic range reduction of the image "camera-man". The results are shown in Table II. In the table, "BT" columns mean the results obtained with simple bit truncation. From the table, we can see that the SMS algorithm is better at preserving both the mean and the standard deviation than simple bit truncation. This is more obvious when the number of bits we keep for a pixel is lower. These results confirm that the proposed SMS algorithm has the same moment preserving property as the AMBTC, although the latter is a block-based algorithm and the former is an image-based algorithm.

VI. CONCLUSION

In this paper we have presented a further study on the recently proposed SMQT algorithm for image enhancement and extended it to a generalized successive mean splitting (SMS) algorithm that can be used both in contrast enhancement and in dynamic range reduction. We have discussed the conditions under which the proposed algorithm is the optimal strategy. We have also discussed the close relationship between the proposed algorithm and the moment preserving quantization algorithms which leads to the nice moment preserving property of the proposed algorithm when used in dynamic range reduction. This property has been verified in our experimental results. While confirming the good performance of the proposed algorithm contrast enhancement, we have also offered the reason for this good performance from the information-keeping perspective.

REFERENCES

[1] W. K. Pratt, *Digital image processing*, 2nd edition, John Wiley & Sons, Inc,1991.

[2] Z. Hussain, *Digital image processing, practical applications of parallel processing techniques*, Redwood Press Ltd, 1991.

[3] J. A. Stark, "Adaptive image contrast enhancement using generalizations of histogram equalization," *IEEE Trans. Image Processing*, vol. 9, no. 5, pp. 889–896, May 2000.

[4] E. Reinhard and K. Devlin, "Dynamic range reduction by photoreceptor physiology," *IEEE Trans. Visualization and Computer Graphics*, vol. 11, no. 1, pp. 13–24, Feb. 2005.

[5] L. R. Moyer, L. J. Maciel, R. D. Brown, M. O. Little, M. C. Wicks, and D. D. Weiner, "Analog clutter cancellation algorithms for dynamic range reduction," *IEEE Aerospace and Electronic Systems Magazine*, vol. 8, no. 10, pp. 10–14, Oct. 1993.

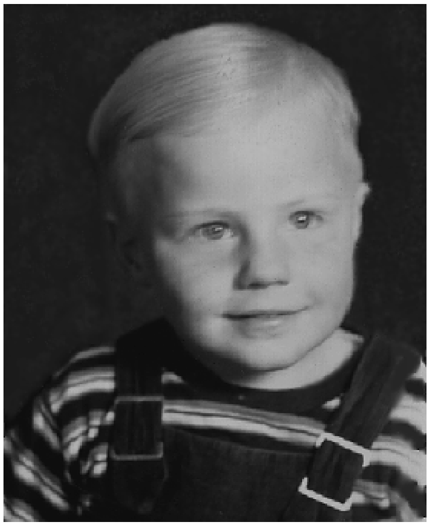
[6] M. Mancuso, R. Poiuzzi, and G. Rizzotto, "A fuzzy filter for dynamic range reduction and contrast enhancement," *Proc. 3rd IEEE Conference on Fuzzy Systems*, IEEE World Congress on Computational Intelligence, vol. 1, pp. 264a–264d, 265–267, June 1994.

[7] M. Nilsson, M. Dahl, and I. Claesson, "Gray-scale image enhancement using the SMQT," *Proc. International Conference on Image Processing*, vol. 1, pp. 933–936, Sep. 2005.

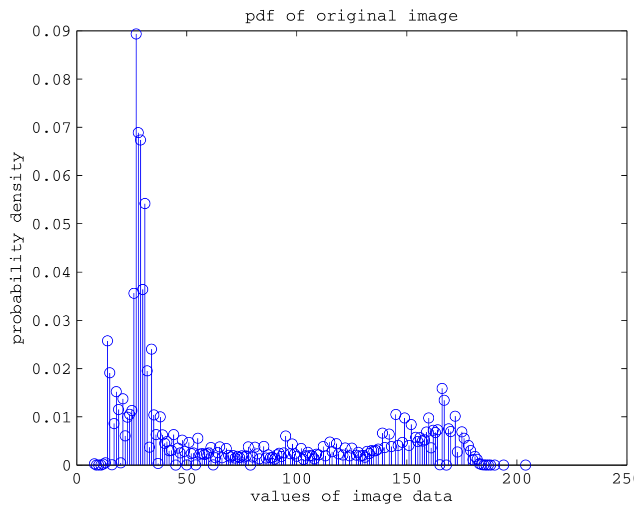
[8] D. J. C. Mackay, *Information Theory, Inference, and Learning Algorithms*, Cambridge University Press, 2002.

[9] E. J. Delp and O. R. Mitchell, "Moment preserving quantization," *IEEE Trans. Commun.*, vol. 39, no. 11, pp. 1549–1558, Nov. 1991.

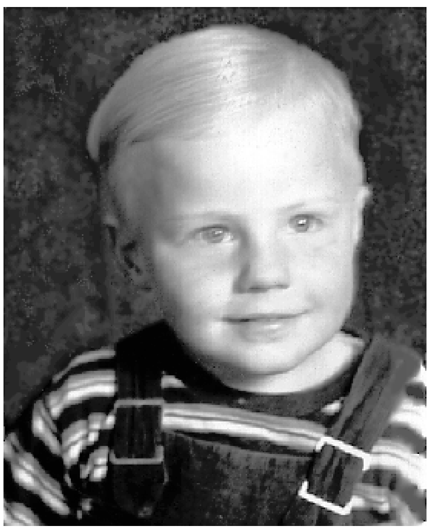
[10] M. D. Lema and O. R. Mitchell, "Absolute moment block truncation coding and its application to color images," *IEEE Trans. Commun.*, vol. 32, no. 10, pp. 1148–1157, Oct. 1984.



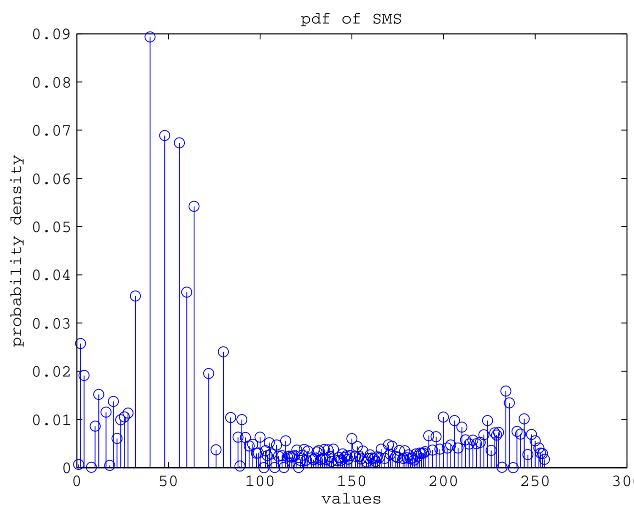
(a) The original image.



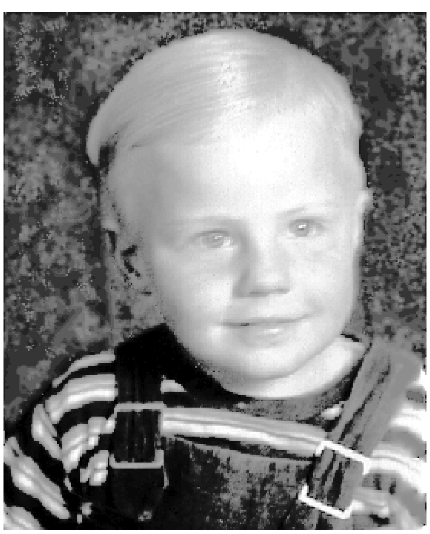
(b) The probability density of the original image.



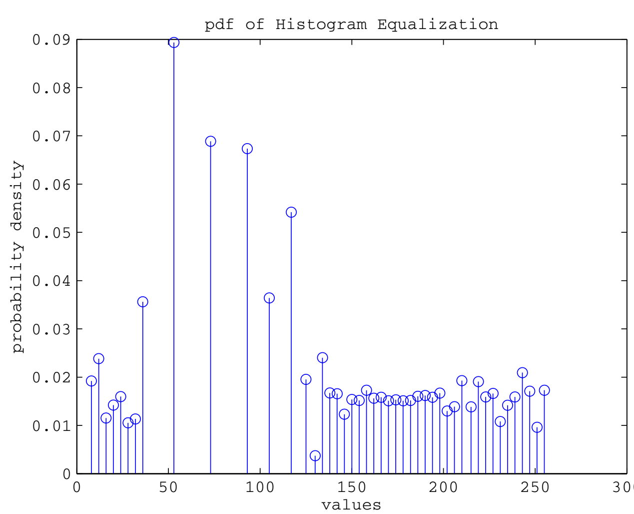
(c) The enhanced image using the SMS.



(d) The probability density of the SMS-enhanced image.



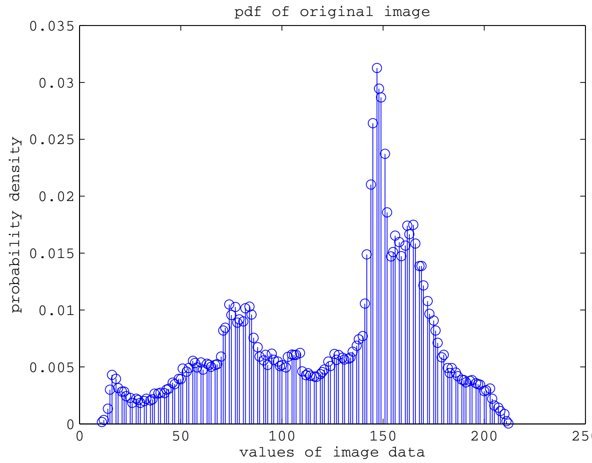
(e) The enhanced image using HE.



(f) The probability density of the HE-enhanced image.

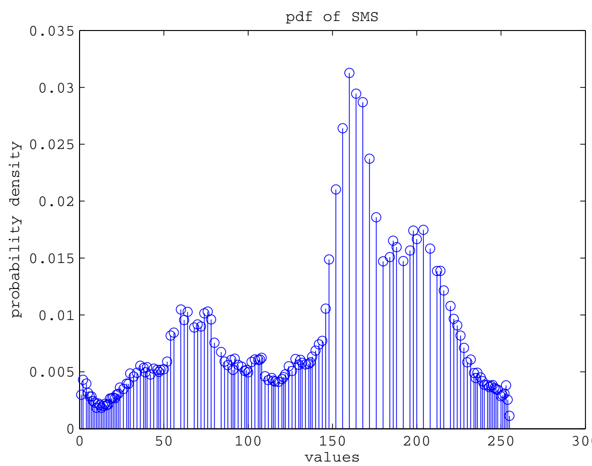
Fig. 2. The contrast enhancement results for the image "Jim".

(a) The original image.



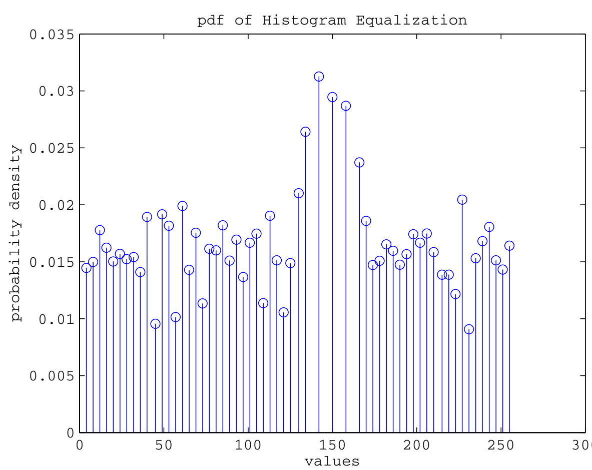
(b) The probability density of the original image.

(c) The enhanced image using the SMS.



(d) The probability density of the SMS-enhanced image.

(e) The enhanced image using HE.



(f) The probability density of the HE-enhanced image.

Fig. 3. The contrast enhancement results for the image “bird”.