Complete Recovery of Synchrotron Wave Fields using Phase-Space Tomography and its Applications in X-ray Imaging

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***Abstract* — We discuss the role of coherence in x-ray imaging and consider how phase-space tomography can be used to extract information about partial coherence. The application of phase-space tomography to x-ray imaging will also be discussed.**

I. INTRODUCTION

Complete statistical information about a wavefield is included in its phase-space description via the phase-space density (PSD) or Wigner distribution function. A measurement of the phase space density will therefore permit maximum possible information extraction from the object.

II. COMPLETE RECOVERY OF SYNCHROTRON WAVE FIELDS

The PSD function, $B(\vec{r},\vec{u})$, of a optical beam describes the beam distribution over space, $\vec{r}$, and momentum, $\vec{u}$, parameters. The phase-space tomography method recognizes that an intensity measurement in a plane can be interpreted as a projection operation over the phase-space distribution of the wavefield

$$I(\vec{r}, z = z_0) \quad = \quad \int B(\vec{r}, \vec{u}, z = z_0) d\vec{u} \qquad (1)$$

The PSD can then be recovered using methods analogous to those used in tomographic imaging [1],[2].

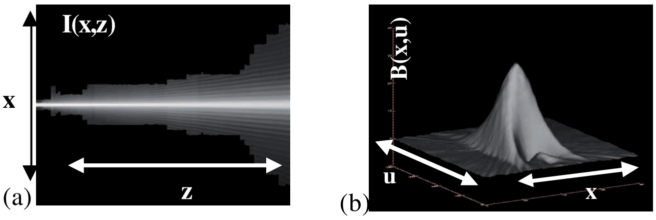

Fig 1: (a) The measured diffracted intensity I(x,z) of a 1.5 keV x-ray beam defined by a single-slit aperture of 25 μm width (logarithmic scale). (b) The PSD function B(x,u) reconstructed from the measured intensity distribution I(x,z) in (a).

The experiment was performed at the 2-ID-B undulator beamline at the Advanced Photon Source. A quasi-monochromatic 1.5 keV x-ray beam was detected using a (5±1) μm wide vertical slit placed in front of an Avalanche Photo Diode (APD) detector. Figure 1(a) shows the resulting intensity I(x,z) on a logarithmic scale. Figure 1(b) shows the PSD function B(x,u) reconstructed from the measured intensity distribution. The result is the first experimental characterisation of an x-ray beam in form of a PSD[3]

III. APPLICATION TO X-RAY IMAGING

Methods for recovering partially coherent fields could be used to recover the information in all the individual coherent data sets. We assume that the object can be described using a two-dimensional complex amplitude transmission function T(r). Define the phase-space transmission function as

$$G(\boldsymbol{r},\boldsymbol{u}) \equiv \left(\frac{k}{2\pi}\right)^2 \int T(\boldsymbol{r}+\boldsymbol{x}/2) T^*(\boldsymbol{r}-\boldsymbol{x}/2) e^{-ik\boldsymbol{u}\bullet\boldsymbol{x}} d\boldsymbol{x} \qquad (2)$$

So that the PSD function immediately behind the object is

$$\mathrm{B}_{out}(r,u) = \int \mathrm{B}_{in}(r,u-u')\,\mathrm{G}(r,u')\,\mathrm{d}u' \qquad (3)$$

Thus, given a knowledge of the PSD function of the incident field, the field produced by diffraction through the aperture can be calculated using Eq(3).

A measured Young's interference fringe and its prediction are shown in Fig. 2(a). The prediction was calculated by combining the phase-space transmission function for the Young's mask with the measured beam coherence function. The equivalent diffraction pattern from a coherent beam was then predicted and the result is shown in Fig. 2(b). It can be seen that the visibility of the fringes is increased to that for a coherent experiment.

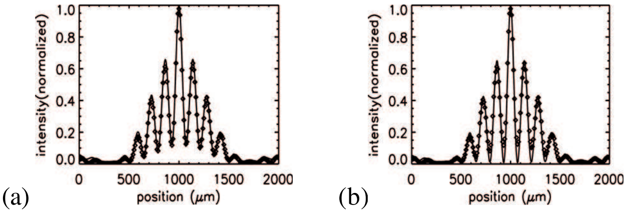

Fig 2: (a) Overlay of the measured Young's fringes (diamonds) with the prediction (solid line) using the known properties of the aperture and the measured coherence function. (b) Predicted fully coherent fringes (line) and the partially coherent experimental data.[4].

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