Phase measurement using x rays (invited)

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This article reviews the developing field of x-ray phase contrast radiography. The underlying principles are outlined and some experimental results are reviewed. The paper then uses simulations to explore the potential application of this method to the imaging of very low Z elements within a very high Z envelope. We predict that it should be possible to image very light elements such as hydrogen within a gold envelope using small laboratory-based sources. © 2004 American Institute of Physics. [DOI: 10.1063/1.1779612]

I. INTRODUCTION

The development of third-generation synchrotrons has made x-ray sources available that provide substantial amounts of coherent flux. Experiments with these sources very quickly showed that nonuniform thickness in the beamline windows introduced significant phase gradients which in turn generated very substantial intensity nonuniformity in what was otherwise a highly uniform x-ray beam. While this represented a substantial problem in the efficient transport of an x-ray beam to the experiment, it was also a demonstration that the highly coherent output of these machines would enable the easy visualization of structure without the need for significant absorption contrast.

It was subsequently noted that phase contrast had the potential to be an important imaging methodology; could yield qualitative three-dimensional tomographic images; and that the information could be used to yield quantitative phase measurements. Quantitative phase tomography was also demonstrated using a number of methods. Imaging of the complex refractive index, and the consequent ability to distinguish between elements within a sample, has also been demonstrated.

However it is not known whether it will be possible to obtain phase contrast from very light elements contained within a high-Z environment. In this article, we explore this question and go on to consider whether these ideas may be applied to the phase contrast imaging of inertial confinement fusion capsules.

II. X-RAY PHASE CONTRAST IMAGING

Consider a coherent electromagnetic wave incident on an object that modifies both the amplitude and phase of the incident wave. The field leaving the object (the “exit wave”) is described by

\[ \psi_{\text{exit}}(\mathbf{r}) = \sqrt{I(\mathbf{r}) \exp[i\Phi(\mathbf{r})]} \]  

The measured intensity leaving the object, which is the domain of conventional radiography, gives the result

\[ I_{\text{measured}}(\mathbf{r}) = |\psi_{\text{exit}}(\mathbf{r})|^2 = I(\mathbf{r}) \].  

It is clear that the phase distribution in the object has no influence on the measured intensity distribution.

The key to phase contrast imaging is to allow the field to propagate some distance so as to allow refraction to reveal the phase distribution. The ability to visualize the phase can be understood using the expression for the conservation of energy. To understand this, we introduce the energy flow vector of the field, which for electromagnetic fields is the Poynting vector. In the case of a coherent field (and we will consider a partially coherent field later), then

\[ \mathbf{S}(\mathbf{r}) = \frac{1}{k} \mathbf{I}(\mathbf{r}) \nabla \Phi(\mathbf{r}), \]

where \( k = 2\pi/\lambda \), and \( \lambda \) is the wavelength of the light. Conservation of energy requires that

\[ \nabla \cdot \mathbf{S}(\mathbf{r}) = 0. \]

Equation (4) describes a three-dimensional divergence. We wish to consider a field predominantly propagating in a given direction (described by the \( z \) axis, or the optical axis of the imaging system). To look at this, we adopt the paraxial approximation, which enables us to write

\[ \psi(\mathbf{r}_{\perp}, z) \approx \sqrt{I(\mathbf{r}_{\perp}, z)} \exp[i(\Phi_{\perp}(\mathbf{r}_{\perp}) - kz)], \]

where \( \mathbf{r} = (\mathbf{r}_{\perp}, z) \). In this case, the Poynting vector can be written

\[ \mathbf{S}(\mathbf{r}_{\perp}, z) = I(\mathbf{r}_{\perp}, z) \left[ \nabla_{\perp} \Phi_{\perp}(\mathbf{r}_{\perp}, z) - k\hat{z} \right], \]

where \( \hat{z} \) is the unit vector along the optical axis, and the conservation of energy equation [Eq. (4)] can be written

\[ k \frac{\partial I(\mathbf{r}_{\perp}, z)}{\partial z} = -\nabla_{\perp} \cdot [I(\mathbf{r}_{\perp}, z) \nabla \Phi_{\perp}(\mathbf{r}_{\perp}, z)]. \]

This is the transport of intensity (ToI) equation. In the remainder of this article we will only discuss paraxial fields recovered in a given plane perpendicular to the optical axis. We can therefore drop the \( \perp \) subscript and the \( z \) argument without introducing any ambiguity. We will refer to the application of Eq. (7) as the ToI approach.

The transport of intensity equation explains the origin of phase contrast introduced via propagation. Let us suppose
defines the phase in the wave field. Thus, a measurement of the material, respectively. We refer to the application of the real part and imaginary parts of the complex refractive index conventionally defined loses meaning. This issue can be addressed soon as this condition is relaxed, the concept of phase as applied to the time averaged Poynting vector performs local focusing or defocusing of the radiation which produces the observed intensity variations. An example of this can be seen in Fig. 1 which shows neutron radiographs taken in contact with the object. The arrows indicate some representative details in the object. The same effects are to be found in x-ray radiography.

that the object is illuminated with a coherent wave and only changes the phase of the wave. That is, the object is completely transparent. In this case Eq. (7) reduces to

\[ k \frac{\partial I(r)}{\partial z} = -I_0 \nabla^2 \Phi(r), \quad (8) \]

where the uniform transmitted intensity has value \( I_0 \). It is clear that the intensity gradient along the optical axis is non-zero, which means that contrast is being generated. Moreover, the contrast is being generated via the Laplacian of the phase distribution. That is, by the curvature in the phase. Physically, this means that curved regions of the phase distribution perform local focusing or defocusing of the radiation which produces the observed intensity variations. An example of this can be seen in Fig. 1 which shows neutron radiographs taken in contact with the object [Fig. 1(a)] and approximately 1 m downstream [Fig. 1(b)].

It can further be shown that provided \( I(r) > 0 \) everywhere then knowledge of both \( I(r) \) and \( \partial I(r)/\partial z \) uniquely defines the phase in the wave field. Thus, a measurement of the differential rate of change of intensity along the optic axis can yield a measurement of the phase, and this has been demonstrated for a wide range of experimental situations.

In the case of objects consisting of a single material the result in Eq. (7) can be modified to give the thickness \( T \) of an object

\[ T(r) = -\frac{1}{2k\beta} \ln \frac{\beta}{(\beta - \delta\pi\lambda d_{F_{\text{pl}}})} \left[ \frac{\delta I(r,z)}{I_0} \right]. \quad (9) \]

where the Fourier transform of its argument, \( u \) is the spatial frequency, and \( \delta \) and \( \beta \) are the decrement to the real part and imaginary parts of the complex refractive index of the material, respectively. We refer to the application of Eq. (9) as the homogenous approach.

The formalism described in the preceding paragraphs has assumed that the incident wave field is fully coherent. As soon as this condition is relaxed, the concept of phase as conventionally defined loses meaning. This issue can be addressed via the time averaged Poynting vector \( \langle S(R) \rangle \). In this case, we may define a partially coherent phase which via the expression

\[ \langle S(R) \rangle = \frac{1}{k} I(r) \nabla \Phi_{pc}(r). \quad (10) \]

It turns out that this definition leads naturally to ideas of topological phases and is a physically appealing generalization of the concept of phase.

Quantitative phase imaging has been demonstrated using laboratory sources and it is Eq. (10) that allows these results to be interpreted. It follows that partially coherent phase measurement can be performed and this allows the application of the ideas to quantitative phase microscopy.

**III. PHASE TOMOGRAPHY**

Quantitative phase imaging allows methods of quantitative phase tomography to be developed. In this case a quantitative phase image is acquired for each projection. It can be shown that, to an excellent approximation, the resulting phase measurement corresponds to the phase acquired as the field passes through the object. More importantly, the ability to acquire phase and amplitude information allows information to be acquired on the complex nature of the wave. This in principle allows some distinction between materials with different complex refractive indices. This approach has also been demonstrated at the European Synchrotron Radiation Facility. If the amplitude and phase of the wave is recovered it can be used to reconstruct the complex refractive index of an object. Figure 2 shows such a three-dimensional reconstruction, where the object consists of a tungsten wire coated in boron. Figure 2 shows the real and imaginary parts of the reconstruction. The light outer coating has a significant...
real part and so a rendered image of this distribution reveals
the boron. On the other hand, the boron has a negligible
imaginary part and so a rendered image of the real part re-
veals the inner tungsten wire. A detailed examination of
these data sets confirms that the reconstructions are quanti-
tatively accurate.

IV. ANALYSIS OF INTERNAL CONFINEMENT FUSION
(ICF) TARGETS

Phase contrast can be routinely observed using appropri-
ate nonsynchrotron x-ray sources in projection mode.5 Quan-
titative phase imaging has also been demonstrated in the im-
ge produced by a microscope,11 including in the x-ray
regime.16 In principle, then, there should be no problem in
acquiring appropriate phase contrast images. The key issue is
whether there will be sufficient sensitivity to allow the im-
aging of a very light (deuterium or tritium) coating within a
gold layer. In this section we report whether this appears
feasible.

We start by demonstrating that phase contrast can be
observed. In Fig. 3 we show $\partial I(\mathbf{r})/\partial z$ at a propagation dis-
tance of 1 m for a range of energies (30–200 keV) where x
rays from a laboratory microfocus x-ray source (source size
of 6 $\mu$m) are allowed to propagate through a simulated ICF
target comprising a 10 $\mu$m thick 300 $\mu$m radius cylindrical
shell of gold coated internally with a 100 $\mu$m layer of frozen
hydrogen (density=0.1 g cm$^{-3}$). The phase contrast signal
due to the hydrogen can be seen across a wide range of
energies. We conclude that it should be possible to see such
very low density structures even if enclosed in a highly ab-
sorbing envelope. We also note that the ability to simply
visualize, rather than measure, the x-ray phase structure may
be sufficient to adequately characterize an ICF target.17

Figure 4 shows the result of applying the ToI approach
to the result in Fig. 3. The gold shell can be seen but the
hydrogen layer is not clearly visible. Note the alternating
contrast bands which we attribute to either a breakdown in
the validity conditions$^{18,19}$ for the ToI, or to a problem in
properly reconstructing the very low spatial frequencies due
to the low sensitivity of the method to small phase curva-
tures.

While it is difficult to detect the hydrogen we can per-
form a differential measurement to detect its presence. Here

FIG. 3. A one dimensional slice of a radiograph taken through a gold cy-
ylinder containing an inner layer of solid hydrogen. The vertical axis is the
energy of the x-ray photons. Refraction by the hydrogen can be seen over
the entire photon energy range.

FIG. 4. Transport of intensity equation phase reconstructions of the data in
Fig. 3. Interestingly, the presence of the hydrogen layer is harder to see,
probably because the signal is dominated by the effects of the gold layer.
The band structure as function of energy arises due to low frequency sensi-
tivities in the ToI reconstruction.

FIG. 5. ToI reconstruction of the difference between a gold container with
and without the hydrogen layer. This differential approach clearly reveals
the presence of the hydrogen. Again, the band structures arises from low
spatial frequency errors.
we simulate the same experiment as for Fig. 3 but where there is no hydrogen layer in the cylinder. The difference between the two results is shown in Fig. 5. The contrast reversals can still be seen in these data. However, the hydrogen shell can now be clearly seen.

The homogeneous approach will not be valid for an object comprised of two different refractive index components but it may still be used to give a qualitative image for the differential method discussed above. The homogeneous approach has some advantages in that it is rather more numerically stable, particularly when applied to experimental data. Figure 6 shows the differential result obtained using the homogeneous approach and the presence of the hydrogen layer is clearly seen.

The additional information present in a tomographic data set means that it may be possible to visualize features where they are difficult to see in a single projection. Figure 7 shows a number of reconstructions using an x-ray energy of 200 keV. Figures 7(a) and 7(b) shows the ToI retrieval for the object with and without hydrogen, respectively. Figure 7(c) shows the tomographic reconstruction of the known phase and Fig. 7(d) shows the differential measurement based on the homogeneous approach. It is particularly interesting that the retrieval using the ToI gives a reconstruction that reproduces the features of the input object, while the homogeneous technique [Fig. 7(d)] is able to clearly highlight the spatial extent of the hydrogen layer.

V. DISCUSSION

This article has reviewed the theory behind propagation-based phase retrieval and has summarized some of the key experimental demonstrations of the approach. It can be seen that this method of phase imaging is robust, easy to use, and very flexible. We then presented some simple simulations of x-ray radiography of gold cylinders containing a hydrogen inner layer. While the simulations do not consider the impact of practical limitations such as the effect of experimental noise, they do indicate that the imaging approach might be surprisingly sensitive and that it might indeed be possible to see the interior of such samples. In particular, we find that a differential technique may permit a very clear tomographic reconstruction of objects with very disparate x-ray optical properties.

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17 D. Montgomery, Rev. Sci. Instrum., these proceedings.