Effects of Phase Quantization on Digital Beamforming for HF Phased Array Radar

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Abstract
Digital beamforming (DBF) are efficient techniques for implementation of reconfigurable radars. Digital implementation can be carried out using finite precision and infinite precision representation of the phased array signal. We consider finite precision representation, since it takes considerably less implementation resources compared with the infinite case. In this paper we evaluate FPGA implementation when quantization is introduced in the phasing network. Digital samples of phased signals are sensitive to finite precision. Finite precision reduces mainlobe level as well as sidelobe gain. For multiple beam generation, it degrades not only the transmitted beam but adjacent beams as well. The quantization effects depend on position of the quantizer. They are different when the quantizer is placed before and after beamforming network. Simulations have been performed to demonstrate the results.

1 Introduction
Quantization is a representation of data samples with certain number of bits per sample after rounding to a suitable level of precision. Quantization errors in a DBF system can be introduced from three sources; one source is input quantization, a second is coefficient quantization and the third is the finite precision in arithmetic operations [1]. The quantization error in the arithmetic operations can be controlled by carefully selecting the size of buffer registers according to the input word length [2]. Quantization errors from phasing network of an ionospheric radar are topic of this article. Tasman International Geospace Environment Radar (TIGER) is an HF radar that investigates ionospheric irregularities in the Southern Hemisphere. Recently a second component of the TIGER is commissioned in New Zealand to acquire higher resolution.

This article is divided into two main sections; quantization effects for digital beamforming (DBF) before and after phasing network. The HF radar comprises phasing array of sixteen antenna elements with modulating frequency range of 8 to 20MHz. The radar antenna geometry is described in section 2. Fixed length samples cause periodic phase quantization error. We discuss finite precision effects on phasing weights in section 3. Quantization problem in phasing networks is significantly addressed in the literature, for example [3]-[7]. Most of the analysis is limited to arrays where steering weights are quantized. In other words beam patterns are calculated using infinite precision and quantization error is modeled after the beamforming network. We discuss effects of quantization in such situation in section 4. Effects of precision levels on the beam patterns and sidelobes are described in section 4.1 and 4.2 respectively. In section 5, quantization effects are discussed when quantization is introduced before beamforming. This is recent issue in digital beamforming where phasing information can be used to calculate the beam pattern in applications such as FPGA. In the last section conclusions are drawn.

2 HF Radar Antenna Geometry
The phasing of signals extends the transmission or detection range of a transmitter or receiver by forming a narrower beam than the individual antennas. Phased array antennas are used to steer a narrow beam over an arc from a fixed antenna array. For each transmitter or receiver the direction is adjusted with systematic phase delays to each of the antennas in the array. In the linear phased array antenna the antenna elements are arranged with uniform spacing, as shown in Figure 1, where \( d \) is inter-element spacing, and \( \theta \) is angle normal to modulated wave. It is obvious from the figure that in the general case, the modulated wave at element \( N - 1 \) will be delayed by a differential distance of \( d \sin \theta \). If we consider the phase of the transmitted signal is zero at the origin, then the phase lead at element \( N \) to that element at origin is \( Nkd \sin \theta \). The operational frequency of the signal varies with the wavelength using the relationship \( f = \frac{c}{\lambda} \), where \( c \) is the velocity of light in the vacuum. The number of array elements and space between them determine the beamwidth and size of sidelobes [3].
3 Periodic Phase Quantization

One source of periodic phase quantization is uniform array structure. Uniform distances between the array elements produce periodic phase quantization when they are steered to certain directions. The periodic quantization error is also caused by factors other than the regular geometry. For example continuous wave transmission and far field operation. In TIGER phased array system, pulsed transmission is employed therefore the quantization error would be reduced because overlapping pulses are reduced compared with continuous transmission [3].

In digital beamforming, weighting coefficients are quantized using a suitable quantizer. For a \( q \)-bit quantizer, minimum step size of phase shifter is \( \pi / 2^q \). In other words the quantization error is distributed between \( -\pi / 2^q \) to \( \pi / 2^q \). Variance of phase quantization error can be written as assuming \( C = \pi / 2^q \) for interval of \([-C C]\) [6]

\[
\sigma_q^2 = \frac{C^2 x^2}{2c} \text{d}x = \frac{C^2}{3}
\]

(1)

Substituting the values of \( C \) again we get

\[
\sigma_q^2 = \frac{\pi^2}{3 \cdot 2^2 q}
\]

(2)

The above expression shows that variance decreases with increase in precision level of quantizer.

Digital phase shifters are employed in digital implementation of phased array beamforming. The size of the phase shifter plays important role in beam accuracy and is dependent on the number of precision levels used in the phase increments [5]. For reduced number of bits, the digital phase shifter coarsely approximate the analog phase shifter as shown in the Figure 2. For higher number of bits, the approximation is close to the ideal phase shifting.

The periodic quantization error depends on step size of the quantizer. For two bit quantizer, the step size is 45 degrees. It is clear from Figure 2 that the quantization error is plus or minus half of the quantization step.

Figure 2 Comparison of ideal and digital phase shifter for two bits (b) periodic quantization error when the phase shift in each array element is quantized by the two bit quantizer.
minimize the quantization effects since higher sampling rate causes less delay errors.

4 Effects of Quantization After Beamforming Network

The quantization of infinite precision samples into fixed word length degrades the phased signals. As described in the previous section, the use of more levels for higher precision decreases the quantization error at the expense of larger hardware resources. For a reduced precision level, quantization error is spread to the main beams and to the grating lobes as well. In this section we study effects of quantization on beam resolution and associated grating lobes.

Quantization causes different beam patterns when it is introduced before beamforming network and after beamforming network. We first look at most common form, when beamforming output is rounded to closest integers, as shown in Figure 3.

![Figure 3 Quantization effects after beamforming network](image)

In this case beamforming is based on infinite precision samples and quantized into suitable quantization levels. Beam steering coefficients are stored in FPGA to realize reconfigurable beam direction and resolution.

In order to get deep understanding we develop mathematical models for array patterns and grating lobes. We can define array factor of a uniform array pointing to a direction $\theta_0$ [7]

$$\text{Array Factor} = \frac{\sin \left( N \left( \frac{2\pi}{\lambda} d \left( \sin \theta - \sin \theta_0 \right) \right) \right)}{N \sin \left( \frac{2\pi}{\lambda} d \left( \sin \theta - \sin \theta_0 \right) \right)}$$

(3)

where $N$ is number of array elements, $d$ is spacing between two adjacent elements, $\lambda$ is wavelength, $\theta$ is phase change in the array and $\theta_0$ is pointing angle. If the nulls of the array occur at directions

$$\text{nulls} = \frac{n\lambda}{2Nd}$$

(4)

where $n$ is an integer. Putting equation (4) into (3)

$$\text{Array Factor} = \frac{\sin \left( N \left( \frac{2\pi}{\lambda} d \left( \sin \theta - \sin \theta_0 \right) \right) \right)}{N \sin \left( \frac{2\pi}{\lambda} d \left( \sin \theta - \sin \theta_0 + \frac{n\lambda}{2Nd} \right) \right)}$$

(5)

Equation (5) can be used to represent beam pattern of a uniform array including grating lobes. The largest grating lobe occur when $n = 1$.

Now we discuss array factor in case of quantization. If the phase progression in the phased array is equal to the least quantized bit of the quantizer then

$$\text{phase progression} = \frac{\pi dN}{\lambda} \left( \sin \theta - \sin \theta_0 \right) = \frac{2\pi}{2^q}$$

(6)

putting into expression (5) we have

$$\text{Array Factor} = \frac{\sin \left( \frac{\pi}{2^q} \right)}{N \sin \left( \frac{\pi}{N} + \frac{n\pi}{N} \right)}$$

(7)

which can be written as

$$\text{Array Factor} = \frac{\sin \left( \frac{\pi}{2^q} \right)}{N \sin \left( \frac{\pi}{N} + \frac{n\pi}{N} \right)} = \frac{\sin \left( \frac{\pi}{2^q} \right)}{N \sin \left( \frac{n\pi}{N} \right)}$$

(8)

We calculate power by approximating numerator angle

$$\text{Power} = \left[ \frac{\left( \frac{\pi}{2^q} \right)}{N \sin \left( \frac{n\pi}{N} \right)} \right]^2$$

(9)

Equation (9) represents approximate power of quantized phased array. This can also be used to approximate grating lobe levels resulting from quantization.

4.1 Quantization Effects on Beam Pattern

Quantized phased signals degrade main beam resolution and sidelobes as well. However non-linearity arises in the sidelobes since quantizer is not adequate to represent resolution of sidelobe levels.

In order to investigate the quantization effects, an example is presented with fixed word length phasing samples. The coefficients of the phasing vector are quantized into six and seven bits; the increased number of bits will reduce the quantization effect. For an actual design the fixed bit width depends on available hardware resources. The quantized beam in Figure 4 shows that the quantizer does not adequately represent the beam pattern and thus introduces quantization noise. The seven bit results will also introduce quantization error, but at a lower level as shown in Figure 4(b).
As can be seen from this simple example, six bit compromises the first and second sidelobes at the -20dB level, while seven bits provides a reasonably faithful reconstruction of the theoretic sidelobe at this level. For a DBF system of more than ten bits, the sidelobe level will be essentially unaffected by the quantization at the -20dB level.

Figure 4 Quantization effects on beam pattern when samples are rounded to (a) six bit precision and (b) seven bits.

4.2 Sensitivity of Sidelobe Levels to Quantization

The quantization causes gain errors in sidelobe levels. Higher resolution in quantization introduces lower quantization error. The graph in Figure 4 shows that the six bit samples introduce quantization error which reduces first sidelobe gain while producing a gain error in the second sidelobe. The quantization error changes the dynamic range of the grating lobes and degrades the adjacent beam resolution for multiple beam systems.

A simulated graph is displayed in Figure 5 to demonstrate non-linear behavior of the quantizer in the sidelobe resolution. For lower order quantizer, the quantization step is not perfectly matched with the sidelobe levels. In the first sidelobe, the quantized resolution is less than the infinite precision and it approaches to floating point value with increased number of quantized levels. Figure 5(a) shows that for four bit quantizer, the first sidelobe resolution is at 12dB and at ten bits it approaches to infinite precision of 13.5dB. Unlike the first sidelobe, the second sidelobe exhibits higher resolution error at lower precision level since the quantizer can not represent the dynamic range adequately. The quantization error reduces with an increase in the number of bits.

Figure 5 Quantization effects on sidelobe levels, (a) first sidelobe (b) second sidelobe.

We can use expressions given above to approximate the grating lobe level. From equation (9), assuming denominator has dominating role to sample at grating lobe peak. For example if we use six bit precision for array of sixteen elements then the first sidelobe level is

\[
\text{Power(GL)} = 10 \log \left[ N \sin \left( \frac{n \pi}{N} \right) \right]^2
\] (10)
\[ \text{Power}(GL) = 10.02 \text{dB} \]

and similarly for second sidelobe we can calculate as

\[ \text{Power}(GL) = 15.80 \text{dB} \]

Comparing with the graphs in Figure 5, second sidelobe is closely matched with the calculations and the first sidelobe is deviated by two dB.

5 Effects of Quantization Before Beamforming

For FPGA implementation of DBF, all weights of the DBF are stored in registers. Therefore we look at the issue when DBF is performed using such weights. Difference with the earlier issue is that beam generation and steering are based on fixed numbers. This problem has not been addressed in the literature partly because technology was not advanced enough to handle large amount of calculation in the digital domain.

A block diagram is shown in Figure 6 to realize the concept of digitizing the phase samples. In this case beamforming calculation is based on rounded integers. As mentioned in the section on periodic quantization, the periodic quantization is now included in the phase calculation. Therefore the digitization error causes higher degradation in the beam resolution compared to the previous case.

![Figure 6 Quantization effects before beamforming network](image)

Simulations have been performed to demonstrate the quantization effects on the beam pattern. An example is shown in Figure 7 where phasing weights are rounded to six and seven bits. A comparison of Figure 7 with Figure 4 shows that for identical precision of the quantizer, beam pattern exhibit higher periodic error in the latter case. The periodic error is reduced with increase in the precision level.

The output beam resolution is adversely affected when the phasing coefficients are quantized before beamforming network. In order to circumvent these components, precision level of the quantizer should be at least greater than ten bits.

![Figure 7 Effect of all weights quantization when samples are quantized to (a) six bits and (b) seven bits.](image)

6 Conclusions

In this paper, effects of fixed word length have been discussed on phasing of the digital TIGER system. Finite precision is described in two scenarios. In first case, conventional approach of quantized weights is addressed after beamforming. Secondly a new approach of quantization before beamforming is adopted. The second scenario exhibit sub optimum results since quantization error accumulates in beamforming calculations. Quantization effects have been reported for beam patterns and for the sidelobe resolution. The quantization error is higher for lower precision level. In order to overcome these effects, precision level of more than ten bits is required.
7 References


